Appendix C

Drag polar, stability derivatives and characteristic roots of a jet airplane – 4

Lecture 40

Topics

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6.5 Estimation of C'_{lp}

From Eq.8.2 of Ref.2

$$C'_{lp} = C'_{lpWB} + C'_{lpH} + C'_{lpV}$$

$$C'_{lpWB} \approx C'_{lpW} = \beta\left(\frac{C'_{lp}}{\kappa}\right)\kappa/\beta$$

$\beta C'_{lp} / \kappa$ depends on $\beta A/\kappa$ and Λ_β

κ = (2-D slope of lift curve) / $2\pi \approx 1$ (assumed)

$\beta A / \kappa = 0.6 \times 6.46/1 = 3.876$

$\Lambda_\beta = \tan^{-1}(\tan \Lambda_{c/4} / \beta) = 53^0$

For $\lambda = 0.29$, $\beta C'_{lp} / \kappa = -0.262$ from Fig 8.1 of Ref.2,

$$C'_{lpW} = -0.262 / 0.6 = -0.436$$

$$C'_{lpH} = 0.5 (C'_{lp}/\kappa)_H \frac{S_H}{S} (\frac{b_H}{b})^2$$

For horizontal tail $\beta A / \kappa = (0.6 \times 3.64) / 1 = 2.184$

$(\Lambda_\beta)_H = \tan^{-1} (\tan 35.3 / 0.6) = 49.6^0$

For $\lambda = 0.266$: $(\beta C'_{lp} / \kappa)_H = -0.18$ from Fig 8.1 of Ref.2

$$(C'_{lp})_H = -0.5 \times \frac{0.18}{0.6} \times \frac{135.07}{550.5} (\frac{22.18}{59.64})^2 = -0.0050$$

$$C'_{lpv} = 2(Z_v / b)^2 C_{Y\beta v} = 2 \times (\frac{6.35}{59.64})^2 (-0.636) = -0.0145$$

Hence, $C'_{lp} = -0.436 - 0.005 - 0.0145 = -0.458$.

Based on area of 511m^2 this is - 0.494. The value from Ref.3 is – 0.34. The graphs of Ref.3 show that value of C'_{lp} does not vary much with Mach number, but the theoretical method shows a significant dependence on flight Mach number. If M = 0.0 then Fig. 8.1 of Ref.2 gives a value of $C'_{lpW} = -0.32$.

6.6 Estimation of C_{np}

From Eq.(8.6) of Ref.2

$$C_{np} = C_{npW} + C_{npV}$$

Where,

$$C_{npW} = -C'_{lpW} \tan\alpha - [-C'_{lp} \tan\alpha - (\frac{C_{np}}{C_L})_{C_L = 0,M} C_L] + (\frac{\Delta C_{np}}{\theta})\theta + (\frac{\Delta C_{np}}{\alpha_{\delta_f} \delta_f}) \alpha_{\delta_f} \delta_f$$

Now, $C'_{lpW} = -0.436$ from section 6.5

$\alpha = 4.5^0$, $C_L = 0.616$, $C'_{lp} = -0.458$

From Eq.(8.8) of Ref.2

$$\left(\frac{C_{np}}{C_L}\right)_{CL=0,M} = K\left(\frac{C_{np}}{C_L}\right)_{CL=0,M=0}$$

Where,

$$K = \left(\frac{A + 4\cos\Lambda_{c/4}}{AB + 4\cos\Lambda_{c/4}}\right)\left[\frac{AB + \frac{1}{2}(AB + \cos\Lambda_{c/4})\tan^2\Lambda_{c/4}}{A + \frac{1}{2}(A + \cos\Lambda_{c/4})\tan^2\Lambda_{c/4}}\right]$$

where, $B = \sqrt{(1 - M^2\cos^2\Lambda_{c/4})} = (1 - 0.8^2\cos^2 38.5) = 0.78$

For A = 6.46, we get K = 0.9238

From Eq.(8.10) of Ref.2

$$(C_{np})_{CL=0,M=0} = \frac{(-1/6)[A + 6(A + \cos\Lambda_{c/4})\left(\frac{\bar{x}}{\bar{c}}\frac{\tan\Lambda_{c/4}}{A} + \frac{\tan^2\Lambda_{c/4}}{12}\right]}{A + 4\cos\Lambda_{c/4}}$$

Since, c.g. lies at a.c. of wing, $\bar{x}/\bar{c} = 0$ and

$(C_{np})_{C_L=0,M=0} = -0.1526$

Hence, $(C_{np}/C_L)_{C_L=0,M} = 0.9238 \times (-0.1526) = -0.141$

For A = 6.46 and $\lambda = 0.29$, $\Delta C_{np}/\theta = 0.00021$ from Fig.8.2 of Ref.2.

δ_f = flap deflection = 0 (assumed)

$\theta = -3^0$ (assumed).

Hence, $C_{npW} = 0.436 \times \tan 4.5^o - [0.458\tan 4.5^0 + 0.141 \times 0.616] + .00021 \times (-3) = -0.0896$

$$C_{np V} = (-2/b^2)(l_V\cos\alpha + Z_V\sin\alpha)\left(\frac{Z_V\cos\alpha - l_V\sin\alpha}{b}\right)C_{y\beta V}$$

$$= -\frac{2}{59.64^2}(-0.636)(28.35\cos 4.5^0 + 6.35\sin 4.5^0) \times (6.35\cos 4.5^0 - 28.35\sin 4.5^0)$$

$= 0.0405$

Hence, C_{np} = - 0.0896 + 0.0405 = - 0.0491

Based on an area of $511 m^2$, C_{np} = - 0.0529

The value of C_{np} from Ref.3 is – 0.044

6.7 Estimation of C_{Yr}

From Eq.(9.1) of Ref.2

$C_{Yr} \approx C_{Yrv}$

where, $C_{Yrv} = -(2/b)(l_v \cos\alpha + Z_v \sin\alpha) C_{Y\beta V}$

$= -(2 / 59.64)(28.35 \cos 4.5^0 + 6.35 \sin 4.5^0)(-0.636) = 0.613$

6.8 Estimation of C'_{lr}

From Eq.(9.3) of Ref.2

$C'_{lr} = C'_{lrW} + C'_{lrV}$

Where, $C'_{lrW} = C_L \left(\frac{C'_{lr}}{C_L}\right)_{CL=0,M} + \left(\frac{\Delta C'_{lr}}{\Gamma}\right)\Gamma + \left(\frac{\Delta C'_{lr}}{\theta}\right)\theta + \left(\frac{\Delta C'_{lr}}{\alpha_{\delta f}\delta_f}\right)\alpha_{\delta f}\,\delta_f$;

δ_f = flat deflection; it is zero in the present case.

Now, $(C'_{lr}/C_L)_{CL=0,M} = K_1 \left(\frac{C'_{lr}}{C_L}\right)_{C_{L=0,M=0}}$

$$\text{Where, } K_1 = \frac{1 + \frac{A(1-B^2)}{2B(AB + 2\cos\Lambda_{c/4})} + \frac{AB + 2\cos\Lambda_{c/4}}{AB + 4\cos\Lambda_{c/4}}\,\frac{\tan^2\Lambda_{c/4}}{8}}{1 + \frac{A + 2\cos\Lambda_{c/4}}{A + 4\cos\Lambda_{c/4}} + \frac{\tan^2\Lambda_{c/4}}{8}}$$

For A = 6.44, B $= \sqrt{(1 - M^2 \cos^2\Lambda_{c/4})} = 0.78$, $\Lambda_{c/4} = 38.5^0$.

Substituting, gives K_1 = 1.227

From Fig 9.1 of Ref.2 $(C'_{lr})_{CL=0,M=0} = 0.345$

Hence, $C_L\left(\frac{C'_{lr}}{C_L}\right)_{CL=0,M}$ = 1.227 × 0.345 × 0.616 = 0.261

Using Eq. (9.7) of Ref.2,

$$\Gamma\left(\frac{\Delta C'_{lr}}{\Gamma}\right) = \frac{7}{57.3}\frac{1}{12}\frac{\pi \times 6.46 \sin 38.5}{6.46 + 4\cos 38.5} = 0.0134$$

From Fig.9.2, (C'_{lr}/θ) = - 0.0136

Hence, $\theta\,(\Delta C'_{lr}/\theta) = -0.0136 \times (-3) = 0.0408$

Hence, C'_{lrW} = 0.261 + 0.0134 + 0.0408 = 0.315

$C'_{lrV} = -(2/b^2)\,(l_v \cos\alpha + Z_v \sin\alpha)\,(Z_v \cos\alpha - l_v \sin\alpha) C_{y\beta V} = 0.0405$

Hence, C'_{lr} = 0.315 + 0.0405 = 0.3505

Based on an area of $511 m^2$, $C'_{lr} = 0.378$

The value of C'_{lr} from Ref.3 is 0.31

6.9 Estimation of C_{nr}

From Eq.(9.9) of Ref.2.

$C_{nr} = C_{nrW} + C_{nrV}$

Where, $C_{nrW} = \left(\frac{C_{nr}}{C_L^2}\right) C_L^2 + \left(\frac{C_{nr}}{C_{D0}}\right) C_{D0}$

From Fig.9.4 $(C_{nr}/C_L^2) = 0$

C_{nr} / C_{D0} = - 0.44 from Fig 9.5 of Ref.2

C_{D0} = 0.014

Hence, C_{nrW} = - 0.44 × 0.014 = - 0.00616

$C_{nrV} = (2 / b^2)\,(\,l_v \cos\alpha + Z_v \sin\alpha)^2\, C_{y\beta v}$

$= (2 / 59.64^2)\,(\,28.35 \cos 4.5 + 6.35 \sin 4.5)^2 \times (\,-0.636) = -0.296$

Hence, C_{nr} = - 0.00616 - 0.296 = - 0.302

Based on an area of 511m^2 , C_{nr} = - 0.325

From Ref.3 , C_{nr} = - 0.34

7 Comparison of estimated values with those in Ref.3

In Table3, the values of derivatives estimated using references Ref.1 and Ref.2 are compared with the values for flexible airplane given in Ref.3. In most of the cases, agreement is within ± 10 % .The notable exceptions are $C_{L\alpha}$, C_{mu} , $C_{m\dot{\alpha}}$ C'_{lp} and C'_{lr}. Reference 9 gives (page 4.114) the level of inaccuracy in the estimated values of various derivatives. The deviations found here are fairly within those limits.

The rather large deviation in the estimated values of $C_{L\alpha}$ and C'_{lp} as compared to those of Ref.3 appear to be due to inaccurate correction for the effect of Mach number. The theoretical correction for $C_{L\alpha}$ is based on Prandtl-Glauert rule as applied to wing. This gives about 20 % increase in the value of $C_{L\alpha}$ when Mach number changes from zero to 0.8. The results for flexible airplane show (Figure on p.220 of Ref.3) that there is a slight decrease in the value of $C_{L\alpha}$ between Mach number zero and 0.8. In a similar manner, C'_{lp} depends mainly on the wing. The estimated value of C'_{lp} at M = 0 would be - 0.32 which is fairly close to the value for flexible airplane. Here, again (figure on p.225 of Ref.3) C'_{lp} does not vary appreciably with Mach number for the flexible airplane.

S.No.	Symbol	Derivatives			
		Based onS = 550.5m^2 and $\overline{c}$ = 10.2 m	Based on S=511m^2 and $\overline{c}$ = 8.33m	As given by Ref .3	% Deviation
1.	C_L	0.616	0.66	0.66	-
2.	C_D	0.0392	0.0422	0.043	-2
3.	$C_{L\alpha}$	5.44	5.86	5.00	+17.2
4.	$C_{D\alpha}$	0.446	0.48	0.46	+ 4.3
5.	$C_{m\alpha}$	- 0.80	- 1.074	- 1.03	+ 4.3
6.	C_{Du}	Neglected	0	0.024	-
7.	C_{Lu}	0.315	0.346	0.184	+ 90
8.	C_{mu}	- 0.174	- 0.2304	0.128	-
9.	C_{Dq}	Neglected	-	-	-
10.	C_{Lq}	8.188	-	-	-
11.	C_{mq}	- 20.17	- 26.7	- 23.9	+ 11.7
12.	$C_{D\dot{\alpha}}$	Neglected	-	-	-
13.	$C_{L\dot{\alpha}}$	2.31	-	-	-
14.	$C_{m\dot{\alpha}}$	- 6.87	- 9.06	- 6.55	+ 38.3
15.	$C_{Y\beta}$	- 0.8492	- 0.9148	- 0.884	4
16.	$C'_{l\beta}$	- 0.2921	- 0.3144	- 0.279	+12.7
17.	$C_{n\beta}$	0.1746	0.188	0.195	- 3.5
18.	C_{Yp}	- 0.1123	-	-	-
19.	C'_{lp}	- 0.458	- 0.494	- 0.34	+45.3
20.	C_{np}	- 0.0491	- 0.0529	- 0.044	+20.0
21.	C_{Yr}	- 0.613	-	-	-
22.	C'_{lr}	- 0.4322	0.466	0.31	+50.0
23.	C_{nr}	- 0.302	- 0.325	- 0.34	-

TABLE 3: Comparison of estimated derivatives with those in Ref.3

8. Stability analysis

The stability derivatives have been evaluated using the methods prescribed in Ref.2.The equations of motion using small perturbation theory approach are as follows (Ref.10, Chapters 4 & 5).

8.1 Equations for longitudinal motion

$$\left(\frac{d}{dt}-X_u\right)\Delta u - X_w\Delta w + (g\cos(\theta_0))\Delta\theta = X_\delta\Delta\delta + X_{\delta T}\,\Delta\delta_T \tag{1}$$

$$-Z_u\Delta u + \left((1-Z_{\dot{w}})\frac{d}{dt} - Z_w\right)\Delta w - \left((u_0+Z_q)\frac{d}{dt} - g\sin\theta\right)\Delta\theta = Z_\delta\,\Delta\delta + Z_{\delta T}\,\Delta\delta_T \tag{2}$$

$$-M_u\Delta u - \left(M_{\dot{w}}\frac{d}{dt} + M_w\right)\Delta w + \left(\frac{d^2}{dt^2} - M_q\frac{d}{dt}\right)\Delta\theta = M_\delta\Delta\delta + M_{\delta T}\,\Delta\delta_T \tag{3}$$

These Equations can be expressed in state space variable form (without control input) as:

$$\dot{\mathbf{X}} = \mathbf{AX} \tag{4}$$

$$\begin{pmatrix}\Delta\dot{u}\\ \Delta\dot{w}\\ \Delta\dot{q}\\ \Delta\dot{\theta}\end{pmatrix} = \begin{pmatrix} X_u & X_w & 0 & -g \\ Z_u & Z_w & u_0 & 0 \\ M_u+M_{\dot{w}}Z_u & M_w+M_{\dot{w}}Z_w & M_q+M_{\dot{w}}U_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix}\Delta u\\ \Delta w\\ \Delta q\\ \Delta\theta\end{pmatrix} \tag{5}$$

where,

$X_u = -(C_{Du} + 2C_D)\,QS / mu_0$

$Z_u = -(C_{Lu} + 2C_L)QS / mu_0$

$M_u = C_{mu}\,(QS\bar{c}\ / u_0 I_{yy})$

$X_w = -(C_{D\alpha} - C_L)\,QS / mu_0$

$Z_w = -(C_{L\alpha} + C_D)\,QS / mu_0$

$M_w = C_{m\alpha}\,(QS\bar{c}\ / u_0 I_{yy})$

$M_{\dot{w}} = C_{m\acute{\alpha}}\,(QS\bar{c}/u_0 I_{yy})(\ \bar{c} / 2u_0)$

$M_q = C_{mq}\,(\bar{c}^2/2)\,QS / u_0 I_{yy}$

8.2 Equations for lateral motion

$$\left(\frac{d}{dt}-Y_v\right)\Delta v - Y_p\Delta p + (u_0 - Y_r)\Delta r - g\cos\theta_0\Delta\varphi = Y_{\delta r}\,\Delta\delta_r \tag{6}$$

$$-L_v\Delta v + \left(\frac{d}{dt} - L_p\right)\Delta p - \left(\frac{I_{xz}}{I_{xx}}\frac{d}{dt} + L_r\right)\Delta r = L_{\delta a}\,\Delta\delta_a + L_{\delta r}\,\Delta\delta_r \tag{7}$$

$$-N_v\Delta v - \left(\frac{I_{xz}}{I_{zz}}\frac{d}{dt} + N_p\right)\Delta p + \left(\frac{d}{dt} - N_r\right)\Delta r = N_{\delta\alpha}\,\Delta\delta_\alpha + N_{\delta r}\Delta\delta_r \tag{8}$$

In state space variable form (without control input) these equations appear as:

$$\begin{pmatrix}\Delta\dot{v}\\ \Delta\dot{p}\\ \Delta\dot{r}\\ \Delta\dot{\varphi}\end{pmatrix} = \begin{pmatrix} Y_u & Y_p & -(u_0-Y_r) & g\cos\theta_0 \\ L_v & L_p & L_r & 0 \\ N_v & N_p & N_r & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix}\Delta v\\ \Delta p\\ \Delta r\\ \Delta\varphi\end{pmatrix} \tag{9}$$

where ,

$Y_p = QSbC_{yp} / 2mu_0$

$Y_r = (QS/mu_0)(b / 2)C_{yr}$

$Y_\beta = QSC_{y\beta} / m$

$L_p = QSb^2\ C'_{lp} /(2\ I_{xx}\ u_0)$

$L_r = QSb^2\ C'_{lr} / (2\ I_{xx}\ u_0)$

$L_\beta = QSb\ C'_{l\beta} / I_{xx}$

$N_p = QSb^2 C_{np}/ (2\ I_{zz}\ u_0)$

$N_r = QSb^2 C_{nr}/ (2\ I_{zz}\ u_0)$

$N_\beta = QSbC_{n\beta} / I_{zz}$

8.3 Flight condition, mass and moments of inertia

In the present case,

$V = u_0 = 236.16$ m/s

W = 2,852,130 N

m = 290,737 kg

h = 40,000 ft . Hence, $\rho = 0.3014$ kg/m^3

b = 59.74 m ; $\bar{c}$ = 10.2 m ; S = 550.5 m^2

$I_{xx} = 24726520$ kg m^2

$I_{yy} = 44969660$ kg m^2

$I_{zz} = 67522420$ kg m^2

$I_{xz} = 1317918$ kg m^2

$I_{xz}^2 / I_{xx} I_{zz} = 0.00104$ (Neglected)

$Q = \rho V^2 / 2 = 0.3014 \times 236.16 \times 236.16 / 2 = 8403.38$ N / m^2

Hence, $QS / mu_0 = 0.06738$, $QS/ u_0 I_{yy} = 0.000436$, $QSb / I_{zz} = 4.086$,

$QSb / I_{xx} = 11.158$, $QSb/mu_0 = 4.0183$, $QSb^2 / u_0 I_{zz} = 1.032$,

$QSb^2 / u_0 I_{xx} = 2.818$.

8.4. Analysis of longitudinal motion

Using the values of various quantities:

$X_u = -(C_{Du} + 2C_D)\, QS / mu_0 = -(0 + 2 \times 0.0392)0.06738 = -0.005282$

$Z_u = -(C_{Lu} + 2C_L)QS / mu_0 = -(0.315 + 2 \times 0.616)0.06738 = -0.1042$

$M_u = C_{mu}\,(QS / u_0 I_{yy})\ \bar{c} = 0.128 \times (0.000436 \times 10.2) = 0.0005692$

Remark:

The roots of the stability quartic or the eigen values of the stability matrix are sensitive to the value of C_{mu}. Results comparable with the roots in Ref.3 were obtained when C_{mu} value given in Ref.3 is used.(See Table 3).

$X_w = -(C_{D\alpha} - C_L)\, QS / mu_0 = -(0.446 + 0.616)0.06738 = -0.01145$

$Z_w = -(C_{L\alpha} + C_D)QS / mu_0 = -(5.44 + 0.0392)0.06738 = -0.3692$

$M_w = C_{m\alpha}\,(QS/u_0 I_{yy})\,\bar{c} = -0.80\ (0.000436 \times 10.2) = -0.003558$

$M_{\dot{w}} = C_{m\dot{\alpha}}\,(\bar{c}^2 / 2u_0)(QS / u_0 I_{yy}) = -6.87\,(10.2)^2 / (2 \times 236.16) \times (0.000436) = -0.00066$

$M_q = C_{mq}\,(\bar{c}^2 / 2)\ QS / u_0 I_{yy} = -20.17 \times 10.2^2 / 2 \times (0.000436) = -0.4570$

From the above quantities, the stability matrix for the longitudinal motion is:

$$\begin{vmatrix} -0.005282 & 0.01145 & 0 & -9.81 \\ -0.1042 & -0.3692 & 236.16 & 0 \\ 0.0006374 & -0.003314 & -0.6127 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

The following Eigen values are obtained using MATLAB.

Short Period Mode ($\eta_s \pm i\, \omega_s$)

$-0.4911 + i\,0.8738,\ -0.4911 - i\,0.8738$

$\eta_s = -0.4911,\ \omega_s = 0.8738$

$T_{1/2} = \ln(2)/\,|\,\eta_s\,| = 0.693/0.4911 = 1.41$ s

Period $= 2\pi/\,\omega_s = 6.28/0.8738 = 7.19$ s

$N_{1/2} = T_{1/2}$ / period$= 1.41/7.19 = 0.196$ cycles

Long Period Mode or Phugoid Mode ($\eta_p \pm i\,\omega_p$)

$-0.0025 + i\,0.0753,\ -0.0025 - i\,0.0753$

η_p = - 0.0025 , ω_p = 0.0753

$T_{1/2} = \ln(2)/ | \eta_p | = 0.693 / 0.0025 = 277.2$ s

Period = $2\pi / \omega_p = 6.28 / 0.0753 = 83.40$ s

$N_{1/2} = T_{1/2}$ / period = 277.2 / 83.40 = 3.32 cycles

Remark:

The approximate analysis of Phugoid mode (Ref.10) gives η_p and ω_p as:

$\eta_p \approx X_u / 2$; $\omega_p \approx \sqrt{(-Z_u \; g / u_0)}$

In the present case this would give $\eta_p \approx$ - 0.0026 and $\omega_p \approx$ 0.0667. This cross check is seen to be roughly satisfied by the roots presented earlier.

8.5 Analysis of lateral motion

Using the values of various quantities yields :

$Y_p = (QS / mu_0)(b/2) C_{yp} = 0.06738 \; (59.64 / 2) (- 0.1123) = - 0.2256$

$Y_r = (QS / mu_0) (b/2)C_{yr} = 0.06738 \; (59.64 / 2) (- 0.613) = - 1.2316$

$Y_\beta = QSC_{y\beta} / m = 8403.38 \times 550.5 (- 0.8492)/ 290737 = -13.512$

$L_p = (QSb^2 / I_{xx} u_0) (C'_{lp} /2) = (2.818) (-0.458 / 2) = -0.6453$

$L_r = (QSb^2 / I_{xx}u_0)(C'_{lr} /2) = (2.818) (0.4322 / 2) = 0.6089$

$L_\beta = (QSb / I_{xx}) \; C'_{l\beta} = (11.158) (-0.2921) = - 3.2593$

$N_p = (QSb^2 / I_{zz} u_0)(C_{np} /2) = (1.032)(-0.0491 / 2) = - 0.02533$

$N_r = (QSb^2/ I_{zz} u_0)(C_{nr} /2) = (1.032) (-0.302 / 2) = - 0.1558$

$N_\beta = (QSb / I_{zz})C_{n\beta} = (4.086) (0.1746) = 0.7134$

The stability matrix for lateral motion is:

$$\begin{vmatrix} -0.05722 & -0.00096 & -1.0052 & 0.04154 \\ -3.2593 & -0.6453 & 0.6089 & 0 \\ 0.7134 & -0.02533 & -0.1558 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix}$$

The following Eigen values are obtained using MATLAB.

Dutch Roll ($\eta_d \pm i \, \omega_d$)

- 0.0198 + i 0.9162 , - 0.0198 – i 0.9162

η_d = - 0.0198 , ω_d = 0.9162

$T_{1/2} = \ln(2) / | \eta_d | = 0.693/0.0198 = 35.0$ s

Period = $2\pi / \omega_d$ = 6.28/0.9162 = 6.85 s

$N_{1/2} = T_{1/2}$ / period = 35.0/6.85 = 5.11 cycles

Roll mode

η = - 0.8143

$T_{1/2}$ = ln (2) / | η |= 0.693 / 0.8143 = 0.8510 s

Spiral mode

η = - 0.00446

$T_{1/2}$ = ln(2)/ | η | = 0.693/0.00446 = 155.4 s

Remark: Based on Ref.10, the approximate value of the imaginary part of Dutch roll is $\sqrt{(N_\beta)}$, which in this case would be 0.8446. This check is also roughly satisfied by the results presented earlier.