Chapter 2

Lecture 9

Longitudinal stick–fixed static stability and control – 6

Topics

Example 2.4

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Reference 2.4 describes the stability and control data for ten airplanes. This includes a general aviation airplane called "Navion". It seems an appropriate case to illustrate the static stability, dynamic stability and response of an airplane without the complications of compressibility effects. This airplane is dealt with in this chapter and also in chapters 8,9 and 10. The three-view drawing of the airplane is shown in Fig.2.29. The geometrical and aerodynamic data and the flight condition are given below. Some additional data given in Ref.1.1, chapter 2 are also included therein. Remaining data are deduced by measuring dimensions from the three-view drawing. Though references 1.1 and 2.4 use FPS units, data are converted to SI units for the sake of uniformity.

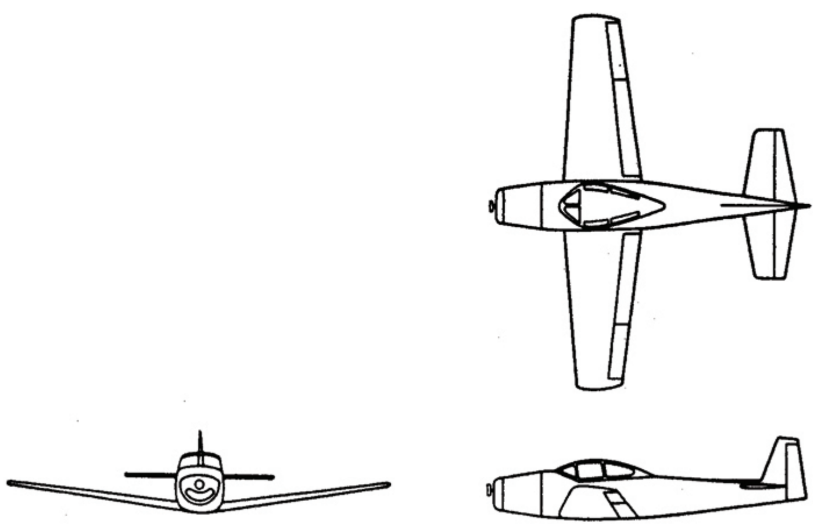

Fig.2.29 Three views of a general aviation airplane

(Adapted from Ref.2.4, section 10)

Wing:

Area (S) = 17.09 m^2, Span (b) = 10.18 m,

Root chord (c_r) = 2.16 m, Tip chord (c_t) = 1.21 m,

Taper ratio (λ) = 0.56, Aspect ratio (A_w) = 6.06,

Mean aerodynamic chord ($\bar{c}$) = 1.737 m,

$i_w = 1^0$ (Ref.1.1, chapter 2):

Characteristics of airfoil used on wing (deduced from Ref.1.1, chapter 2):

C_{mac} = -0.116, $C_{l\alpha w}$ = 0.097 deg^{-1} = 5.56 rad^{-1}, α_{olw} = - 6^0,

a.c. location = 0.25$\bar{c}$.

Fuselage:

Length (l_f) = 8.23 m,

Width of fuselage at maximum cross section = 1.4 m,

Height of fuselage at maximum cross section = 1.6 m,

The widths at different locations along the length of fuselage are shown in Tables E 2.4.1 & E 2.4.2.

Horizontal tail:

Area (S_t) = 4.73 m^2, Span (b_t) = 4.01 m,

Root chord (c_{rt}) = 1.54 m, Tip chord (c_{rt}) = 0.82 m.

Aspect ratio of tail (A_t)= 3.4,

Distance between quarter chord of the mean aerodynamic chords of wing and tail = 4.63 m, Distance l_h as shown in Fig.2.22 is 3.17 m.

Characteristics of the airfoil used on tail (deduced from Ref.1.1, chapter 2):

C_{mac} = 0, $C_{l\alpha t}$ = 0.1 deg^{-1}= 5.73 rad^{-1}, i_t = -1^0.

Flight condition:

Weight = 12232.6 N.

Altitude : sea level, ρ = 1.225 kg/m^3, speed of sound = 340.29 m/s.

flight velocity = 53.64 m/s ; Mach no. (M) = 0.158.

$$\text{Lift coefficient} = C_L = \frac{W}{\frac{1}{2}\rho V^2 S} = \frac{12232.6}{0.5\times1.225\times53.64^2\times17.09} = 0.406;$$

Ref.2.4 gives :

C_L = 0.41 , c.g. location: 0.295$\bar{c}$.

$(C_{L\alpha})_{airplane} = 4.44$, $(C_{m\alpha})_{airplane} = -0.683$.

Obtain : (i) Contributions of wing, horizontal tail, fuselage and power plant to the moment about c.g. (ii) $C_{m\alpha}$ of the airplane (iii) location of the neutral point and (iv) static margin.

Solution:

i) Slopes of lift curve for wing, tail and airplane

From Ref.1.8b, the slope of lift curve for unswept wing at low subsonic Mach number is given by :

$$C_{L\alpha} = \frac{2\pi A}{2+\sqrt{\frac{A^2}{K^2}+4}}; K = \frac{C_{l\alpha}}{2\pi}$$

where, $C_{l\alpha}$ is the lift curve slope of the airfoil.

For wing:

$$C_{L\alpha w} = \frac{2\pi\times 6.06}{2+\sqrt{\frac{6.06^2}{(5.56/6.28)^2}+4}} = 4.17\,rad^{-1}$$

For horizontal tail:

$$C_{L\alpha t} = \frac{2\pi\times 3.4}{2+\sqrt{\frac{3.4^2}{(5.73/6.28)^2}+4}} = 3.43\,rad^{-1}$$

The slope of lift curve of the airplane ($C_{L\alpha}$) is obtained using Eq.(2.60b):

$$C_{L\alpha} = C_{L\alpha w} + \eta\frac{S_t}{S}\, C_{L\alpha t}(1-\frac{d\varepsilon}{d\alpha})$$

$\eta = 0.9$ is assumed.

$d\varepsilon/d\alpha$ is estimated by the approximate method i.e.

$$\frac{d\varepsilon}{d\alpha} = \frac{2C_{L\alpha w}}{\pi A_w} = \frac{2\times 4.17}{3.14\times 6.06} = 0.438$$

Hence,

$$C_{L\alpha} = 4.17+0.9\times\frac{4.73}{17.09}\times 3.43(1-0.438) = 4.65\,rad^{-1}$$

This estimated value of $C_{L\alpha}$ is only 4.7%higher than the actual values of 4.44 rad^{-1} given in Ref.2.4. Thus, the values of $C_{L\alpha w}$, $C_{L\alpha t}$, $d\varepsilon/d\alpha$ and η are considered to be reasonably accurate (see also Appendix C).

II) Wing contribution:

Following the simplified approach, the wing contributions to C_{m0} and $C_{m\alpha}$ are obtained from Eqs.(2.19) and (2.20):

$$C_{m0w} = C_{macw} + C_{L0w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)$$

$$C_{m\alpha w} = C_{L\alpha w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)$$

$$C_{Low} = C_{L\alpha w}(i_w - \alpha_{0Lw}) = 4.17\,\frac{\{1-(-6)\}}{57.3} = 0.51$$

$$C_{mow} = C_{macw} + C_{Low}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) = -\,0.116 + 0.51(0.295 - 0.25) = -0.093$$

$$C_{m\alpha w} = C_{L\alpha w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) = 4.17\,(0.295-0.25) = 0.1887 \;\; rad^{-1}$$

Remark:

For this particular airplane and for the given configuration, the wing contribution to $C_{m\alpha}$ is positive or destabilizing (Note: c.g. is behind a.c.).

III) Horizontal tail contribution:

The tail contributions to C_{m0} and $C_{m\alpha}$ are obtained from the following equations:

$$C_{m0t} = \eta\; V_H\; C_{L\alpha t}\;(i_t - \varepsilon_0)$$

$$(C_{m\alpha t})_{stick\text{-}fixed} = -\eta\, V_H\, C_{L\alpha t}\;(1 - \frac{d\varepsilon}{d\alpha})$$

The tail volume ratio is given by:

$$V_H = \frac{S_t}{S}\,\frac{l_t}{\bar{c}} = \frac{4.73}{17.09} \times \frac{4.63}{1.737} = 0.738$$

As estimated earlier : $d\varepsilon/d\alpha = 0.438$

$$\varepsilon_0 = \frac{d\varepsilon}{d\alpha}(i_w - \alpha_{0lw}) = 0.438\{1-(-6)\} = 3.07^0$$

$$C_{m\alpha t} = -V_H\,\eta\, C_{L\alpha t}(1 - \frac{d\varepsilon}{d\alpha}) = 0.738\times0.9\times3.43\times(1-0.438) = -1.28 \;\; rad^{-1}$$

$$C_{m0t} = -V_H\,\eta\, C_{Lt}\,(i_t - \varepsilon_0) = -0.738\times0.9\times3.43(\frac{-1-3.07}{57.3}) = 0.162$$

IV) Fuselage contribution:

The contributions of fuselage to C_{m0} and $C_{m\alpha}$ are obtained using the method explained in section 2.5.3 and 2.5.4. To obtain C_{mof} we divide the fuselage into nine equal divisions as shown in Fig. 2.30.

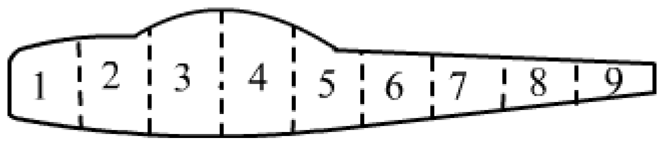

Fig.2.30 Subdivisions of fuselage for calculating C_{mof}

Table E 2.4.1 presents Δx and w_f at various stations along the fuselage. The quantity α_{0Lf} is $i_w + \alpha_{0Lw}$ which equals 1-6 = -5^0. As the fuselage has no camber i_f is taken as zero. Hence, $\alpha_{0Lf} + i_f$ equals -5^0 .The quantity $w_f^2(\alpha_{oLf} + i_f)\Delta x$ is given in the last column of the table E 2.4.1. The sum $\sum w_f^2(\alpha_{0Lf} + i_f)\Delta x$ is - 47.155.

Station	Δx (m)	w_f (m)	$\alpha_{0Lf} + i_f$	$w_f^2(\alpha_{0Lf} + i_f)\, \Delta x$
1	0.914634	1.097561	-5	-5.497689383
2	0.914634	1.402439	-5	-8.983337807
3	0.914634	1.402439	-5	-8.983337807
4	0.914634	1.402439	-5	-8.983337807
5	0.914634	1.25	-5	-7.141328478
6	0.914634	0.945122	-5	-4.08075913
7	0.914634	0.70122	-5	-2.2387498
8	0.914634	0.457317	-5	-0.963512572
9	0.914634	0.243902	-5	-0.283386051
i_f = 0 at every station				Sum= -47.15543884

Table E 2.4.1 Estimation of C_{m0f}

To obtain the term (k_2-k_1) from Fig.2.19, requires the fineness ratio of the fuselage which is obtained below.

The area of the maximum fuselage cross section (A_{fmax}) is :

$$A_{fmax} = 1.4 \times 1.6 = 2.24\ m^2$$

Hence, equivalent diameter (d_e) is:

$$d_e = \sqrt{A_{fmax}/(\pi/4)} = \sqrt{2.24/(\pi/4)} = 1.69\,m$$

Consequently, fineness ratio = l_f/d_e = 8.23/1.69 = 4.87.   From Fig.2.19, (k_2-k_1) corresponding to fineness ratio of 4.87 is 0.82. Substituting various values , C_{m0f} is given as:

$$C_{m0f} = \frac{k_2 - k_1}{36.5S\bar{c}}\sum_{x=0}^{l_f} w_f^2(\alpha_{0Lf} + i_f)\Delta x = \frac{0.82\times(-47.155)}{36.5\times17.07\times1.737} = -0.0357$$

To obtain $C_{m\alpha f}$ the fuselage is subdivided as shown in Fig. 2.31. The portion of the fuselage ahead of the root chord is divided into four equidistant portions each of length 0.4573 m. These subdivisions are denoted as 1, 2, 3 and 4. The portion of fuselage aft of the root chord is divided into five equidistant sections each of length 0.8841m and denoted as 5,6,7,8 and 9. The root chord (Fig.2.31) has length c = 1.98 m. Thus, the total fuselage length of 8.23 m is thus divided as: (0.4573 x 4 +1.98 + 0.8841x 5). The length l_h as shown in Fig.2.22 is the distance of the aerodynamic centre of horizontal tail behind the root chord of the wing. It is 3.17m. The calculations of the quantities needed to obtain $C_{m\alpha f}$ are shown in Table E2.4.2. The second column shows Δx which is the length of each subdivision of the fuselage. The third column gives the width of the fuselage in the middle of the subsection (see Fig.2.22). The fourth column gives the distance x for the section 4 as defined in Fig.2.22. For rows 3, 2 and 1 of this column the distance is x_i is as defined in Fig.2.22. For rows 5 to 9 of this column the distance x_i is as shown in Fig.2.22. The fifth column shows $x/\bar{c}$ for the fourth row and $x_i/\bar{c}$ for other rows. The sixth column is dε/dα – the upwash and downwash at the subdivision. For row four the upwash value is based on curve 'b' of Fig.2.23. For rows 3, 2 and 1 the upwash value is based on curve 'a' of Fig.2.23.

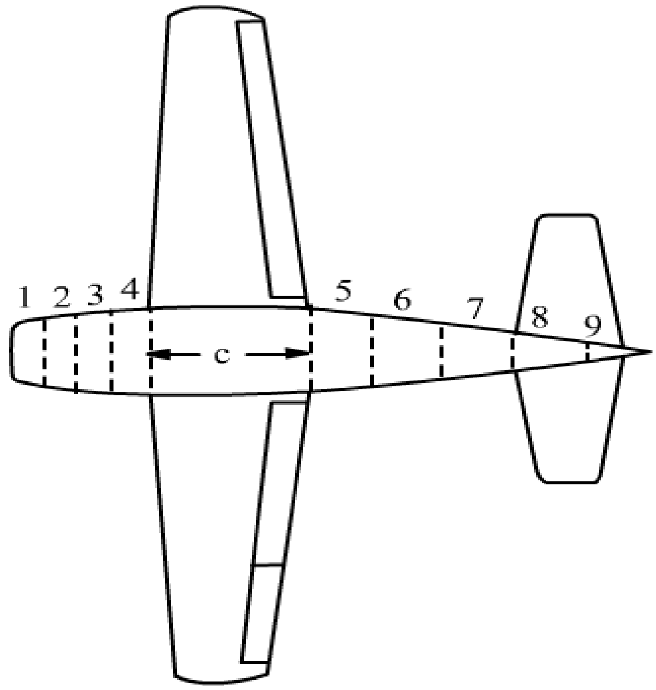

Fig.2.31 Subdivisions of fuselage for estimating $C_{m\alpha f}$

Station	Δx (m)	w_f (m)	x_i or x	(x_i or x)/c	dε/dα*	w_f^2(dε/dα)Δx
1	0.4573	0.914634	1.60061	0.808	1.20	0.4591
2	0.4573	1.036585	1.14329	0.577	1.34	0.6584
3	0.4573	1.158537	0.68598	0.346	1.56	0.9575
4	0.4573	1.280488	0.45731	0.231	3.20	2.4023
5	0.8841	1.158537	0.44207	0.223	0.078	0.0921
6	0.8841	0.945122	1.32622	0.667	0.2341	0.1843
7	0.8841	0.70122	2.21036	1.114	0.3910	0.1715
8	0.8841	0.457317	3.09451	1.561	0.548	0.1011
9	0.8841	0.243902	3.97865	2.008	0.7048	0.03632
c =1.98m		l_h =3.17m			Sum = 5.061	

*For $C_{L\alpha W}$ = 0.0785/deg^{-1}. See Remarks (ii) at the end of section 2.5.4

Table E2.4.2 Estimation of $C_{m\alpha f}$

The rows 5 to 9 of this column show the downwash for the corresponding subdivisions. As given in Fig.2.22 and by Eq.(2.58) , dε/dα at the subdivisions behind the root chord is given by:

$$\frac{d\varepsilon}{d\alpha} = \frac{x_i}{l_h}[1 - (\frac{d\varepsilon}{d\alpha})_{tail}]$$

We note that dε/dα at tail is 0.438 for this airplane. Using values of x_i and l_h the values of downwash are tabulated in column 6. The last column shows values of $w_f^2(d\varepsilon/d\alpha)\Delta x$. The sum , $\Sigma w_f^2(d\varepsilon/d\alpha)\Delta x$ is 5.061.

Since, $C_{L\alpha W}$ = 4.17/rad = 0.0728/degree, the actual value of the sum is (see Remark (ii) at the end of section 2.5.4):

5.061 x (0.0728/0.0785) = 4.694.

Finally,

$$C_{m\alpha f} = \frac{k_2 - k_1}{36.5S\bar{c}}\sum_{x=0}^{l_f} w_f^2 \frac{d\varepsilon}{d\alpha}\Delta x = \frac{0.82\times4.694\times57.3}{36.5\times16.56\times1.72} = 0.212 \text{ rad}^{-1}$$

V) Contribution of power plant:

It is difficult to estimate this contribution accurately. As mentioned in Remark (ii) in section 2.6.2, this contribution is taken as 0.04 $C_{L\alpha}$ = 0.04 x 4.65 = 0.186.

VI) C_{m0} and $C_{m\alpha}$

The contributions to C_{m0} and $C_{m\alpha}$ from the wing, the horizontal tail, the fuselage and the power plant are shown in Table E 2.4.3. The values of C_{m0} and $C_{m\alpha}$ for the entire airplane are the sums of the values for the components. These are also shown in Table E 2.4.3.

Item	C_{m0}	$C_{m\alpha}$
Wing	-0.093	0.1877
Fuselage	-0.0357	0.212
Power	-	0.186
H.tail	0.162	-1.28
Airplane	0.0333	-0.694

Table E 2.4.3 C_{m0} and $C_{m\alpha}$ due to components and for the entire airplane

Figure 2.27 shows the contributions graphically. C_{mcg} and $C_{m\alpha}$ for the airplane are:

$$C_{mcg} = -0.093 + 0.1877\alpha - 0.0357 + 0.212\alpha + 0.186\alpha + 0.1620 - 1.28\alpha$$
$$= 0.0333 - 0.694\,\alpha$$

Hence, $(C_{m\alpha})_{stick\text{-}fixed} = -0.694$

It is very interesting to note that the value of $C_{m\alpha}$ given in Ref.2.4 is -0.683. Thus the estimates of the contributions of various components to static stability can be considered to be reasonably accurate.

VII) Neutral point location:

The neutral point is given by:

$$\frac{x_{NP}}{\bar{c}} = \frac{x_{ac}}{\bar{c}} - \frac{1}{C_{L\alpha w}}\{(C_{m\alpha})_{f,n,p} - V_H\ \eta\ C_{L\alpha t}(1-\frac{d\varepsilon}{d\alpha})\}$$

Substituting various values, $\frac{x_{NP}}{\bar{c}}$ is given as:

$$\frac{x_{NP}}{\bar{c}} = 0.25 - \frac{1}{4.17}\{0.212 + 0.186 - 1.28\}$$
$$= 0.25 + 0.2115 = 0.4615$$

VIII) The static margin:

The static margin when c.g. is at $0.295\bar{c}$ is :

$$\frac{x_{NP}}{\bar{c}} - \frac{x_{cg}}{\bar{c}} = 0.4615 - 0.295 = 0.1665.$$

Hence, $(dC_m/\ dC_L) = -(\text{static margin}) = -0.1665$

$C_{m\alpha} = C_{L\alpha w}\,(dC_m / dC_L) = 4.17 \times (-0.1665) = -0.694$ as it should be.