Chapter 2

Lecture 5

Longitudinal stick–fixed static stability and control – 2

Topics

2.2 C_{mcg} and $C_{m\alpha}$ as sum of the contributions of various component

2.3 Contributions of wing to C_{mcg} and $C_{m\alpha}$

2.3.1 Correction to $C_{m\alpha w}$ for effects of horizontal components of lift and drag – secondary effect of wing location on static stability

Example 2.1

Example 2.2

Example 2.3

2.2 C_{mcg} and $C_{m\alpha}$ expressed as sum of the contributions of various components of the airplane

Using wind tunnel tests on a model of an airplane or by Computational Fluid Dynamics (CFD), the C_{mcg} vs α curve for the entire airplane can be obtained. However, CFD has not yet advanced enough to give accurate values of the moments and these computations are not inexpensive. Wind tunnel tests are very expensive and are resorted to only at the later stages of airplane design. Hence, the usual practice to obtain the C_{mcg} vs α curve is to add the contributions of major components of the airplane and at the same time take into account the interference effects. The contributions of individual components are based on the wind tunnel data or the analyses available in literature. References 1.1,1.8,1.9, 1.12, 2.1 and 2.2 are some of the sources of data.

The contributions to C_{mcg} and $C_{m\alpha}$ are due to the wing, the fuselage, the power plant and the horizontal tail. Figure 2.8 shows the forces and moments produced by the wing and the horizontal tail. The contributions of fuselage, nacelle and the power plant are shown as moments about c.g. and denoted by $M_{f,n,p}$. The fuselage reference line is denoted by FRL. It may be recalled that the angle of attack (α) of the airplane is the angle between free stream velocity (V) and FRL. The c.g. of the airplane is also shown in the figure. The wing is represented by its mean aerodynamic chord (m.a.c.). It is set at an angle of incidence i_w to the FRL. Hence, the angle of attack of wing (α_w) is $\alpha + i_w$. Following the usual practice, the lift of the wing (L_W) is placed at the aerodynamic centre of the wing (a.c.) along with a pitching moment (M_{acw}). The drag of the wing (D_w) is also taken to act at the aerodynamic centre of the wing. The wing a.c. is located at a distance x_{ac} from the leading edge of the m.a.c. The airplane c.g. is at a distance x_{cg} from the leading edge of the m.a.c.

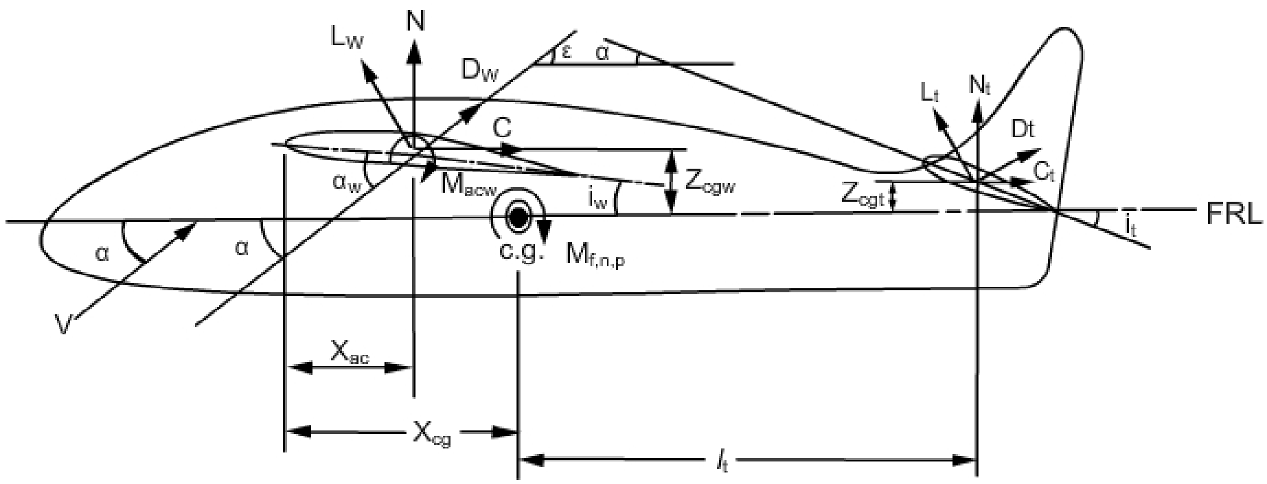

Fig.2.8 Contributions of major components to C_{mcg}

The horizontal tail is also represented by its mean aerodynamic chord. The aerodynamic centre of the tail is located at a distance l_t behind the c.g. The tail is mounted at an angle i_t with respect to the FRL. The lift, drag and pitching moment due to the tail are L_t, D_t and M_{act} respectively. As the air flows past the wing, it experiences a downwash ε which is shown schematically in Fig.2.8. Owing to this the angle of attack of the horizontal tail would be ($\alpha + i_t - \varepsilon$). Further, due to the interference effects the tail would experience a dynamic pressure different from the free stream dynamic pressure. These aspects will be elaborated in section 2.4.2 and 2.4.3. With this background the pitching moment about the c.g. can be expressed as:

$$M_{cg} = (M_{cg})_w + (M_{cg})_f + (M_{cg})_n + (M_{cg})_p + (M_{cg})_t \tag{2.11}$$

$$C_{mcg} = \frac{M_{cg}}{\frac{1}{2}\rho V^2 S\bar{c}} = (C_{mcg})_w + (C_{mcg})_{f,n,p} + (C_{mcg})_t \tag{2.12}$$

$$C_{m\alpha} = (C_{m\alpha})_w + (C_{m\alpha})_{f,n,p} + (C_{m\alpha})_t \tag{2.13}$$

Note:

(i) For convenience the derivative of C_{mcg} with α is denoted as $C_{m\alpha}$.

(ii) In Fig.2.8 the angle 'i_t' is shown positive for the sake of indicating the notation; generally 'i_t' is negative.

The contributions to C_{mcg} and $C_{m\alpha}$ of the individual components are described in the next four sections.

2.3 Contributions of wing to C_{mcg} and $C_{m\alpha}$

Figure 2.9 schematically shows the forces (lift, L_w and drag, D_w) and the moment (M_{acw}) due to the wing and the relative locations of the c.g. of the airplane and the aerodynamic centre of the wing.

The following may be recalled / noted.

i) The angle of attack of the airplane is the angle between the relative wind and the fuselage reference line (FRL). This angle is denoted by α.

ii) The wing is represented by its mean aerodynamic chord (m.a.c.).

iii) The wing is set at an angle i_w to the FRL. This is done so that the fuselage is horizontal during cruising flight. Thus, $\alpha_w = \alpha + i_w$ or $\alpha = \alpha_w - i_w$.

iv) x_{ac} is the distance of the a.c. from the leading edge of the m.a.c..

v) x_{cg} is the distance of the c.g. from the leading edge of the m.a.c..

vi) Z_{cgw} is the distance of the a.c. below c.g.

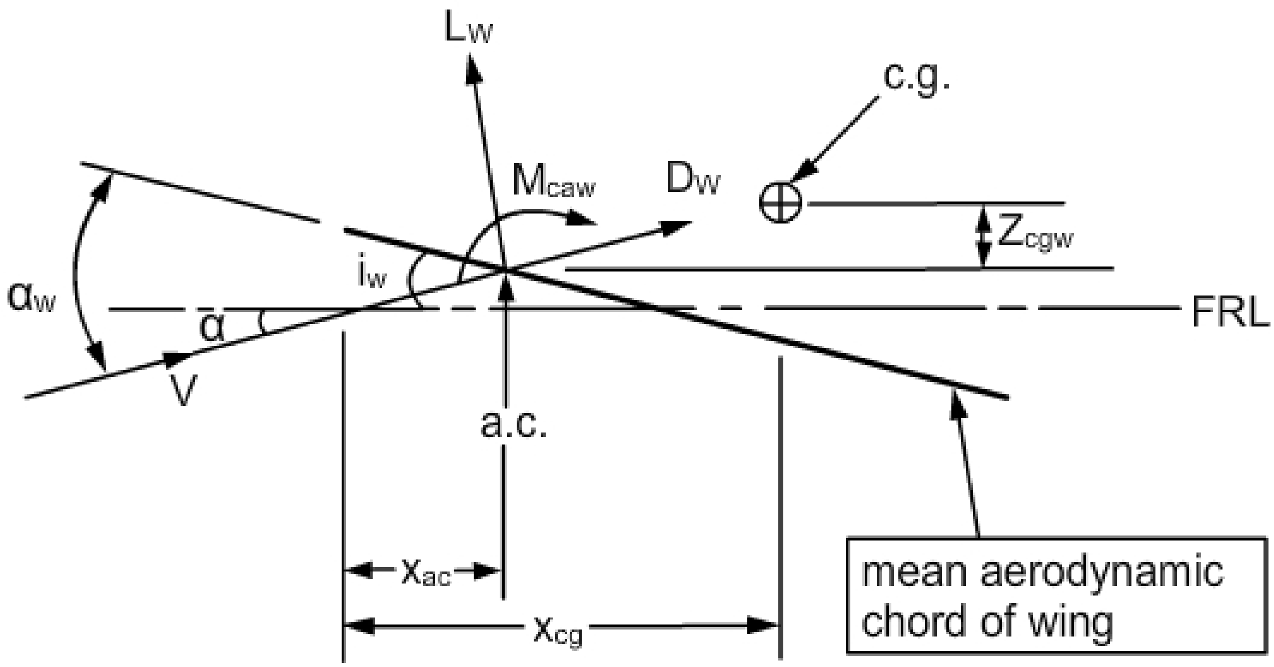

Fig.2.9 Wing contribution

Taking moment about c.g., gives the contribution of wing (M_{cgw}) to the moment about c.g as:

$$M_{cgw} = L_w\cos(\alpha_w - i_w)[x_{cg} - x_{ac}] + D_w\sin(\alpha_w - i_w)[x_{cg} - x_{ac}]$$
$$+ L_w\sin(\alpha_w - i_w)Z_{cgw} - D_w\cos(\alpha_w - i_w)Z_{cgw} + M_{acw} \tag{2.14}$$

Noting that,

$$C_{mcgw} = \frac{M_{cgw}}{\frac{1}{2}\rho V^2 S\bar{c}};\ C_{Lw} = \frac{L_w}{\frac{1}{2}\rho V^2 S};\ C_{Dw} = \frac{D_w}{\frac{1}{2}\rho V^2 S};\ C_{macw} = \frac{M_{acw}}{\frac{1}{2}\rho V^2 S\bar{c}}, \tag{2.15}$$

yields:

$$C_{mcgw} = C_{Lw}\cos(\alpha_w - i_w)[\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}] + C_{Dw}\sin(\alpha_w - i_w)[\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}]$$
$$+ C_{Lw}\sin(\alpha_w - i_w)\frac{Z_{cgw}}{\bar{c}} - C_{Dw}\cos(\alpha_w - i_w)\frac{Z_{cgw}}{\bar{c}} + C_{macw} \tag{2.16}$$

Remark:

$(\alpha_w - i_w)$ is generally less than 10^0.Hence, cos $(\alpha_w - i_w) \approx 1$; and $\sin(\alpha_w - i_w) \approx (\alpha_w - i_w)$. Further $C_L >> C_D$.

Neglecting the products of small quantities, Eq.(2.16) reduces to:

$$C_{mcgw} = C_{macw} + C_{Lw}[\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}] \tag{2.17}$$

Now,

$$C_{Lw} = C_{L\alpha w}(\alpha_w - \alpha_{0Lw})$$
$$= C_{L\alpha w}(\alpha + i_w - \alpha_{0Lw})$$
$$= C_{L\alpha w}(i_w - \alpha_{0Lw}) + C_{L\alpha w}\alpha$$
$$= C_{L0w} + C_{L\alpha w}\alpha \tag{2.18}$$

where, α_{0Lw} is the zero lift angle of the wing and

$$C_{L0w} = C_{L\alpha w}(i_w - \alpha_{0Lw})$$

Hence,

$$C_{mcgw} = C_{macw} + C_{L0w}[\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}] + C_{L\alpha w}\,\alpha\,[\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}] \tag{2.19}$$

Differentiating with respect to α , gives the contribution of wing to $C_{m\alpha}$ as :

$$C_{m\alpha w} = C_{L\alpha w}[\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}] \tag{2.20}$$

Remark:

The contribution of wing (C_{mcgw}) as approximately calculated above and given by Eq.(2.19) is linear with α. When the a.c. is ahead of c.g., the term $[\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}]$ is positive and consequently $C_{m\alpha w}$ is positive (Eq.2.20). Since, $C_{m\alpha}$ should be negative for static stability, a positive contribution to $C_{m\alpha}$ is called destabilizing contribution. When the a.c. is ahead of c.g. the wing contribution is destabilizing. Figure 2.10 shows C_{mcgw} vs α in this case.

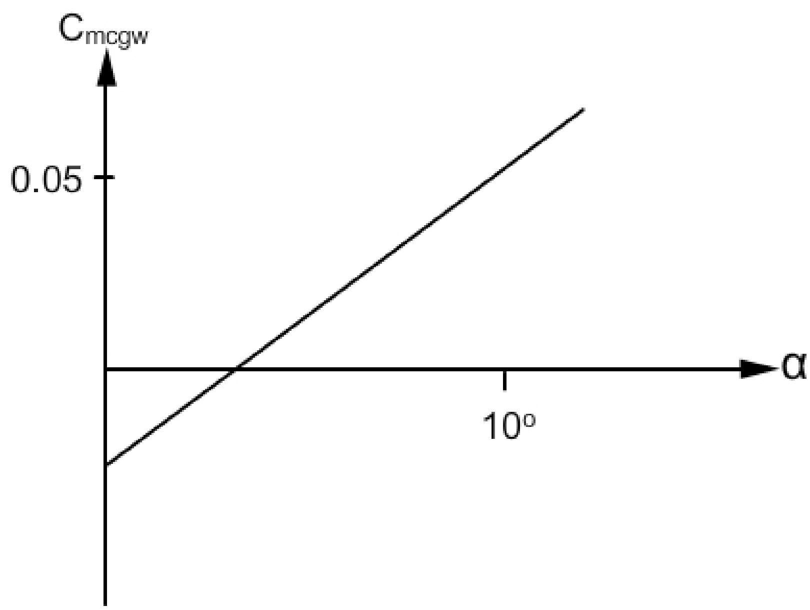

Fig.2.10 Approximate contribution of wing to C_{mcg}

2.3.1 Correction to $C_{m\alpha w}$ for effects of horizontal components of lift and drag – secondary effect of wing location on static stability

In the simplified analysis for the contribution of wing to C_{mcg} , the contributions of the horizontal components of lift and drag to the moment about c.g. , have been ignored (compare Eqs. 2.16 and 2.17).Let, the neglected terms be denoted by M_{cgwh}. Equation (2.14) gives the following expression for M_{mcgwh}

$$M_{cgwh} = L_w \sin(\alpha_w - i_w) Z_{cgw} - D_w \cos(\alpha_w - i_w) Z_{cgw} \quad (2.21)$$

Dividing by ½ $\rho V^2 S \bar{c}$ and noting that $\cos(\alpha_w - i_w) \approx 1$ yields :

$$C_{mcgwh} = [C_{Lw} \sin(\alpha_w - i_w) - C_{Dw}] \frac{Z_{cgw}}{\bar{c}}; \quad (2.22)$$

Differentiating Eq.(2.22) with α gives:

$$C_{m\alpha wh} = [\frac{dC_{Lw}}{d\alpha} \sin(\alpha_w - i_w) + C_{Lw} \cos(\alpha_w - i_w) - \frac{dC_{Dw}}{d\alpha}] \frac{Z_{cgw}}{\bar{c}} \quad (2.23)$$

Now , $\frac{dC_{Lw}}{d\alpha} \sin(\alpha_w - i_w) \approx C_{L\alpha w}(\alpha_w - i_w)$

$$C_{L\alpha w}(\alpha_w - i_w) = C_{L\alpha w}(\alpha_w - \alpha_{0L}) - C_{L\alpha w}(i_w - \alpha_{0L}) = C_{Lw} - C_{L0w}$$

Further, $C_{Lw}\cos(\alpha_w - i_w) \approx C_{Lw}$

and $$\frac{dC_{Dw}}{d\alpha} = \frac{dC_{Dw}}{dC_L}\frac{dC_L}{d\alpha} = C_{L\alpha w}\frac{dC_{Dw}}{dC_L} \qquad (2.24)$$

Thus, $$C_{m\alpha wh} = [2C_{Lw} - C_{L0w} - C_{L\alpha w}\frac{dC_{Dw}}{dC_L}]\frac{Z_{cgw}}{\bar{c}} \qquad (2.25)$$

The drag polar of the wing can be assumed as :

$$C_{Dw} = C_{D0w} + \frac{C_{Lw}^2}{\pi Ae},$$

Then, $$\frac{dC_{Dw}}{dC_L} = \frac{2C_{Lw}}{\pi Ae}$$

Substituting this in Eq.(2.25) yields:

$$C_{m\alpha wh} = [2C_{Lw} - C_{L0w} - C_{L\alpha w}\frac{2C_{Lw}}{\pi Ae}]\frac{Z_{cgw}}{\bar{c}}$$

$$C_{m\alpha wh} = [2C_{Lw}\{1 - \frac{C_{L\alpha w}}{\pi Ae}\} - C_{L0w}]\frac{Z_{cgw}}{\bar{c}} \qquad (2.26)$$

The term $[1 - (2C_{L\alpha w} / \pi Ae)]$ is generally positive. This can be seen as follows. An approximate expression for $C_{L\alpha w}$ is:

$$C_{L\alpha w} = 2\pi\frac{A}{A+2}\text{ ; A = Aspect ratio of wing.}$$

Hence,

$$\frac{C_{L\alpha w}}{\pi Ae} = 2\pi\frac{A}{A+2}\frac{1}{\pi Ae} = \frac{2}{(A+2)e} \qquad (2.27)$$

2/{(A+2)e} is less than 1 for typical values of A and e. Further, for low wing aircraft, where the a.c of the wing is below c.g., the term $Z_{cgw}/\bar{c}$ is positive (Fig.2.9) . Hence, $C_{m\alpha wh}$ as given by Eq.(2.26) is positive or destabilizing (Fig.2.11). For high wing aircraft, $Z_{cgw}/\bar{c}$ is negative consequently $C_{m\alpha wh}$ is negative and hence stabilizing (Fig.2.11).

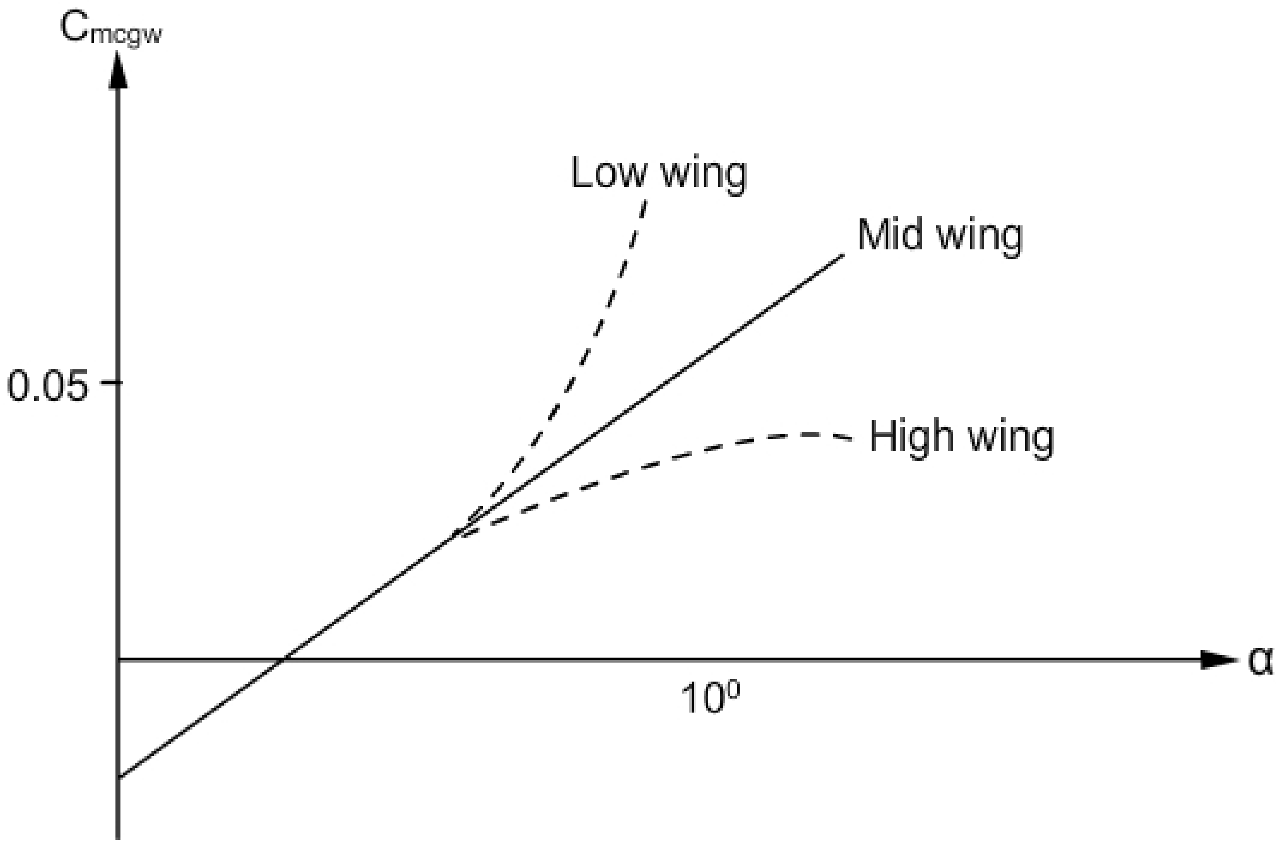

Fig.2.11 Effect of wing location on C_{mcgw}

An important aspect of the above derivation may be pointed out here. The expression for $C_{m\alpha wh}$ involves C_L or the slope of C_{mcgw} vs α curve depends on C_L or α (see example 2.3) . Hence, C_{mcgw} become slightly non-linear. The usual practice, is to ignore the contributions of the horizontal components to $C_{m\alpha w}$. However, the following aspects may be pointed out. (a) A high wing configuration is slightly more stable than a mid-wing configuration. A low wing configuration is slightly less stable than the mid-wing configuration. (b) In the simpler analysis the C_{mcgw} vs α curve is treated as straight line but the C_{mcg} vs α curves, obtained from flight tests on airplanes, are found to be slightly non-linear. One of the reasons for the non-linearity in actual curves is the term M_{egwh}.

Example 2.1

Given a rectangular wing of aspect ratio 6 and area 55.8 m^2. The wing section employed is an NACA 4412 airfoil with aerodynamic centre at 0.24 $\bar{c}$ and C_{mac} = -0.088.The c.g. of the wing lies on the wing chord, but 15 cm ahead of the a.c. Calculate the following.

(a) The lift coefficient for which the wing would be in equilibrium (C_{mcg} = 0). Is this lift coefficient useful? Is the equilibrium statically stable?

(b) Calculate the position of c.g. for equilibrium at C_L = 0.4. Is this equilibrium statically stable?

Solution:

The given data for the wing are : A = 6, S = 55.8m^2, Airfoil: NACA 4412; a.c. at 0.24 $\bar{c}$, C_{mac} = -0.088

Before solving the problem we workout the additional data needed for the solution.

($dC_l/d\alpha$) or $C_{l\alpha}$ or a_0 of the given airfoil: From Ref.1.7 p.484 a_0 is 0.106/degree

For a_0 = 0.106 and A = 6, from Fig.5.5 of Ref.1.7, $C_{L\alpha w}$ = 0.081/degree.

Note: Using $C_{L\alpha}$ = (A/A+2) $C_{l\alpha}$, we would get:

$C_{L\alpha}$ = {6/(6+2)}(0.106) = 0.0795 deg^{-1}

For a rectangular wing, $\bar{c}$ = S/b

Further A = b^2 / S.

Hence, b = $(AxS)^{1/2}$ = $(6x55.8)^{1/2}$ = 18.30 m

Consequently, $\bar{c}$ = 55.8/18.3 = 3.05 m.

Hence, x_{ac} = 0.24x3.05 = 0.732 m, x_{cg} = 0.732 - 0.15 = 0.582 m

The configuration is shown in Fig.E2.1

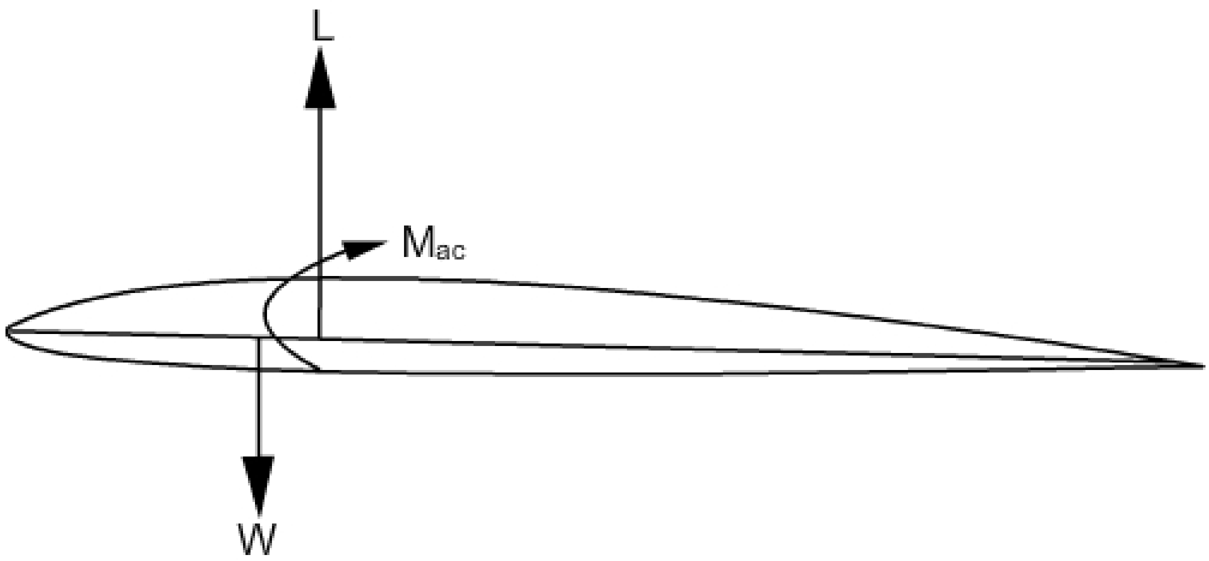

Fig.E2.1 Configuration for example 2.1

(a) For equilibrium

$L - W = 0$; $M_{cg} = - L \times 0.15 + \frac{1}{2} \rho V^2 S \bar{c}\ C_{mac} = 0$

Or $- C_L \times 0.15 + \bar{c}(- 0.088) = 0$

Hence, $C_L = - 0.088 \times 3.05/0.15 = - 1.77$

This lift coefficient is not useful.

The equilibrium is stable as c.g. is ahead of a.c.

(b) Calculation of c.g. location for moment equilibrium at $C_L = 0.4$

$$C_{mcg} = 0.4 \times (x_{cg} - x_{ac}) + \bar{c}\ (- 0.088) = 0$$

$$x_{cg} - x_{ac} = 3.05 + \frac{0.088}{0.4} = 0.671\text{m}$$

$$\text{or } \frac{x_{cg}}{\bar{c}} = \frac{x_{ac}}{\bar{c}} + \frac{0.671}{\bar{c}} = (0.24 + 0.22) = 0.46$$

This equilibrium is unstable as a.c. is ahead of c.g.

Example 2.2

If the wing given example 2.1 is rebuilt maintaining the same planform, but using reflex cambered airfoil section such that $C_{mac} = 0.02$, with the a.c. still at $0.24\bar{c}$. Calculate the c.g. position for equilibrium at $C_L = 0.4$. Is this equilibrium statically stable?

Solution:

For equilibrium at C_L = 0.4 with C_{mac} = 0.02;

$$C_{mcg} = 0.4\,(x_{cg} - x_{ac}) + \bar{c}\,(0.02) = 0$$

$$\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}} = -\frac{0.02}{0.4}3.05 = -0.1525\,\text{m}$$

$$\frac{x_{cg}}{\bar{c}} = 0.24 - \frac{0.1525}{3.05} = 0.19$$

Equilibrium is stable as c.g. is ahead of a.c.

Remark: From the above two examples we draw interesting conclusions about an airplane which has an all wing configuration. (a) For such a configuration, the static stability consideration requires that c.g. should be ahead of a.c.. (b) C_{mac} should be positive.

Example 2.3

An airplane is equipped with a wing of aspect ratio 6 ($C_{l\alpha w}$ = 0.095) and span efficiency factor e of 0.78, with an airfoil section giving C_{mac} = 0.02. Calculate, for C_L between 0 and 1.2, the pitching moment coefficient of the wing about the c.g. which is located $0.05\bar{c}$ ahead of a.c. and $0.06\bar{c}$ under a.c.. Repeat the calculations when chord wise force component is neglected. Assume C_{D0w} = 0.008, $\alpha_{oLw} = 1^0$, $i_w = 5^0$.

Solution:

The given data about the wing are: A = 6, $C_{l\alpha}$ = 0.095, e = 0.78, C_{mac} = 0.02, $\alpha_{oLw} = 1^0$, C_{D0w} = 0.008, $i_w = 5^0$,

From Fig.5.5 of Ref.1.7, $C_{L\alpha w}$ = 0.074 deg^{-1} = 4.24 rad^{-1}

C_{L0w} = 0.074 (5-1) = 0.296.

$$C_{Dw} = 0.008 + \frac{C_L^2}{\pi A\, e} = 0.008 + \frac{C_L^2}{3.14\times6\times0.78} = 0.008 + 0.068\ C_L^2$$

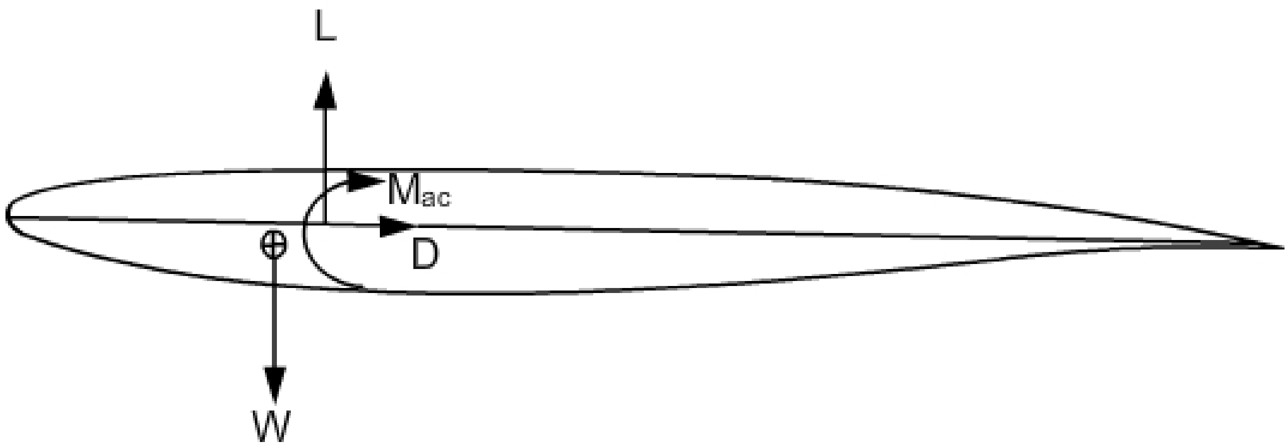

Fig.E2.3 Schematic of configuration for example 2.3

Combining Eqs.(2.20) and (2.26),

$$C_{m\alpha w} = \frac{(x_{cg} - x_{ac})}{\bar{c}} C_{L\alpha w} + [2C_{Lw}\{1 - \frac{C_{L\alpha w}}{\pi A e}\} - C_{L0w}]\frac{Z_{cgw}}{\bar{c}};$$

$$= -0.05 \times 4.24 + [2C_L\{1 - \frac{4.24}{3.14 \times 6 \times 0.78}\} - 0.296](-0.06)$$

$$= -0.212 - 0.0854\, C_L + 0.0178 = -0.1942 - 0.0854\, C_L$$

Hence, $C_{mcgw} = 0.02 + (-0.1942 - 0.0854\, C_L)\alpha$

$$= 0.02 + (-0.1942 - 0.0854\, C_L)\{(C_L - C_{Low}) / 4.24\}$$

$$= 0.0336 - 0.0399C_L - 0.0201\, C_L^2$$

The values of C_{mcgw} for different values of C_L are presented in table E2.3.

The approximate contribution of wing after neglecting the horizontal component from Eq.(2.17) is :

$$C_{mcgw} = C_{macw} + C_{Lw}[\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}]$$

or $(C_{mcgw})_{approximate} = 0.02 - 0.05C_L$. These values are also included in table E2.3.

C_L	(C_{mcgw}) without horizontal component	(C_{mcgw}) with horizontal component
0	0.02	0.0336
0.4	0	0.0141
0.8	-0.02	-0.0112
1.2	-0.04	-0.04314

Table E2.3 contribution of wing to C_{mg}

Remark:

The c.g. is ahead of a.c , hence the contribution of wing, even without considering horizontal component, is stabilizing. Further the c.g. is below a.c. hence the contribution, considering the horizontal component, becomes more stabilizing.