Appendix C

Drag polar, stability derivatives and characteristic roots of a jet airplane - 3

Lecture 39

Topics

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6.3 Estimation of $C_{n\beta}$

6.4 Estimation of C_{yp}

5.5 Estimation of C_{Lu}

From Eq.(4.2) of Ref.2, $C_{Lu} = \dfrac{M^2}{1-M^2}C_L = \dfrac{0.8^2}{1-0.8^2} \times 0.616 = 1.17$

Remark:

According to Ref. 9, p.4-68, this formula for C_{Lu} is valid only for low values of C_L. The value of C_{Lu} lies between -0.2 to 0.6. From figure on p.222 of Ref.3 one finds $C_{LM} = 0.23$

Hence, $C_{Lu} = M \times C_{LM} = 0.8 \times 0.23 = 0.184$.

An alternate procedure for C_{LM} is as follows (Ref.12, chapter 4, see also Ref..13, section 7.11.2).

$C_L = C_{L\alpha}(\alpha - \alpha_{oLW})$; or $C_{LM} = C_{L\alpha M}(\alpha - \alpha_{0LW})$

or $C_{Lu} = M\ C_{L\alpha M}(\alpha - \alpha_{0LW})$; α_{0LW} = Zero lift angle of wing

In a manner similar to the calculation of $C_{L\alpha}$ at M = 0.8, the values of $C_{L\alpha}$ at M = 0.82 and M = 0.78 are obtained as 4.9645 and 4.8391 respectively.

Then, $C_{L\alpha M}$= (4.9645 - 4.8391) / (0.82 - 0.78) = 3.135

$(\alpha - \alpha_{0LW}) = C_L / a_w = 0.616 / 4.9$

Hence, $C_{Lu} = 0.8 \times 3.135 \times 0.616 / 4.9 = 0.315$

Based on the area of 511m^2 this becomes:

$$C_{Lu} = 0.315 \times \frac{550.5}{511} = 0.346$$

Remarks:

i) Reference 11 volume VI, p.376 gives:

$$C_{Lu} = \{M_1^2 \times (\cos\Lambda_{c/4})^2 \times C_{L1}\} / \{1 - M_1^2 (\cos\Lambda_{c/4})^2\}$$

Where, M_1 & C_{L1} are the Mach number and lift coefficient in chosen flight condition. In the present case:

$$C_{Lu} = \{0.8^2 \times (\cos 38.5)^2 \times 0.616\} / \{1 - 0.8^2 (\cos 38.5)^2\} = 0.3971$$

ii) For stability analysis of longitudinal motion, in section 8.4, C_{Lu} = 0.315 is used.

5.6 Estimation of C_{mu}

From Eq.(4.3) of Ref.2

$$C_{mu} = - C_L \frac{\partial \bar{X}_{acw}}{\partial M}$$

To calculate this quantity, $\bar{X}_{acW}$ must be obtained at Mach numbers close to the flight Mach number. Adopting the procedure outlined in section 5.3, the following values are obtained.

M	0.78	0.82
$\bar{X}_{acW}$	0.3243	0.3356

Hence, $\partial \bar{X}_{acw} / \partial M$ = (0.3356 - 0.3243) / 0.04 = 0.2825

Hence, C_{mu} = - 0.616 × 0.2825 = - 0.174

Based on the area of 511 m^2 and reference chord of 8.33 m

C_{mu} = - 0.1740 × 550.5 × 10.2 / (511 × 8.33) = - 0.2304

Alternatively (Ref.13, section 7.11.3),$C_{mu} = M\, C_{mM}$

From figure on p.222 of R.3 C_{mM} = 0.16, consequently

C_{mu} = 0.8 × 0.16 = 0.128.

It is seen that the calculated value of C_{mu} (i.e. -0.2304) and that given in Ref.3 (i.e. 0.128) are not only different but have different signs. It may be pointed out that from Ref.3 it is noticed that C_{mM}, depends on Mach number and also on C_L . Refer also to remark in section 8.4.

5.7 Estimation of C_{Dq}

It is generally small and hence neglected.

5.8 Estimation of C_{Lq}

From Eq. (5.1) of Ref.2

$$C_{Lq} = C_{Lqw} + C_{LqH}$$

$$(C_{Lqw})_M = \left(\frac{A + 2\cos\Lambda_{c/4}}{AB + 2\cos\Lambda_{c/4}}\right)(C_{Lqw})_{M=0}$$

$$(C_{Lqw})_{M=0} = \left(\frac{1}{2} + \frac{2X_W}{\bar{c}}\right)(C_{L\alpha w})_{M=0}$$

where, X_W is the distance between c.g and a.c. In the present case it is zero. Noting that $(C_{L\alpha w})_{M=0}$ is 4.005, gives:

$$(C_{Lqw})_{M=0} = \left(\frac{1}{2} + 0\right) 4.005 = 2.003$$

A = aspect ratio

$B = \sqrt{\{1-M^2 \cos^2 \Lambda_{c/4}\}} = \sqrt{\{1-0.8^2 \cos^2 38.5\}} = 0.78$

Consequently, $(C_{Lqw})_M = \dfrac{6.46 + 2\cos 38.5}{6.46 \times 0.78 + 2\cos 38.5} \times 2.003 = 2.447$

$$(C_{LqH})_M = 2C_{L\alpha H}\,\eta_H \bar{V}_H; \quad \bar{V}_H = \frac{X_H}{\bar{c}}\frac{S_H}{S}$$

X_H = distance from c.g to a.c of tail = 61.75 – 31.41 = 30.34 ; $X_H / \bar{c}$ = 2.975

$$\bar{V}_H = \frac{30.34}{10.2} \times \frac{135.08}{550.5} = 0.731$$

$$(C_{LqH})_M = 2 \times 4.135 \times 0.95 \times 0.731 = 5.741$$

$$C_{Lq} = 2.447 + 5.741 = 8.188$$

5.9 Estimation of C_{mq}

From Eq.(5.6) of Ref.2

$$C_{mq} = C_{mqW} + C_{mqH}$$

$$(C_{mqW})_M = (C_{mqW})_{M=0}\left[\frac{\dfrac{A^3\tan^2\Lambda_{c/4}}{AB+6\cos\Lambda_{c/4}}+\dfrac{3}{B}}{\dfrac{A^3\tan^2\Lambda_{c/4}}{A+6\cos\Lambda_{c/4}}+3}\right]$$

$$(C_{mqW})_{M=0} = -KC_{l\alpha W}\cos\Lambda_{c/4}\left(\frac{A\left(2\left(\dfrac{\bar{X}_W}{\bar{c}}\right)^2+\dfrac{1}{2}\left(\dfrac{\bar{X}_W}{\bar{c}}\right)\right)}{A+2\cos\Lambda_{c/4}}+\frac{1}{24}\frac{A^3\tan^2\Lambda_{c/4}}{A+6\cos\Lambda_{c/4}}+\frac{1}{8}\right)$$

From Fig 5.1 of Ref.2, K = 0.705 for A = 6.46, $C_{l\alpha W} = 2\pi = 6.28$ and as evaluated in section 5.8, and B = 0.78.

$$(C_{mqW})_{M=0} = -0.705\times 6.28\cos 38.5\{0+1/24\frac{6.46^3\tan^2 38.5}{6.46+6\cos 38.5}+1/8\} = -2.641$$

$$(C_{mqW})_M = -2.641\left[\frac{\dfrac{6.46^3\tan^2 38.5}{6.46\times 0.78+6\times\cos 38.5}+\dfrac{3}{0.78}}{\dfrac{6.46^3\tan^2 38.5}{6.46+6\times\cos 38.5}+3}\right] = -3.086$$

$$C_{mqH} = -\frac{2C_{L\alpha H}\,\eta_H\,\bar{V}_H\,X_H}{\bar{c}} = -2\times 4.135\times 0.95\times 0.731\times 2.975 = -17.09$$

Hence, $C_{mq} = -17.09 - 3.086 = -20.17$

Based on an area of 511m^2 and $\bar{c}$ = 8.33 m C_{mq} is :

$$C_{mq} = -20.17\times\frac{550.5}{511}\times\frac{10.2}{8.33} = -26.7$$

From p.221 of Ref.3 $C_{mq} = -23.9$. The estimated value is higher perhaps because the value of $C_{L\alpha}$ given by theory is higher.

5.10 Estimation of $C_{D\dot{\alpha}}$

This is taken as zero.

5.11 Estimation of $C_{L\dot{\alpha}}$

From Eq.(6.1) of Ref.2.

$C_{L\dot{\alpha}} = C_{L\dot{\alpha}W} + C_{L\dot{\alpha}H}$

Expressions are given in Ref.2 for $C_{L\dot{\alpha}W}$ and $C_{L\dot{\alpha}H}$. However, from reference 11 vol.VI p. 387.

$C_{L\dot{\alpha}} \approx -C_{m\dot{\alpha}} (\bar{c} / X_H)$

Hence, $C_{L\dot{\alpha}}$ is estimated after evaluating $C_{m\dot{\alpha}}$.

5.12 Estimation of $C_{m\dot{\alpha}}$

From Eq.(6.4) of Ref.2

$C_{m\dot{\alpha}} = C_{m\dot{\alpha}W} + C_{m\dot{\alpha}H}$

$C_{m\dot{\alpha}W} = 0$

$C_{m\dot{\alpha}H} = - 2\, C_{L\alpha H}\, \eta_H\, \bar{V}_H\, (X_H / \bar{c})\, (d\varepsilon/d\alpha)$

$= - 2 \times 4.135 \times 0.95 \times 0.731 \times 2.975 \times 0.432 = - 6.87$

Hence, $C_{m\dot{\alpha}} = - 6.87$

The value of $C_{m\dot{\alpha}}$ based on the area of $511m^2$ and $\bar{c}$ of 8.33m is :

$$C_{m\dot{\alpha}} = -6.87 \times \frac{550.5}{511} \times \frac{10.2}{8.33} = -9.06$$

Remarks:

i) Value of $C_{m\dot{\alpha}}$ given in Ref.3 is - 6.55.

ii) Value of $C_{L\dot{\alpha}}$ is : -(- 6.87 / 2.975) = 2.31

6. Estimation of lateral stability derivatives

These derivatives can be classified as angle of sideslip derivatives $C_{Y\beta}$, $C'_{l\beta}$, $C_{n\beta}$; roll rate derivatives C_{Yp}, C'_{lp}, C_{np} and yaw rate derivatives C_{Yr}, C'_{lr}, C_{nr}

6.1 Estimation of $C_{Y\beta}$

From Eq. (7.1) of Ref.2

$C_{Y\beta} = C_{Y\beta W} + C_{Y\beta B} + C_{Y\beta V}$

$C_{Y\beta W} = - 0.0001\ \Gamma \times 57.3$

$\Gamma = 7^0$, hence $C_{Y\beta W} = -0.0001 \times 7 \times 57.3 = -0.0401\ rad^{-1}$

$C_{Y\beta B} = -2K_i \ (S_0 / S)$

K_i is given in Fig 7.1 of Ref.2. It depends on the position of wing on fuselage and on $(2 z_w / d)$

z_w = distance from body centre to c /4 of exposed wing root chord = 3.5 m (estimated from Fig.1).

d = maximum diameter at wing body junction = 7.2 m

Hence, $2z_w / d = 3.5 \times 2 / 7.2 = 0.972$

K_i = 1.48 from Fig 7.1 of Ref.2.

S_0 = cross sectional area of the fuselage at the point where dS/dx is maximum. This is estimated to occur at 36.5 m from nose. At this location S_0 is 32.2 m^2.

$$C_{Y\beta B} = -2 \times 1.48 \ \frac{32.2}{550.5} = -0.1731$$

$C_{Y\beta V}$:

$$C_{Y\beta V} = -k \, C_{L\alpha V} \, (1 + \frac{d\sigma}{d\beta}) \, \eta_V \, \frac{S_V}{S}$$

The factor k, given in Fig.7.3 of R.2, depends on $(b_V/2r_1)$. See section 2.5 for definition of 'r_1'.Here $b_V / 2 \, r_1 = 11.6 / 5.0 = 2.32$. Hence, k = 0.82

$C_{L\alpha v}$**:** It depends on the effective aspect ratio of the vertical tail (A_{Veff}).

$$A_{Veff} = \frac{A_{VB}}{A_V} A_V \{1 + K_H \ (\frac{A_{VHB}}{A_{VB}} - 1)\}$$

From section 2.5, $A_V = 1.38$, $\lambda_V = 0.296$.

From Fig.7.5 of Ref.2

$$\frac{A_{VB}}{A_V} = 1.58$$

$\frac{A_{VHB}}{A_{VB}}$ depends on z_H / b_v . z_H is defined in Fig 7.6 of Ref.2.

It is taken as zero because a.c. of the horizontal tail lies below the root chord of the vertical tail. X is the distance of the a.c. of the horizontal tail from the leading edge of the vertical tail root chord = 61.75 – 52.0 = 9.75 m.

Hence, $X / c_v = 9.75/13 = 0.75$; C_v is taken equal to $(c_r)_v$

From Fig.7.6 of Ref.2

$$\frac{A_{VHB}}{A_{VB}} = 1.29$$

S_H / S_V =135.08/ 97.73 = 1.372. From Fig.7.7 of Ref.2, K_H = 1.03

Then, A_{Veff} = 1.58 × 1.38 [1 + 1.03 (1.29 -1)] = 2.83

The term $C_{L\alpha v}$ is calculated from A_{Veff} in the same manner like that for a wing of this aspect ratio. Using Eq.(3.8) of Ref.2 , $C_{L\alpha v}$ = 3.40.

From Eq.(7.5) of Ref.2

$$\{1 + (d\sigma / d\beta)\}\eta_V = 0.724 + 3.06\frac{97.73 / 550.5}{1 + \cos 39.8} + 0.4(\frac{3.5}{7.2}) + 0.009 \times 6.46 = 1.284$$

Hence, $C_{Y\beta V}$ = - 0.82 × 3.40 × 1.284 × (97.73 / 550.5) = - 0.636

And

$C_{Y\beta}$ = - 0.636 – 0.1731 – 0.0401 = - 0.8492

Based on an area of 511m^2 , $C_{Y\beta}$ = .8492 × (550.5 / 511) = - 0.9148

From Ref.3 the value of $C_{Y\beta}$ is - 0.884.

6.2 Estimation of $C'_{1\beta}$

From Eq.(7.8) of Ref.2

$$C'_{1\beta} = C'_{1\beta WB} + C'_{1\beta H} + C'_{1\beta V}$$

Now,

$$C'_{1\beta WB} = 57.3\,[\{C_{LWB}(\frac{C'_{1\beta}}{C_L})_{\Lambda_{C/2}} K_{M\Lambda}K_f + (\frac{C'_{1\beta}}{C_L})_A\} + \Gamma\{\frac{C'_{1\beta}}{\Gamma}K_{M\Gamma} + \frac{\Delta C'_{1\beta}}{\Gamma}\}$$

$$+ (\Delta C'_{1\beta})_{ZW} + \theta \tan\Lambda_{c/4}\,(\frac{\Delta C'_{1\beta}}{\theta\tan\Lambda_{c/4}})]$$

$C_{LWB} \approx C_L$ = 0.616

For A = 6.46, λ = 0.29 and $\Lambda_{C/2}$ = 35.0^0, from Fig 7.11 of Ref.2

$$(\frac{C'_{1\beta}}{C_L})\ \Lambda_{C/2} = - 0.0029$$

$$M \cos\Lambda_{c/2} = 0.8 \cos 35 = 0.655\ ; \quad \frac{A}{\cos\Lambda_{c/2}} = \frac{6.46}{\cos 35} = 7.86$$

From Fig 7.12 of Ref.2, $K_{M\Lambda}$ = 1.23.

The parameter K_f depends on l_f / b. The length l_f is shown in Fig. 7.13 of Ref.2. Based on Fig 2 of this appendix, l_f = 45.16 m.

l_f / b = 45.16 /59.64 = 0.757.

From Fig.7.13 of Ref.2, K_f = 0.884

From Fig 7.14 of Ref.2, $(C'_{l\beta}/C_L)_A = -0.0002$

For $\Lambda_{c/2} = 35^0$ and $\lambda = 0.29$, from Fig 7.15, $(C'_{l\beta}/\Gamma) = -0.0001825$

From Fig 7.16, $K_{M\Gamma} = 1.15$

From Eq.(7.10) of Ref.2

$$\frac{\Delta C'_{l\beta}}{\Gamma} = -0.0005\sqrt{A}\,(d/b)^2$$

$$d = \sqrt{(\text{average fuselage cross sectional area } / 0.7854)} = 6.41\text{m}$$

Hence, $\Delta C'_{l\beta}/\Gamma = -0.0005\times\sqrt{6.46}\,(6.41/59.64)^2 = -0.0000146$

From Eq.(7.12) of Ref.2

$$(\Delta C'_{l\beta})_{ZW} = -\frac{1.2\sqrt{A}}{57.3}\left(\frac{Z_W}{b}\right)\left(\frac{2d}{b}\right) = -\frac{1.2\times\sqrt{6.46}}{57.3}\left(\frac{3.5}{59.64}\right)\left(\frac{2\times 6.41}{59.64}\right) = -0.000671$$

From Fig 7.17 of Ref.2

$$\frac{\Delta C'_{l\beta}}{\theta\tan\Lambda_{c/4}} = -0.000031$$

The quantity θ is the wing twist which is assumed to be equal to -3^0

Now, $C'_{l\beta WB}$ = 57.3 [0.616 (- 0.0029 × 1.23 × 0.884 - 0.0002)

+ 7 (-0.0001825 × 1.15 - 0.0000146)- 0.000671 + (-3) (-0.000031) tan 38.5] = - 0.2431

From Eq.(7.13) of Ref.2 : $C'_{l\beta H} = C'_{l\beta HB}\dfrac{S_H b_H}{S\,b}$

$C'_{l\beta HB}$ can be evaluated in the same manner as $C'_{l\beta WB}$. It is simply taken equal to $C'_{l\beta WB}$

i.e $C'_{l\beta HB} = -0.2431$.

Then,

$$C'_{l\beta H} = -\frac{0.2431\times 135.07\times 22.17}{550.5\times 59.64} = -0.0222$$

From Eq.(7.14) of Ref.2

$$C'_{l\beta V} = C_{Y\beta V} \left(\frac{Z_V \cos\alpha - l_V \sin\alpha}{b}\right)$$

l_v = distance between c.g and the a.c of vertical tail

Reference 2 shows the distances l_v and Z_v in Fig.7.18. However, it does not give a procedure to obtain the location of the aerodynamic centre of vertical tail from the leading edge of its root chord (the distance $\frac{X'_{ac}}{c_r}$ in section 5.3 of this appendix). Hence, the procedure to obtain $\frac{X'_{ac}}{c_r}$ for the wing is adopted considering the vertical tail as a hypothetical wing. For this purpose, the span of a hypothetical wing (b_h) is taken as 2 b_v or b_H = 2 x 11.6 = 23.2 m . The root chord , tip chord and taper ratio of the hypothetical wing are: $(c_r)_h = (c_r)_V$ = 13.00 m, and $(c_t)_h = (c_t)_V$ = 3.85 m, and

$\lambda_h = \lambda_V$ = 0.297 . This hypothetical wing thus has an area (S_h) of (23.2/2) (13 + 3.85) = 195.46 m^2 and an aspect ratio (A_h) of 23.2^2/195.46 = 2.75. Hence, the parameters $A_h \tan \Lambda_{LE}$ and $\beta / \tan \Lambda_{LE}$ are 2.75 tan 46.26 and 0.6 / tan 26.26 or 2.878 and 0.574.

From Fig 3.9 of Ref.2, $\frac{X'_{ac}}{(c_r)}$ for the hypothetical wing is 0.597.

Consequently, for the vertical tail, $X'_{ac} = 0.597 \times 13 = 7.76$ m.

Since, leading edge of the vertical tail is at 52 m from the nose, the location of a.c. from nose is (52 + 7.76) m or 59.76 m

Noting that the c.g. of the airplane is at 31.41 m from nose, gives :

l_v = 59.76 – 31.41 = 28.35 m.

Z_v = distance of vertical tail a.c above c.g. Assuming c.g to lie on the centre line of the cylindrical portion of the fuselage this is = 6.35 m

α = angle of attack. Assuming $\alpha_{0L} = -2^0$ one gets :

$$C_L = C_{L\alpha}(\alpha - \alpha_{0L})$$

or $0.616 = (5.44 / 57.3)(\alpha + 2)$

Hence, $\alpha = 4.5^0$

Note : The value of α given in Ref.3 is 4^0

Then,

$$C'_{l\beta V} = -6.36\ \frac{6.35\cos 4.5^0 - 28.35\sin 4.5^0}{59.64} = -0.042$$

Hence,

$$C'_{l\beta} = -0.2279 - 0.0222 - 0.042 = -0.2921$$

With reference to wing area of 511m^2, $C'_{l\beta}$ = - 0.3144. The value from figures of Ref.3 is - 0.279

6.3 Estimation of $C_{n\beta}$

From Eq.(7.13) of Ref.2

$$C_{n\beta} = C_{n\beta W} + C_{n\beta B} + C_{n\beta v}$$

$C_{n\beta W}$ = 0 as angle of attack is not high

$$C_{n\beta B} = -57.3 K_N K_{R1} \frac{S_{BS}}{S} \frac{l_B}{b}$$

The quantity K_N is shown in Fig.7.19 of Ref.2. It depends on three parameters namely l_B^2 / S_{BS} , $\sqrt{(h_1 / h_2)}$ and h /W.

l_B = 68.63 m , S_{BS} = body side area = 416 m^2

X_m = c.g location from nose = 31.41m

X_m / l_B = 31.41 / 68.63 = 0.4576

h_1 and h_2 are fuselage heights at l_B / 4 and 3 l_B / 4. From Fig. 3 these are 7.8 m and 5.8 m.

Hence, $\sqrt{h_1 / h_2} = \sqrt{7.8 / 5.8} = 1.16$

h = maximum height of fuselage = 8.4 m

W= body maximum width = 6.41m

Hence, h / W = 8.4/6.41 =1.31

l^2_B / S_{BS} = 68.63^2 / 416 = 11.32

Then, from Fig 7.19 of Ref.2 , K_N = 0.00124

K_{Rl} depends on fuselage Reynolds number. Which is 258.1×10^6.

From Fig 7.20 of Ref.2, $K_{Rl} = 2.14$

Consequently, $C_{n\beta B} = -57.3\times0.00124\times2.14\times(416/550.5)\times\frac{68.63}{59.64} = -0.1321$

$$C_{n\beta V} = -C_{Y\beta V}\left(\frac{l_V\cos\alpha + Z_V\sin\alpha}{b}\right)$$

$$= 0.636\left(\frac{28.35\cos 4.5^0 + 6.35\sin 4.5^0}{59.64}\right) = 0.3067$$

Hence, $C_{n\beta} = -0.1321 + 0.3067 = 0.1746$

Based on an area of $511m^2$ this would be = 0.188

From Ref.3 the value of $C_{n\beta} = 0.195$

6.4 Estimation of C_{Yp}

$$C_{Yp} \approx C_{Ypv} = 2\left(\frac{Z_V\cos\alpha - l_V\sin\alpha}{b}\right)C_{Y\beta V}$$

$$= 2\left(\frac{6.35\cos 4.5^0 - 28.35\sin 4.5^0}{59.64}\right) \times (-0.636) = -0.0841$$