# Chapter 2

# Longitudinal stick–fixed static stability and control

## (Lectures 4 to 11)

**Keywords :** Criteria for longitudinal static stability and control ; contributions of wing, horizontal tail, fuselage and power to pitching moment coefficient ($C_{mcg}$) and its derivative with respect to angle of attack ($C_{m\alpha}$) ; stick-fixed neutral point and static margin ; elevator angle for trim; limitations on forward and rearward movements of c.g. ; determination of neutral point from flight tests.

**Topics**

# Chapter 2

## Lecture 4

## Longitudinal stick–fixed static stability and control - 1

### Topics

### 2.1 Introduction

For a gradual development of the stability and control analysis, the subject has been subdivided into various topics (see subsection 1.3.7 and Fig.1.17). This chapter deals with longitudinal static stability and control in stick-fixed case. The following three items from chapter 1 may be recalled.

(a)In static stability analysis, the forces and moments brought about as a result of the disturbance are considered to examine whether moments tend to bring the airplane back to the equilibrium state or not.

(b)The longitudinal stability analysis deals with the motions in the plane of symmetry i.e. along x- and z- axes and about y-axis.

(c) By stick-fixed case, we imply that even after the disturbance is applied, the control stick is held fixed or the control surface maintains its deflection as in the undisturbed state.

### 2.1.1 Equilibrium state during flight in the plane of symmetry

To analyze the stability, we must consider that the airplane is initially in equilibrium state i.e. it is moving with constant speed along a straight line. If we consider the steady level flight, the conditions for equilibrium are:

$$T - D = 0 \tag{2.1}$$

$$L - W = 0 \tag{2.2}$$

$$M_{cg} = 0 \tag{2.3}$$

These equations imply that:

(a) the thrust must balance the drag by proper setting of the engine throttle,

(b) the lift must balance the weight by proper choice of the lift coefficient at the chosen speed and altitude of flight and

(c ) the pitching moment produced by the wing, the fuselage, the tail and other components must be counterbalanced by the moment produced by the elevator.

Thus, longitudinal control implies the ability to bring $M_{cg}$ to zero by suitable control deflection. In order that this is achieved under different flight conditions, the elevator must have sufficient area and adequate range of deflections. For longitudinal static stability, we need to primarily examine the rotation about the y-axis. The moment about the y-axis is the pitching moment (M or $M_{cg}$). A nose up moment is taken as positive (Fig.2.1).

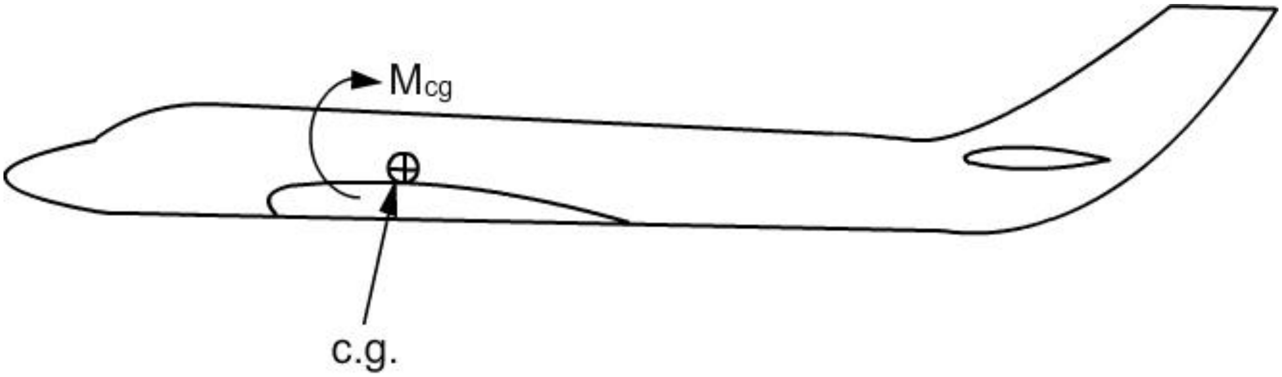

Fig.2.1 Convention for pitching moment

It is convenient to work in terms of pitching moment coefficient ($C_{mcg}$), which is defined as:

$$C_{mcg} = \frac{M_{cg}}{\frac{1}{2}\rho V^2 S\bar{c}} \tag{2.4}$$

where, ρ = ambient density; V = flight velocity; S = wing plan form area; $\bar{c}$ = mean aerodynamic chord of wing.

### 2.1.2 Mean aerodynamic chord

It may be recalled that the mean aerodynamic chord $\bar{c}$ is defined as:

$$\bar{c} = \frac{1}{S}\int_{-b/2}^{b/2} c^2\, dy \tag{2.5}$$

where, c is the local chord of wing (Fig.2.2) and b is the wing span.

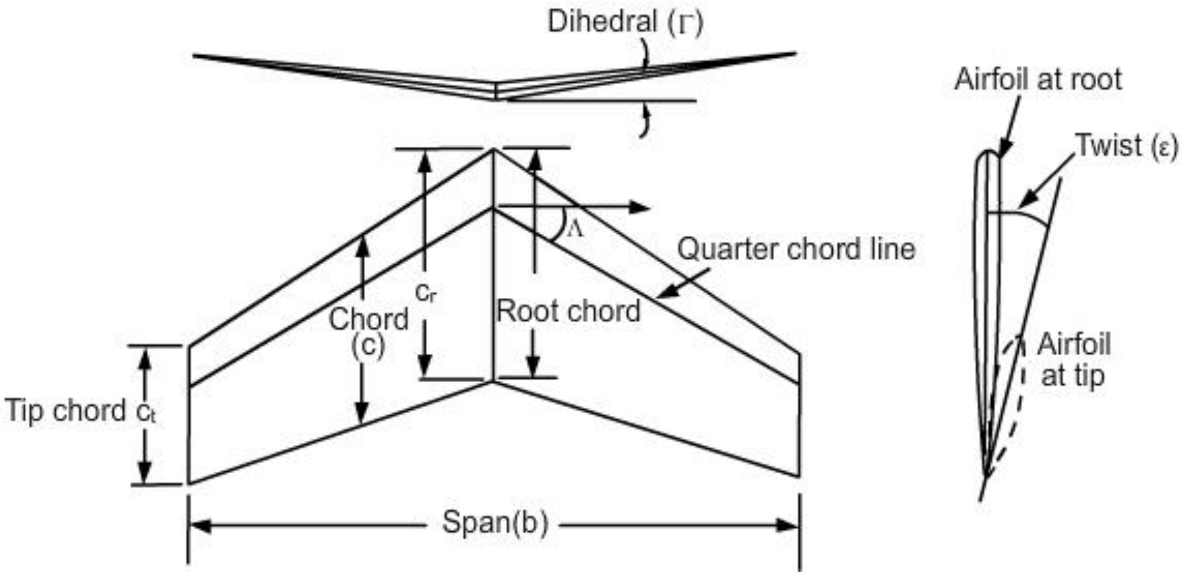

Fig.2.2 Geometric parameters of a wing

For a trapezoidal wing the mean aerodynamic chord is given by the following expression, the derivation is left as an exercise to the reader.

$$\bar{c} = \frac{2}{3}\frac{c_r(1+\lambda+\lambda^2)}{1+\lambda} \tag{2.5a}$$

where, λ = taper ratio = $c_t/c_r$ .

### 2.1.3 Criterion for longitudinal control and trim in pitch

For equilibrium, $C_{mcg} = 0$ (2.6)

When $C_{mcg}$ is made zero by proper control deflection, the airplane is said to be trimmed in pitch.

### 2.1.4 Criterion for longitudinal static stability

The criterion for longitudinal static stability is that when an airplane is disturbed in the plane of symmetry, it has a tendency to return to its equilibrium state. In longitudinal static stability analysis, the effects of perturbations $\Delta u$ and $\Delta w$ are negligible. The change in the angle of attack ($\Delta\alpha$) is considered as the perturbation. Its effect on change in pitching moment, $\Delta M_{cg}$ , is examined to assess the longitudinal static stability. The change in the angle of attack of the airplane could be due to:

(a) the airplane encountering a vertical gust of velocity ($V_{gu}$) or

(b) the pilot deflecting the elevator by a small angle causing a moment and then bringing the elevator to its position in the undisturbed flight .

The gust would change the direction of relative wind and cause a change in the angle of attack (Fig.2.3) given by:

$$\Delta\alpha = V_{gu} / V$$

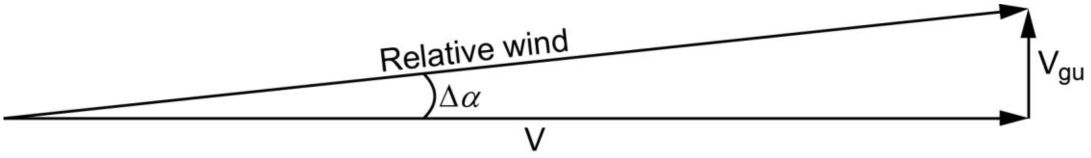

Fig.2.3 change in angle of attack due to gust

The deflection of elevator by the pilot will also lead to change in the angle of attack. Note that the convention for $\Delta\alpha$ is the same as that for the angle of attack i.e. measured from the relative wind towards the fuselage reference line (FRL) and taken positive clock wise (Fig.2.4). With the above conventions for $M_{cg}$ and $\Delta\alpha$, if the airplane is to have static stability, then in response to a positive $\Delta\alpha$ caused by the disturbance, the airplane should produce a $\Delta M_{cg}$

which is negative. Similarly, a disturbance causing negative Δα should result in positive $\Delta M_{cg}$ or $(dM_{cg}/ d\alpha)$ should be negative.

### 2.1.5 Alternate explanation of criterion for a longitudinal static stability

The above argument can be explained in an alternative manner. Consider that the airplane is flying in level flight at angle of attack α i.e. the c.g. moves along a horizontal line and the fuselage reference line (FRL) makes an angle α to the flight direction (Fig.2.4). Now, imagine that the airplane is disturbed and acquires an additional angle of Δα i.e. its angle of attack becomes (α+Δα). The airplane in the changed attitude is shown by dotted lines in Fig.2.4. Now, if the airplane has static stability, then it should produce $\Delta M_{cg}$ such that the airplane returns to the original angle of attack i.e. a disturbance which causes positive Δα, should result in negative $\Delta M_{cg}$ . Similarly, a disturbance that causes negative Δα should be accompanied by positive $\Delta M_{cg}$ . This again means that $(dM_{cg} / d\alpha)$ or $(dC_{mcg} /d\alpha)$ should be negative. If $(dC_{mcg} / d\alpha)$ is zero then the airplane has no tendency to come back and is neutrally stable. If $(dC_{mcg} / d\alpha)$ is positive then the moment produced as a result of positive Δα is also positive and would take the airplane to increased Δα. This means that the airplane is unstable.

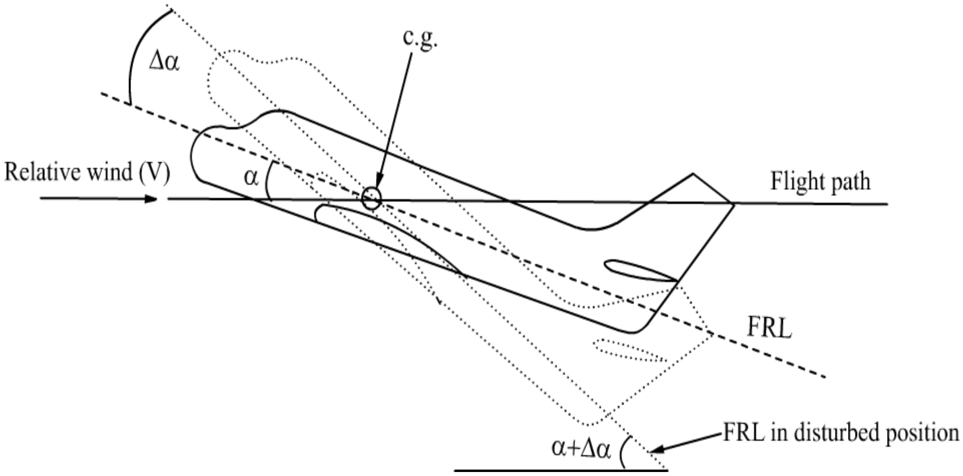

Fig.2.4 Airplane in disturbed position

Thus, the criterion for longitudinal static stability is:

$(dM_{cg}/d\alpha)$ $< 0$ for static stability

$> 0$ for instability (2.7)

$= 0$ for neutral stability

Or, $dC_{mcg}/d\alpha$ or $C_{m\alpha}$ $< 0$ for static stability

$> 0$ for instability (2.8)

$= 0$ for neutral stability

**Remark:**

The angle $\Delta\alpha$ in Fig.2.4 is portrayed big for the sake of clarity. In static stability analysis $\Delta\alpha$ would be small.

### 2.1.6 Desirable values of $C_{m0}$ and $C_{m\alpha}$

Figure 2.5 shows $C_{mcg}$ vs $\alpha$ curves for two airplanes A and B. Both configurations A and B are in trim (i.e. $C_{mcg} = 0$) at point P without deflection of control surface (elevator). However, from stability criterion given in Eq.(2.8), the configuration A, with $C_{m\alpha} < 0$ , is stable and the configuration B, with $C_{m\alpha} > 0$, is unstable. This figure also shows that for airplane A the value of $C_{m0}$ (i.e. value of $C_{mcg}$ when $\alpha$ is zero) is positive. These factors indicate that, for an airplane to be both stable and give trim at realistic values of $C_L$ , requires that :

(a) $C_{m0}$ should be positive and

(b) $C_{m\alpha}$ should be negative.

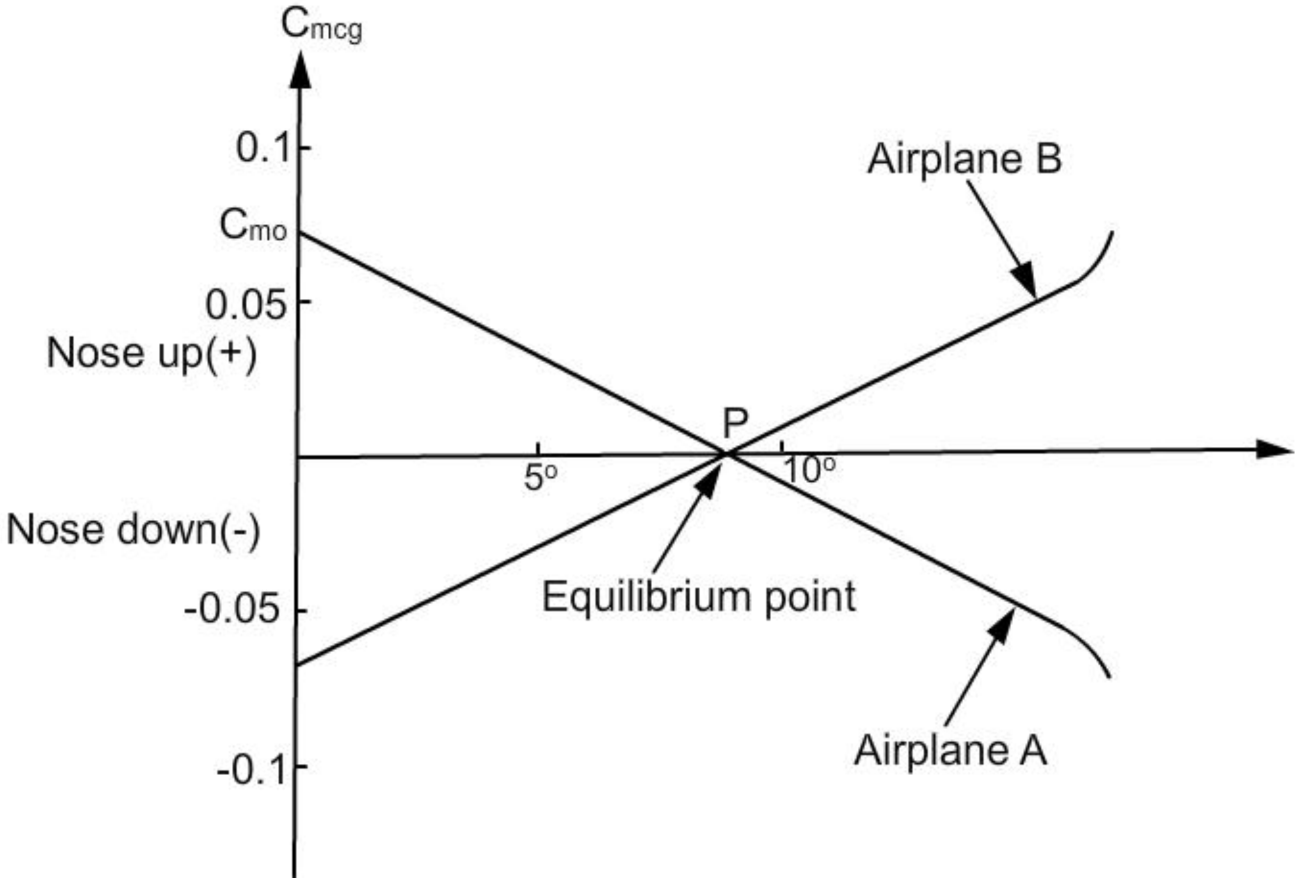

Fig.2.5 $C_{mcg}$ vs α variations for stable and unstable airplanes

### 2.1.7 Effect of elevator deflection on $C_{mcg}$ vs α curve

When an elevator is deflected it produces a moment about c.g. Then the value of $C_{m0}$ of the airplane changes and the $C_{mcg}$ vs α curve is shifted (Fig.2.6). However, $C_{m\alpha}$ does not change due to the elevator deflection and the slope of the curve is same as that with zero elevator deflection (see section 2.12.3 and example 2.7). This figure also indicates that elevator deflection brings about change in the value of α at which $C_{mcg}$ is zero or the airplane is in trim. It may be pointed out that the elevator deflection is denoted by $\delta_e$ and downward deflection of elevator is taken positive (see section 2.4.5 for further details).

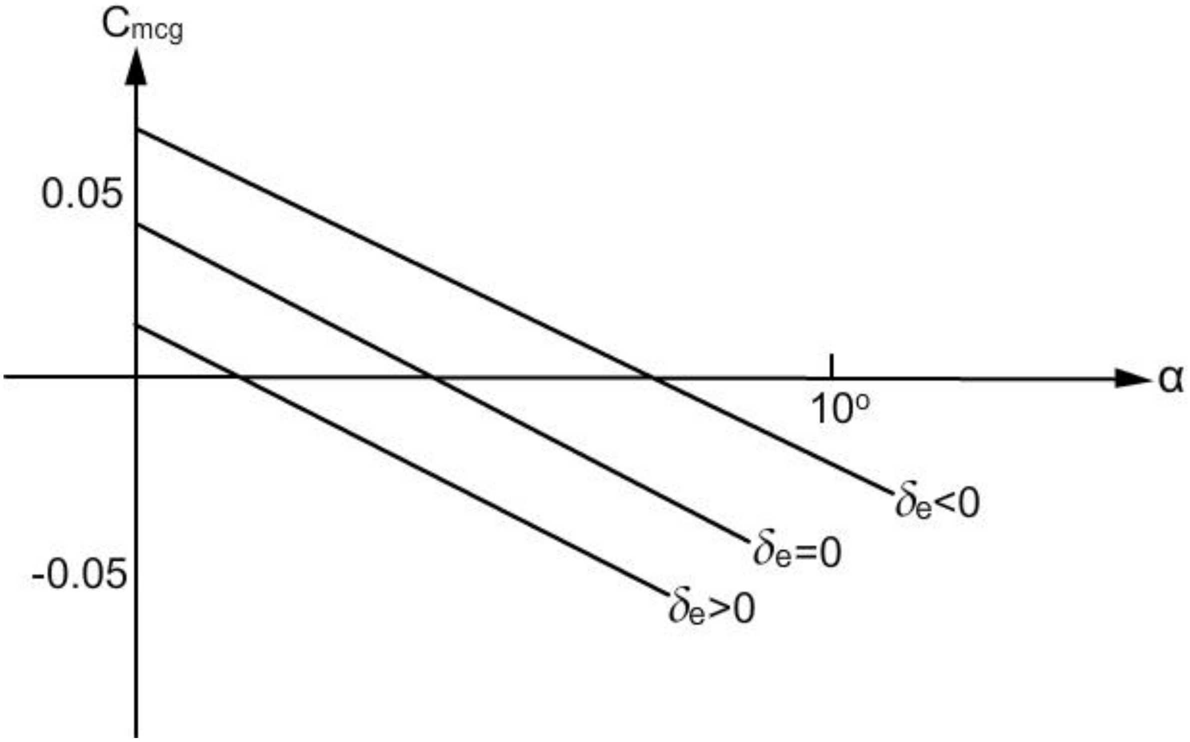

Fig.2.6 Effect of elevator deflection on trim

### 2.1.8 $C_{mcg}$ expressed as function of $C_L$

When the angle of attack (α) is not near stalling angle, the $C_L$ vs α curve of the airplane is nearly linear (Fig.2.7). In this situation $C_{mcg}$ can be plotted as functions of $C_L$ and $dC_m/dC_L$ can be used as criterion for static stability instead of $C_{m\alpha}$ .This was the practice in older literature, on stability and control e.g. Ref.1.7.

Noting that $\frac{dC_m}{d\alpha} = \frac{dC_m}{dC_L}\frac{dC_L}{d\alpha} = C_{L\alpha}\frac{dC_m}{dC_L}$

yields:

$$\frac{dC_m}{dC_L} = \frac{C_{m\alpha}}{C_{L\alpha}} \tag{2.9}$$

When $C_L$ vs α is linear, the longitudinal static stability criterion is:

$$\begin{aligned} dC_m / dC_L &< 0 \text{ for static stability} \\ &> 0 \text{ for instability} \\ &= 0 \text{ for neutral stability} \end{aligned} \tag{2.10}$$

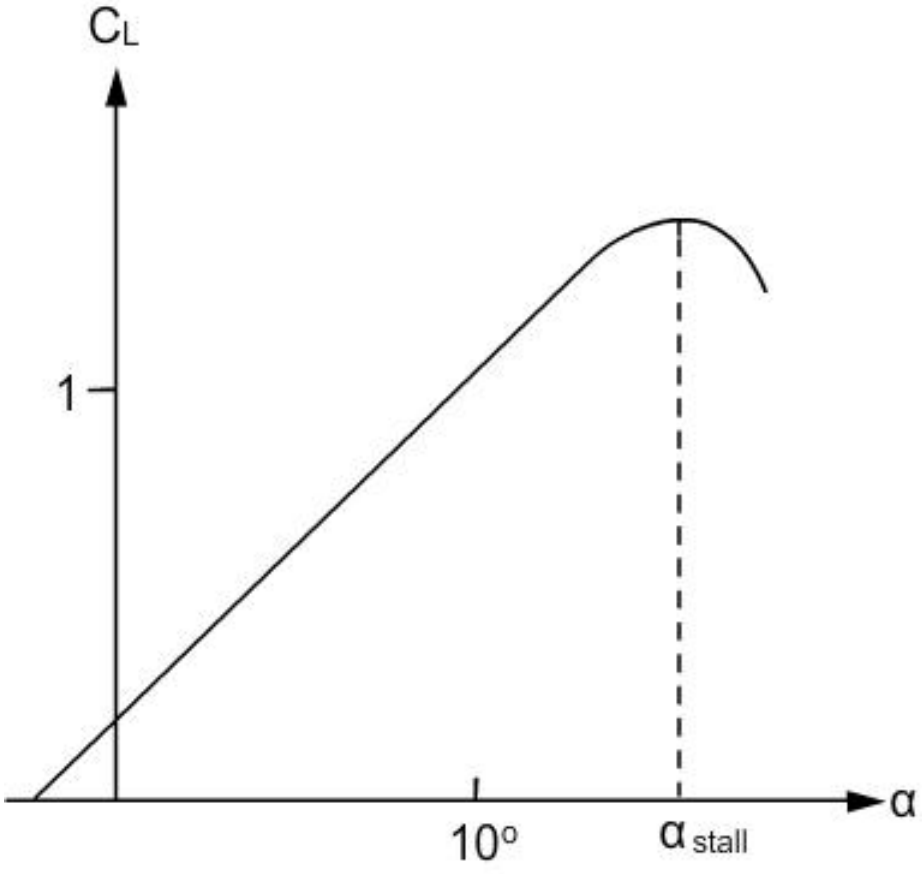

Fig.2.7 $C_L$ vs α