Appendix C

Drag polar, stability derivatives and characteristic roots of a jet airplane – 2

Lecture 38

Topics

4.3 Estimation of $(C_{D0})_N$

The procedure for estimating drag of nacelle as given in section 3.4.4 of Ref.1 is not applicable for the by pass engines used on this aircraft. For want of better information a value of $(C_{D0})_N$ equal to 0.006, based on wetted area, is taken (table 2.3 of Ref.1)

Wetted area = 36.97 m^2 (each nacelle)

$(C_{DO})_N$ for four nacelles:

$$\frac{0.006 \times 4 \times 36.97}{550.5} = 0.00162$$

4.4 C_{D0} for complete airplane

$$C_{DO} = (C_{D0})_{WB} + (C_{D0})_H + (C_{D0})_V + (C_{D0})_N + (C_{D0})_{MISC}$$

Where, $(C_{D0})_{MISC}$ is drag due to miscellaneous roughnesses and protuberances. From section 3.4.6 of Ref.1, for a jet airplane this quantity is about 2% of the sum of the C_{D0}'s for all components. Hence,

$$C_{D0} = 1.02\,[(C_{D0})_{WB} + (C_{D0})_H + (C_{D0})_V + (C_{D0})_N]$$

$$= 1.02\,[0.00936 + 0.00174 + 0.00096 + 0.00162] = 0.01395 = 0.014$$

4.5 Induced drag

Section 3.3 of Ref.1 gives methods for estimating induced drag of wing body combination. However, the calculation procedure requires knowledge of airfoil shape, leading edge radius etc. The value of e, the Oswald or airplane efficiency factor, as given by Eq.(3.18) of Ref.1 is sensitive to a parameter R which depends on the value of leading radius of the airfoil. Since the latter information is not available, the value of e is calculated using simpler procedure of section 2.3 of Ref.1, according to which

$$\frac{1}{e} = \frac{1}{e_{wing}} + \frac{1}{e_{fuselage}} + \frac{1}{e_{other}}$$

To estimate e_{wing}, Ref.1 suggests use of Fig.2.4. However this figure, taken from Ref.5, is applicable only for unswept wings. Reference 6 has suggested a procedure (see p.7.8) which can be interpreted as follows. Calculate the value of e_{wing} for an unswept wing of same aspect ratio and taper ratio. Then,

$$e_{wing} = (e_{wing})_{\Lambda=0} \cos(\Lambda_{c/4} - 5)$$

For A = 6.46 and λ = 0.29 chapter 1 Ref.8, gives $(e_{wing})_{\Lambda=0} = 0.995$.

Hence, $e_{wing} = 0.995 \cos(38.5 - 5) = 0.830$

The value of $\dfrac{\frac{1}{e_{fuselage}}}{S_B / S}$ is given in Fig 2.5 of Ref.1. For circular fuselage it is 0.85 and for rectangular fuselage it is 2.0. Since, the fuselage in the present case is of rounded cross-section, a mean of these values is taken i.e.,

$$\frac{\frac{1}{e_{fuselage}}}{S_B / S} = \frac{0.85 + 2.0}{2} = 1.475$$

$$\frac{1}{e_{fuselage}} = 1.475 \times \frac{32.96}{550.5} = 0.095$$

Ref.1 prescribes $\dfrac{1}{e_{other}} = 0.05$

Hence, $\dfrac{1}{e} = \dfrac{1}{0.83} + 0.095 + 0.05 = 1.35$

or e = 0.741

Consequently, $C_{Di} = \dfrac{C_L^2}{\pi \times 6.46 \times 0.741} = 0.0665\,C_L^2$

4.6 Drag polar

The drag polar of the complete airplane in flight at M = 0.8 and with clean configuration, based on wing area of 550.5m^2, is:

$C_D = 0.014 + 0.0665\,C_L^2$

Remarks:

i) In flight at M = 0.8 at h = 40,000 ft (or 12,200 m) with W = 2,852,129N, the lift co-efficient is:

$$C_L = \frac{2852129 \times 2}{1.225 \times 0.2460 \times 236.16^2 \times 550.5} = 0.616$$

The drag co-efficient at this C_L is:

$C_D = 0.014 + 0.0665 \times 0.616^2 = 0.0392$

Then, C_L based area of 511 m^2 = $0.616 \times \dfrac{550.5}{511} = 0.66$ and

C_D based on wing area of 511 m^2 = $0.0392 \times \dfrac{550.5}{511} = 0.0422$

ii) From Ref.3 it is noted that for the actual airplane the values of C_L and C_D, under the above flight conditions, are 0.66 and 0.043. Further, the value of $C_{D\alpha}$ estimated in section 5.2, also compares closely with the actual value. These comparisons indicate that the estimated drag polar is fairly accurate.

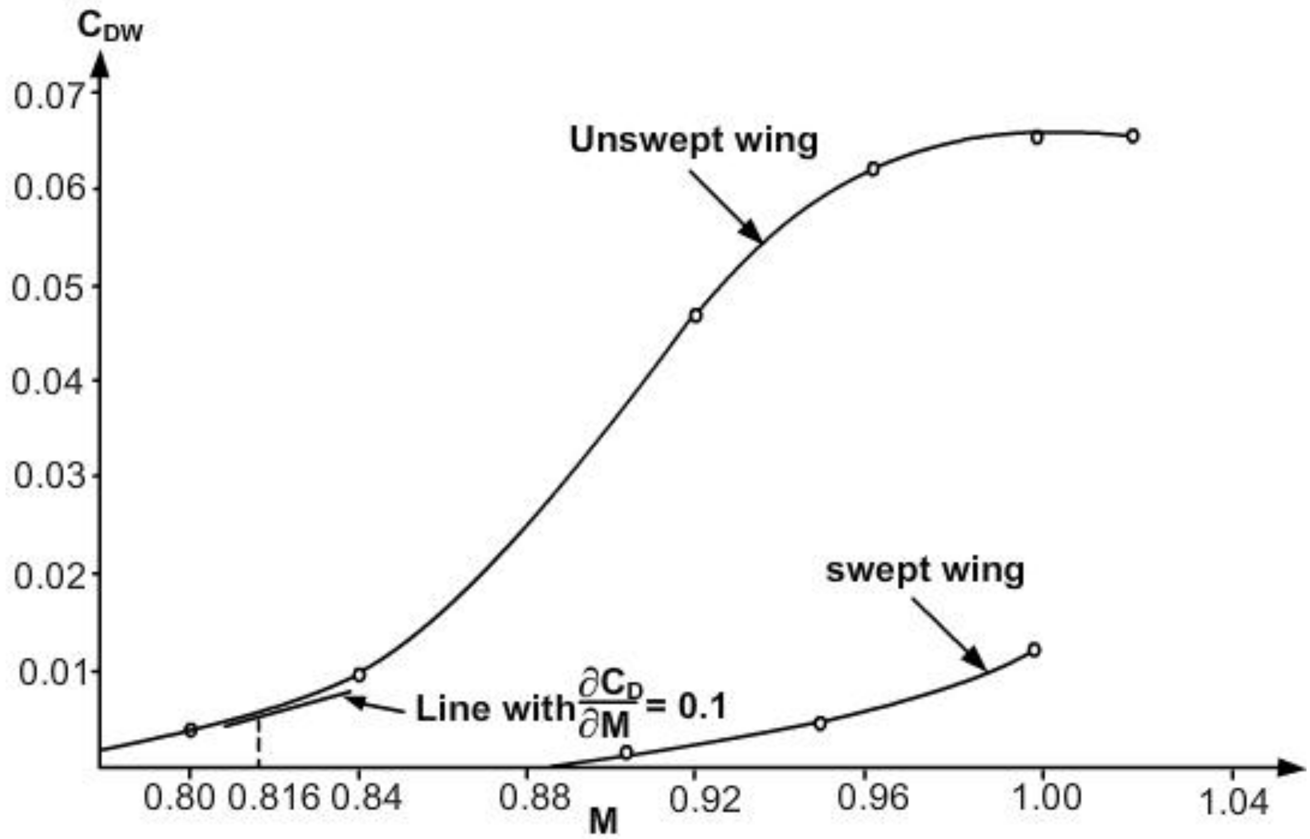

Fig.6 Transonic wave drag

5. Estimation of longitudinal stability derivatives

These derivatives are estimated mainly based on the methods given in Ref.2. These derivatives can be classified as angle of attack derivatives ($C_{D\alpha}$, $C_{L\alpha}$, $C_{m\alpha}$) speed derivatives (C_{Du} , C_{Lu} , C_{mu}), pitch rate derivatives (C_{Dq} ,C_{Lq} , C_{mq}) and angle of attack rate derivatives $(C_{D\dot{\alpha}}, C_{L\dot{\alpha}}, C_{m\dot{\alpha}})$.

5.1 Estimation of $C_{L\alpha}$

From Eq.(3.5) of Ref.2,

$$C_{L\alpha} = C_{L\alpha WB} + C_{L\alpha H}\ \eta_H \frac{S_H}{S}(1-\frac{d\varepsilon}{d\alpha})\ ;\ C_{L\alpha WB} = K_{WB}\ C_{L\alpha W}$$

For b/d > 2 , where d is the width of fuselage at wing root K_{WB} is given by :

$$K_{WB} = 1 - 0.25(\frac{d}{b})^2 + 0.025(\frac{d}{b}) = 1 - 0.25(\frac{6.48}{59.64})^2 + 0.025(\frac{6.48}{59.64}) = 0.9998 \approx 1.0$$

$\mathbf{C_{L\alpha W}}$:

From Eq.(3.8) of Ref.2

$$C_{L\alpha} = \frac{2\pi A}{2+\sqrt{\frac{A^2\beta^2}{\kappa^2}\left(1+\frac{\tan^2\Lambda_{c/2}}{\beta^2}\right)+4}}$$

$$\beta = (1-M^2)^{1/2} = (1-0.8^2)^{1/2} = 0.6$$

$$\kappa = \frac{\text{average 2-D lift curve slope of airfoil}}{2\pi}$$

κ is taken equal to unity.

Substituting various quantities,

$$C_{L\alpha W} = \frac{2\pi \times 6.64}{2+\sqrt{\frac{6.46^2 \times 0.6^2}{1.0}\left(1+\frac{\tan^2 35}{0.6^2}\right)+4}} = 4.90$$

Hence, $C_{L\alpha WB} = 0.9998 \times 4.90 = 4.90$ / radian

$\mathbf{C_{L\alpha H}}$:

In a manner similar to that for wing :

$$C_{L\alpha H} = \frac{2\pi \times 3.642}{2+\sqrt{\frac{3.642^2 \times 0.6^2}{1.0}\left(1+\frac{\tan^2 28.5}{0.6^2}\right)+4}} = 4.135 \text{ / radian}$$

$\boldsymbol{\eta_H}$:

An estimate of this quantity, which is the ratio of the dynamic pressure at tail to the free stream dynamic pressure, can be obtained using R.Ae.S data sheets (now called Engineering Sciences Data Unit, ESDU). This calculation needs the values of C_{DW}, A_W, λ_W etc. and the location of the tail with respect to wing. Reference 2 recommends a value between 0.9 and 1.0 A value of 0.95 has been assumed.

$\frac{d\varepsilon}{d\alpha}$: From Eqs. (3.11) to (3.15) of Ref.2

$$\left|\frac{d\varepsilon}{d\alpha}\right|_M = \left|\frac{d\varepsilon}{d\alpha}\right|_{M=0} \frac{C_{L\alpha W}|_M}{C_{L\alpha W}|_{M=0}}$$

$$\left|\frac{d\varepsilon}{d\alpha}\right|_{M=0} = 4.44\left[K_A K_\lambda K_H \sqrt{(\cos\Lambda_{c/4})}\right]^{1.19}$$

$$K_A = \frac{1}{A} - \frac{1}{1 + A^{1.7}} = \frac{1}{6.46} - \frac{1}{1 + 6.46^{1.7}} = 0.1145$$

$$K = \frac{10 - 3\lambda}{7} = \frac{10 - 3 \times 0.29}{7} = 1.3043$$

$$K_H = 1 - \frac{h_H / b}{\left(2\frac{l_H}{b}\right)^{1/3}}$$

h_H and l_H are defined in Fig 3.7 of Ref.2.

From Figs. 1, 2 and 3 and sections 2.1 and 2.4 :

$l_H = 59.7 + (6.77 / 4) - 27.48 - (10.2 / 4) = 31.36$ m

$h_H = 4.40$ m (estimated from Fig.1)

$$K_H = \frac{1 - (4.4/59.64)}{2(31.36/59.64)^{1/3}} = 0.911$$

$$\left.\frac{d\varepsilon}{d\alpha}\right|_{M=0} = 4.44\left[0.1135 \times 1.3043 \times 0.911 \times \sqrt{\cos 38.5}\right]^{1.19} = 0.3535$$

$$C_{L\alpha W}\big|_{M=0} = \frac{2\pi \times 6.46}{2 + \sqrt{\left[\frac{6.46^2}{1}\left(1 + \frac{\tan^2 35}{1}\right) + 4\right]}} = 4.005/\text{radian}$$

Hence,

$$\left.\frac{d\varepsilon}{d\alpha}\right|_{M=0.8} = 0.3535 \times \frac{4.90}{4.005} = 0.432$$

$$C_{L\alpha} = 4.90 + 4.135 \times 0.95 \times \frac{135.08}{550.5}(1 - 0.432) = 5.44/\text{radian}$$

$C_{L\alpha}$ based on wing area of 511 m^2 is: $5.44 \times \frac{550.5}{511} = 5.86/\text{radian}$

Remark:

The value of $C_{L\alpha}$ given in Ref.3 is 5.0. The calculated value is 17.2% higher. In this connection it may be noted that Ref.3 shows a slight decrease in $C_{L\alpha}$ for $0 \le M \le 0.8$ whereas the theory indicates significant increase in $C_{L\alpha}$ with Mach number (for wing $C_{L\alpha} = 4.005$ at M = 0 and $C_{L\alpha} = 4.90$ at M = 0.8). The difference between the theoretical and actual values is attributable to the flexibility of the airplane. For the sake of consistency, the theoretical value is used in subsequent calculations.

5.2 Estimation of $C_{D\alpha}$

$$C_{D\alpha} = \frac{\partial C_{D0}}{\partial \alpha} + \frac{2C_L C_{L\alpha}}{\pi Ae}$$

$\partial C_{Do} / \partial\alpha = 0$ at M = .8

Hence, $C_{D\alpha} = 2 \times 0.616 \times 5.44 \times 0.0665 = 0.446$

$C_{D\alpha}$ based on wing area of $511m^2 = 0.446 \times 550.5 / 511 = 0.480$

The value of $C_{D\alpha}$ from Ref.3 is 0.46

5.3 Estimation of $C_{m\alpha}$

From Eq.(3.16) of Ref.2

$$C_{m\alpha} = (\frac{dC_m}{dC_L})C_{L\alpha}$$

$$\frac{dC_m}{dC_L} = \bar{X}_{cg} - \bar{X}_{ac}; \ \bar{X}_{cg} = \text{(distance of c.g. from leading edge of m.a.c.)}/\bar{c}$$

$\bar{X}_{ac}$ = (distance of a.c. of airplane from leading edge of m.a.c.) / $\bar{c}$

$\bar{X}_{ac}$, without the contribution of power plant is given by (Eq.3.18 of Ref.2) i.e.,

$$\bar{X}_{ac} = \frac{\bar{X}_{acWB} + \frac{C_{L\alpha H}}{C_{L\alpha WB}} \eta_H \frac{S_H}{S} \bar{X}_{acH}\left(1 - \frac{d\varepsilon}{d\alpha}\right)}{1 + \frac{C_{L\alpha H}}{C_{L\alpha WB}} \eta_H \frac{S_H}{S}\left(1 - \frac{d\varepsilon}{d\alpha}\right)}$$

$$\bar{X}_{acWB} = \bar{X}_{acW} + \Delta\bar{X}_{acB}$$

$\bar{X}_{acW}$ is the distance of the aerodynamic centre of wing behind the leading edge of the mean aerodynamic chord divided by $\bar{c}$. $\Delta\bar{X}_{acB}$ is the shift in aerodynamic center due to fuselage.

The mean aerodynamic chord here equals 10.2 m.

The quantity $\bar{X}_{acW}$ depends on Mach number and the wing parameters. The steps are as follows.

(i) From Fig.3.9 of Ref.2, (X'_{ac}/c_r) is obtained. X'_{ac} is the distance of a.c. of the wing behind the leading edge of the wing root chord.

Here, $\beta / \tan\Lambda_{LE} = 0.6 / \tan 40.7 = 0.698$ and

$A \tan\Lambda_{LE} = 6.46 \tan 40.7 = 5.56$

Figure 3.9 of Ref.2 gives X'_{ac} / c_r for a few values of taper ratio (λ)

For $\lambda = 0.25$, $\frac{X'_{ac}}{c_r} = 0.95$ and for $\lambda = 0.33$, $\frac{X'_{ac}}{c_r} = 1.04$

For the present case of $\lambda = 0.29$, $\frac{X'_{ac}}{c_r} = 0.995$

(ii) $\bar{X}_{acW} = K_1 \left[\frac{X'_{ac}}{c_r} - K_2 \right]$

From Figs 3.10 and 3.11 of Ref.2, for $\lambda = 0.29$, $K_1 = 1.41$, and $K_2 = 0.759$

Hence, $\bar{X}_{acW} = 1.41[0.995 - 0.759] = 0.333$.

$\frac{X'_{ac}}{c_r}$ gives the location of the aerodynamic centre from the leading edge of the root chord.

The location of the leading edge of the root chord from nose is 17.08 m (Fig.2).

Hence, location of wing a.c from nose is: $17.08 + 0.995 \times 14.4 = 31.41$m.

$\mathbf{\bar{X}_{acH}}$:

$\bar{X}_{acH}$ is the shift in aerodynamic center due to horizontal tail $= X_{acH} / \bar{c}$

X_{acH} = Distance from leading edge of wing m.a.c. to a.c. to h.tail (Fig. 3.8 of Ref.2)

To determine X_{acH} , the quantity $\frac{X'_{ac}}{c_r}$ is obtained for h.tail. The procedure to determine

$\frac{X'_{ac}}{c_r}$ for h.tail is similar to that for $\frac{X'_{ac}}{c_r}$ of wing.

For h.tail , $\beta / \tan \wedge_{LE} = 0.6 / \tan 41^0 = 0.696$ and $A \tan \wedge_{LE} = 3.642 \times \tan 41^0 = 3.166$

From Fig.3.9 of Ref.2 , for $\lambda = 0.266$ $\frac{X'_{ac}}{c_r} = 0.618$ for the horizontal tail.

Further, in this case, $K_1 = 1.42$, $K_2 = 0.41$.

Hence, the location of the a.c of h.tail from the leading edge of h.tail mean aerodynamic chord is:

$1.42\ (0.618 - 0.41) = 0.295$ of $\bar{c}_H$.

The location of the leading edge of the root chord of the horizontal tail from nose is 55.8 m (Fig.5). Hence, the location of the a.c of h.tail behind the nose is:

(note $(c_r)_H$ is 9.62 m)

$$55.8 + 0.618 \times 9.62 = 61.75 \text{ m}$$

X_{acH} = distance from leading edge of wing m.a.c to a.c of tail

$$= 61.75 - (31.41 - 10.2 \times 0.333) = 33.74 \text{ m}$$

$$\bar{X}_{acH} = X_{acH}/\bar{c} = 33.74/10.2 = 3.307$$

$\Delta\bar{\mathbf{X}}_{\mathbf{acB}}$:

$$\Delta\bar{X}_{acB} = -\frac{(dM/d\alpha)_{fuselage+nacelle}}{\bar{q}\ S\ \bar{c}\ C_{L\alpha W}}\ ;\ \bar{q} = \frac{1}{2}\rho V^2 \text{ and } (dM/d\alpha)_{fuselage} = \frac{\bar{q}}{36.5}\sum_{i=1}^{n} W_f^2\ (X_i)\ \frac{d\varepsilon}{d\alpha}\Delta X_i$$

The procedure to calculate $(dM/d\alpha)_{fuselage}$ is explained in Fig 3.12 of Ref.2 .The calculations are done as presented in Table 2.

Here, c_f = 14.8 m, l_H = 27.1m.

Note: To obtain c_f and l_H the actual wing on the fuselage is considered (Fig. 2). These distances and X_i, ΔX_i and W_f in the table below are obtained using Figs. 2, 3 and 4.

i	X_i (m)	W_f (m)	ΔX_i (m)	X_i/c_f	$d\varepsilon/d\alpha$ for $C_{L\alpha W}$=0.080 per deg *	$d\varepsilon/d\alpha$ for $C_{L\alpha W}$=0.0855 per degree $	W^2_f $d\varepsilon/d\alpha$ ΔX
1	17.94	2.33	3.88	1.212	1.15	1.229	25.89
2	14.0	4.95	4.0	0.946	1.20	1.283	125.71
3	10.0	5.92	4.0	0.676	1.275	1.363	191.06
4	6.0	6.41	4.0	0.405	1.45	1.550	254.73
5	2.0	6.41	4.0	0.270	3.1	3.134	544.61
6	2.83	6.41	5.66	0.104	0.0593	0.0634	14.74
7	8.49	6.41	5.66	0.313	0.178	0.190	44.23
8	14.15	6.41	5.66	0.522	0.297	0.317	73.72
9	17.80	5.67	5.66	0.657	0.415	0.444	80.72
10	25.47	3.52	5.66	0.940	0.536	0.571	40.02
11	31.13	1.41	5.65	1.149	0.652	0.697	7.83

$$\sum W^2_f (d\varepsilon/d\alpha)\Delta X = 1403$$

* $d\varepsilon/d\alpha$ for $C_{L\alpha W}$ = 0.08 per deg is taken from Fig. 3.13 of Ref.2.

$ In the present case $C_{L\alpha W}$ is 4.9 / rad or 0.0855/ deg. The quantity $d\varepsilon/d\alpha$ in this case is evaluated as follows.

$$\left(\frac{d\varepsilon}{d\alpha}\right)_{C_{L\alpha W} = 0.0855} = \left(\frac{d\varepsilon}{d\alpha}\right)_{C_{L\alpha W} = 0.080} \times \frac{0.0855}{0.080}$$

Table 2 Calculation of fuselage contribution to $C_{m\alpha}$

Remark:

It is suggested that the location of region 5 in Fig.3 be taken in such a manner that X_i / c_f for this section is larger than 0.22. It is the smallest value (of X_i / c_f) for which $d\varepsilon/d\alpha$ is given in Fig 3.13 of Ref.2.

$(dM/d\alpha)_{nacelle}$:

This quantity is neglected as the nacelles are, in the side view, nearly in the region of the root chord (Fig.1).

$$(dM/d\alpha)_{fuselage+nacelle} \approx (dM/d\alpha)_{fuselage}$$

$$(dM/d\alpha)_{fuselage} = (\bar{q}/36.5)(1403) = 38.45\,\bar{q}$$

$$\Delta\bar{X}_{acB} = -\frac{38.45\,\bar{q} \times 57.3}{\bar{q} \times 550.5 \times 10.2 \times 4.9} = -0.080$$

$$\bar{X}_{acWB} = 0.333 - 0.080 = 0.253$$

Substituting various values yields :

$$\bar{X}_{ac} = \frac{0.253 + \frac{4.135}{4.90} \times 0.95 \times \frac{135.08}{550.5} \times 3.307(1 - 0.432)}{1 + \frac{4.135}{4.90} \times 0.95 \times \frac{135.08}{550.5} \times (1 - 0.432)} = \frac{0.253 + 0.3695}{1 + 0.1117} = 0.560$$

The location of c.g in Ref.3 is given as 0.25 of reference chord. The reference chord is 8.33m long. The quarter chord of this reference chord lies very close to the aerodynamic centre of wing. Hence, it is assumed that c.g is located at a.c..

Hence, $\bar{X}_{cg} = 0.333$

(dC_m / dC_L) for power-off condition:

$$\bar{X}_{cg} - \bar{X}_{ac} = 0.333 - 0.560 = -0.227$$

To calculate the correction for the effect of power, on dC_m / dC_L , one requires the knowledge of mass flow through the engine. In the absence of this data it is assumed that $(dC_m / dC_L) = 0.02$ per engine (Ref.7, section 5.7).

Hence, $dC_m / dC_L = -0.227 + 0.02 \times 4 = -0.147$

Therefore $C_{m\alpha} = -0.147 \times 5.44 = -0.814$ / rad.

$C_{m\alpha}$ based on $\bar{c}$ of 8.33 m and S of 511 m^2 is:

$$C_{m\alpha} = -0.814 \times \frac{550.5}{511} \times \frac{10.2}{8.33} = -1.074$$

Remark:

The value of $C_{m\alpha}$ from Ref.3 is -1.03 which is fairly close to the estimated value.

5.4 Estimation of C_{Du}

From p.4.1 of Ref.2, $C_{Du} = M \, \partial C_D / \partial M$

From Fig.6, the change in C_D with M at M = 0.8 is negligible.

Hence, $C_{Du} = 0$. From p.233 of Ref.3 $C_{DM} = 0.03$. This would give

$$C_{Du} = 0.8 \times 0.03 = 0.024$$