Chapter 2

Lecture 7

Longitudinal stick–fixed static stability and control – 4

Topics

2.4.6 Revised expression for C_{mcgt}

Substituting for C_{Lt} in Eq.(2.37) gives:

$$C_{mcgt} = -V_H \eta\, C_{L\alpha t}[i_t - \varepsilon_0 + \alpha(1-\frac{d\varepsilon}{d\alpha}) + \tau\, \delta_e + \tau_{tab}\delta_{tab}]$$

$$= -C_{mot} - V_H\, \eta\, C_{L\alpha t}\,[\alpha(1-\frac{d\varepsilon}{d\alpha}) + \tau\, \delta_e + \tau_{tab}\, \delta_{tab}] \qquad (2.47)$$

$$\text{where, } \tau = \frac{\partial C_{Lt}}{\partial \delta_e} / \frac{\partial C_{Lt}}{\partial \alpha};\quad \tau_{tab} = \frac{\partial C_{Lt}}{\partial \delta_t} / \frac{\partial C_{Lt}}{\partial \alpha} \qquad (2.48)$$

$$\text{and } C_{mot} = -V_H \eta C_{L\alpha t}(i_t - \varepsilon_0) \qquad (2.49)$$

2.4.7 $C_{m\alpha t}$ in stick-fixed case

It may be pointed out that the pilot moves the elevator through the forward and backward movements of the control stick. Further, depending on the values of chosen flight speed and altitude, the pilot adjusts the positions of the elevator to make C_{mcg} equal to zero. In small airplanes, like the general aviation airplanes, the pilot continues to hold the stick and maintain the elevator deflection. In this background the analysis of the static stability of an airplane where the control deflection remains same even after disturbance is called static stability stick-fixed.

Hence, to obtain an expression for $C_{m\alpha t}$ in stick-fixed case it is assumed that the elevator deflection (δ_e) and the tab deflection (δ_t) remain unchanged after the disturbance. Accordingly when Eq.(2.47) is differentiated with respect to α ,the derivatives of δ_e and δ_t are zero i.e., in this case, $(d\delta_e /d\alpha) = (d\delta_t /d\alpha) = 0$ and the following result is obtained:

$$(C_{m\alpha t})_{\text{stick fixed}} = - V_H \eta \, C_{L\alpha t}(1-\frac{d\varepsilon}{d\alpha}) \qquad (2.50)$$

Remarks:

i) $C_{m\alpha t}$ is negative. To illustrate this, consider typical values as: $\eta = 0.9$, $V_H = 0.5$, $C_{L\alpha t} = 4.0$ per radian and $d\varepsilon/d\alpha = 0.4$. Then,

$C_{m\alpha t} = -0.5 \times 0.9 \times 4 \times (1-0.4) = -1.08$/radian.

ii) V_H depends on (S_t/S) and $(l_t/\bar{c})$. Hence, the contribution of tail to stability ($C_{m\alpha t}$) can be increased in magnitude by increasing (S_t/S) or $(l_t/\bar{c})$ i.e. by increasing the area of the horizontal tail or by shifting the tail backwards.

iii) C_{m0} is the value of C_{mcg} when α is zero. It (C_{m0}) is the sum of terms like C_{mow},C_{mot} etc. This value (C_{m0}) can be adjusted by changing C_{m0t}. In this context we observe from Eq.(2.49) that:

$$C_{m0t} = - V_H \eta C_{L\alpha t}(i_t - \varepsilon_0)$$

This suggests that by choosing a suitable value of i_t , the value of C_{mo} can be adjusted. This would permit trim, with zero elevator deflection, at a chosen value of lift coefficient (see Fig.2.6 and example 2.5). The chosen value of C_L for this purpose is invariably the value of C_L during cruise. This serves as criterion for selecting tail setting.

iv) In the beginning of this section a reason for examination of the stick-fixed stability was given by considering the case of general aviation airplane. However, the analysis of stick-fixed stability is carried out for all airplanes and the level of $(C_{m\alpha t})_{\text{stick-fixed}}$ decides the elevator deflection required in steady flight and in manoeuvres (see subsections 2.12.3 and 4.2).

2.5 Contributions of fuselage to C_{mcg} and $C_{m\alpha}$

Fuselage and nacelle are classified as bodies. The steps for estimating the contributions of a body to C_{mcg} and $C_{m\alpha}$ are based on the descriptions in chapter 2 , of Ref.1.1, chapter 5 of Ref.1.7 and chapter 3 of Ref.1.12. In this approach, the contribution of the body to $C_{m\alpha}$ is estimated based on the slender body theory and subsequently applying corrections for the effects of (a) finite fineness ratio, (b) non-circular cross section, (c) fuselage camber and (d) downwash due to wing.

2.5.1 Contribution of body to $C_{m\alpha}$ based on slender body theory

The potential flow past and an axisymmetric slender body was studied by Munk in 1924 (see Ref.1.1, chapter 2 for bibliographic details). He showed that a body at an angle of attack has a pressure distribution as shown in Fig.2.18 and produces no net force, but a moment. He showed that the rate of change of moment with angle of attack α, in radians, is given by:

$$\frac{dM}{d\alpha} = 2q \times \text{volume of body}\,;\, q = \frac{1}{2}\rho V^2$$

Alternatively when α is in degrees,

$$\frac{dM}{d\alpha} = \frac{\text{Volume of body} \times q}{28.7} \qquad (2.51)$$

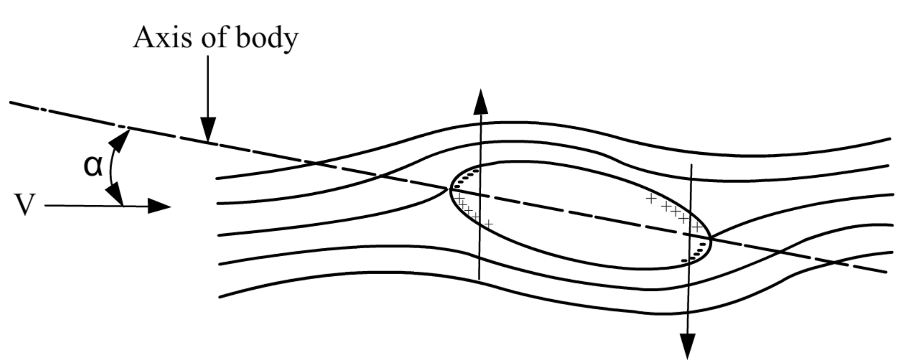

Fig.2.18 Streamlines and potential flow pressure distribution on an axisymmetric body; the negative and positive signs indicate respectively that the local pressure is lower or higher than the free stream pressure

Remark:

In a viscous flow, the pressure distribution about the body changes and it experiences lift and drag.

2.5.2 Correction to moment contribution of fuselage for fineness ratio

Generally the fuselage has a finite length. For such a fuselage Multhopp in 1942 (see Ref.1.1, chapter 2 for bibliographic details) suggested the following correction to Eq.(2.51).

$$\frac{dM}{d\alpha} = \frac{(k_2-k_1)}{28.7} q \times \text{volume of body} \qquad (2.52)$$

Where, (k_2-k_1) is a factor which depends on the fineness ratio (l_f/d_e) of the body; l_f is the length of the body and d_e is the equivalent diameter defined as:

$(\pi/4)d_e^2$ = max. cross sectional area of fuselage.

Figure 2.19 presents variation of (k_2-k_1) with fineness ratio

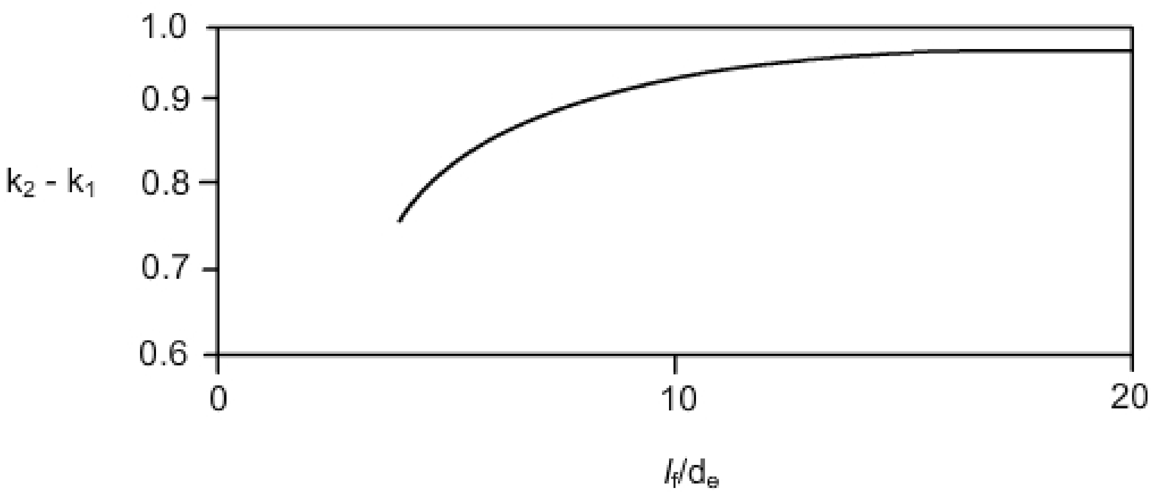

Fig.2.19 Correction to moment contribution of fuselage for fineness ratio
(Adapted from Ref.2.2, section 4.2.1.1)

2.5.3 Correction to moment contribution of fuselage for non-circular cross section

For a fuselage of non-circular cross-section, Eq.(2.52) is modified as:

$$\frac{dM}{d\alpha} = \frac{(k_2-k_1)q}{28.7}\int_0^{l_f}\frac{\pi}{4}w_f^2\,dx = \frac{(k_2-k_1)q}{36.5}\int_0^{l_f}w_f^2\,dx \qquad (2.53)$$

where, w_f is the local width of the fuselage.

Hence, the contribution of fuselage to $C_{m\alpha}$ can be expressed as :

$$C_{m\alpha f} = \frac{\frac{dM}{d\alpha}}{\frac{1}{2}\rho V^2 S\bar{c}} \qquad (2.54)$$

2.5.4 Correction to moment contribution due to fuselage for fuselage camber and downwash due to wing

In an airplane, the flow past a fuselage is affected by the upwash-downwash field of the wing (Fig.2.12). Further, the midpoints of the fuselage cross sections may not lie in a straight line. In such a case the fuselage is said to have a camber (Fig.2.20). A fuselage with camber would produce a pitching moment coefficient (C_{mof}) even when FRL is at zero angle of attack.

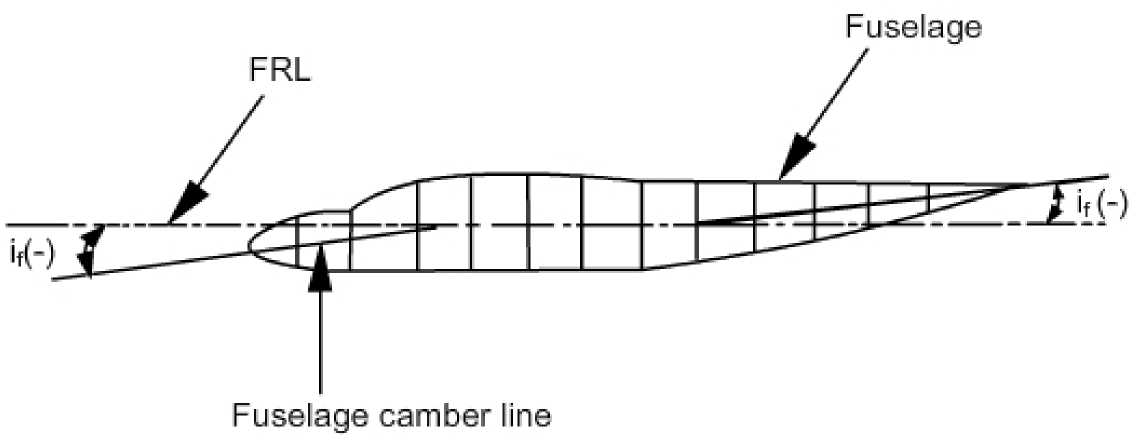

Fig.2.20 Fuselage with camber

For a fuselage with camber, C_{mcgf} is expressed as:

$$C_{mcgf} = C_{m0f} + C_{m\alpha f}\alpha \qquad (2.55)$$

with

$$C_{m0f} = \frac{k_2 - k_1}{36.5S\bar{c}} \sum_{x=0}^{l_f} w_f^2(\alpha_{0Lf} + i_f)\Delta x \text{ and} \qquad (2.56)$$

$$C_{m\alpha f} = \frac{k_2 - k_1}{36.5 S \bar{c}} \sum_{x=0}^{l_f} w_f^2 \frac{d\varepsilon}{d\alpha} \Delta x \qquad (2.57)$$

where, (a) w_f is the average width over a length Δx of fuselage (Fig.2.21) (b) i_f is the incidence angle of fuselage camber line with respect to FRL. It is taken negative when there is nosedrop or aft upsweep (Fig.2.20). (c) α_{0Lf} is the zero lift angle of wing relative to FRL i.e. $\alpha_{0Lf} = \alpha_{0Lw} + i_w$ and (d) dε/dα is the derivative with α of the local value of upwash / downwash along the fuselage.

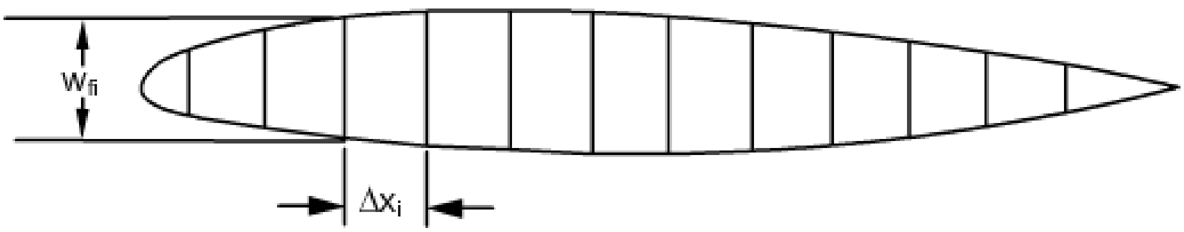

Fig.2.21 Division of fuselage for calculation of C_{m0f}

Though dε/dα along the fuselage can be calculated from an approach like the lifting line theory, the following emprical procedure is generally regarded adequate for evaluating $C_{m\alpha f}$.

a) The fuselage is divided into segments as shown in Fig.(2.22).

b) The local value of, dε/dα ahead of the wing is denoted by $d\varepsilon_u$ /dα . It is estimated from Fig.(2.23). For the segment immediately ahead of the wing (section 5 in Fig.2.22) the value of $d\varepsilon_u$ /dα varies rapidly and is estimated from the curve 'b' in Fig.2.23 (see example 2.4). For other segments ahead of wing, the curve 'a' in the same figure is used to estimate $d\varepsilon_u$ /dα.

(c) For the portion of the fuselage covered by the wing root (length 'c' indicated in Fig.2.22) dε/dα is taken as zero. Actually, the contribution of this portion is taken to be zero as, this portion is accounted for under the wing area.

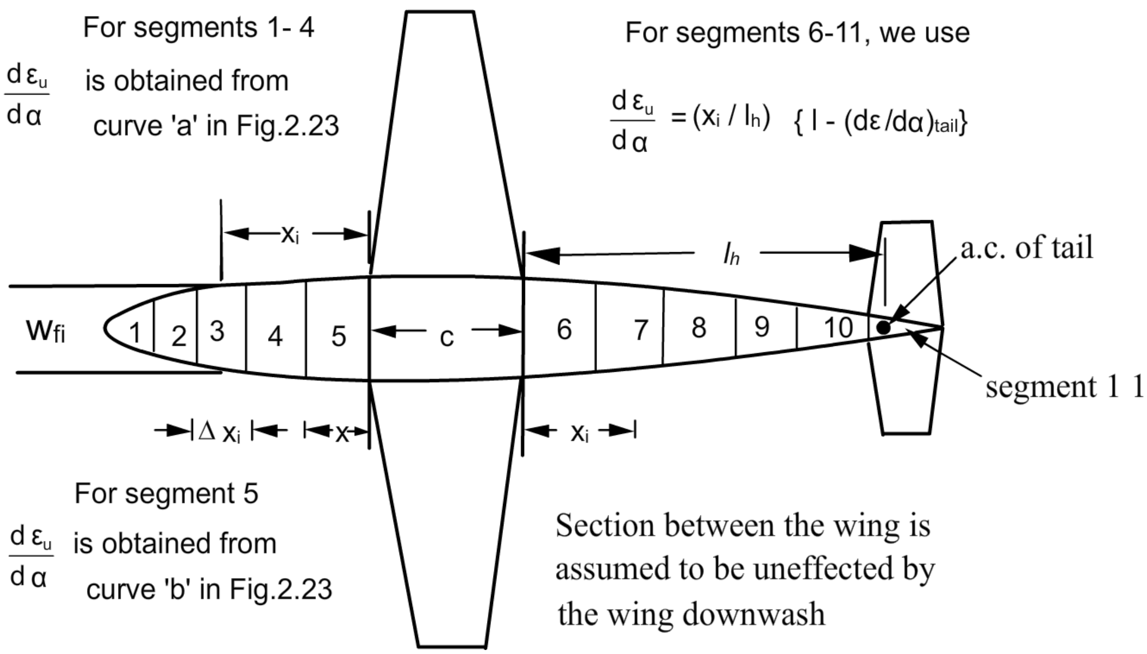

Fig.2.22 Division of fuselage for calculation of $C_{m\alpha f}$

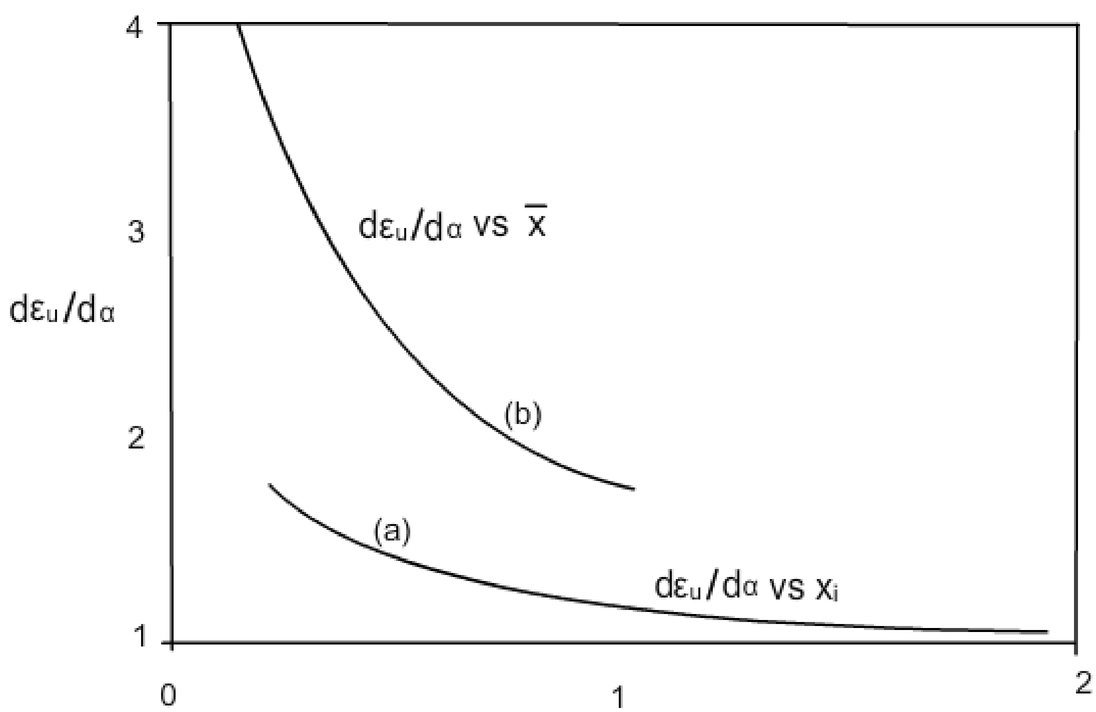

Fig.2.23 Upwash field ahead of wing

(Adapted from Ref.2.2, section 4.2.2.1)

(d) For the portion of the fuselage behind the wing (segments 6 to 11 in Fig.2.22), $d\varepsilon/d\alpha$ is assumed to vary linearly from 0 to $\{1- (d\varepsilon/d\alpha)_{tail}\}$ where $(d\varepsilon/d\alpha)_{tail}$ is the value of $(d\varepsilon/d\alpha)$ at a.c. of tail.

Hence, $$\frac{d\varepsilon}{d\alpha} = \frac{x_i}{l_t}\left[1-\left(\frac{d\varepsilon}{d\alpha}\right)_{tail}\right] \qquad (2.58)$$

The procedure is illustrated in example 2.4 for a low speed airplane and in Appendix 'C' for a jet airplane

Remarks:

i) In Ref.2.2, the quantity $d\varepsilon/d\alpha$ of Eq.(2.57) is written as $(1+ d\varepsilon/d\alpha)$ and values of $d\varepsilon/d\alpha$ therein are accordingly lower by one as compared to those in Fig. 2.23

ii) The values in Fig.2.23 are for a $C_{L\alpha WB}$ of 0.0785/deg. $C_{L\alpha WB}$ is the slope of lift curve of the wing-body combination which is roughly equal to $C_{L\alpha W}$ when the aspect ratio of the wing is greater than five. For other values of $C_{L\alpha W}$ multiply the values of $d\varepsilon/d\alpha$ by a factor of $(C_{L\alpha W} /0.0785)$. Note that $C_{L\alpha W}$ is in deg^{-1}. See also example 2.4.

2.5.5 Contribution of nacelle to $C_{m\alpha}$

The contribution of nacelle to $C_{m\alpha}$ can be calculated in a manner similar to that for the fuselage. Generally it is neglected.