Contents

Appendix C

Drag polar,stability derivatives and characteristic roots of a jet airplane - 1

Lecture 37

Topics

1. Introduction

Estimation of the drag polar and stability derivatives is essential for calculation of the performance of the airplane and to know whether the airplane would be stable under various flight conditions. References1 and 2 contain methods for estimating the drag polar and stability derivatives of conventional subsonic airplanes. Though the presentation of the methods is very systematic, the students trying to use these books do find certain difficulties, as there is no worked out example to illustrate the procedure. The aim of this report is to fulfill this need. Towards this end the drag polar and stability derivatives of Boeing 747 are worked out in a flight at Mach number of 0.8 at 12,200 m (40,000 ft) altitude. This aircraft has been chosen as the details regarding the geometry and moments of inertia are readily available. Further Ref.3 has given the values of the stability derivatives from simulator data. These serve as a cross-check on the present estimates. Using the values of the stability derivatives the characteristic equations for longitudinal and lateral modes are formulated. The roots of the characteristic equations are also obtained.

2. Geometrical parameters of the airplane

The three-view drawing of the airplane as taken from Ref.3 is shown in the Fig.1. The overall dimensions are:

Overall length = 70.51 m, Fuselage length = 68.63 m,

Wing span = 59.64 m, Overall height = 19.33 m,

Wheel track = 11.02 m, Wheel base = 25.59 m.

The geometrical parameters of the major components are given below. During the subsequent calculations many geometrical parameters like root chord, tip chord etc. were needed. Wherever, they are not available in references 3 or 4 they (the dimensions) are obtained by measuring from the figures therein.

2.1. Wing geometry

The actual wing geometry is shown in Fig.2 by solid lines. However, the methods available for estimating aerodynamic characteristics of wing and tail assume that these surfaces are trapezoidal in shape. Figures 2.2 and 2.3 of Ref.2 illustrate the procedure for obtaining an equivalent trapezoidal shape.

Reference 3 gives the value for wing reference area and wing reference chord. It is assumed that these values are used in Ref.3, as reference area and length for the purpose of non-dimensionalization.

Reference area (S_R) = 511 m^2

Reference chord ($\bar{c}_R$) = 8.33 m

Wing span (b) = 59.64 m

Aspect ratio based on reference area = $59.64^2 / 511 = 6.96$

The following values are for the equivalent trapezoidal wing (Fig.2)

Root Chord (c_r) = 14.4 m

Tip chord (c_t) = 4.06 m.

Planform area (S) = (59.64 / 2) (14.4 + 4.06) = 550.5 m^2

Taper ratio (λ) = 0.29

Mean aerodynamic chord (m.a.c or $\bar{c}$) $= (2/3)c_r \dfrac{1+\lambda+\lambda^2}{1+\lambda} = 10.2$ m

Aspect ratio based on planform area = 6.46

Leading edge sweep ($\wedge_{LE}$) = 41.7^0

Quarter-chord sweep($\wedge_{c/4}$) = 38.5^0

Half-chord sweep ($\wedge_{c/2}$) = 35.0^0

Dihedral angle (Γ) from Ref.4 = 7^0

Assumed value of twist (θ) = -3^0 (linear)

Airfoil thickness ratio (Ref.4): Inboard = 13.44 %, Midspan = 7.8 % Outboard = 8%

Hence, average thickness ratio = ½ [(13.44 + 7.8) / 2 + (7.8 + 8) / 2)] = 9.26 %

Estimated wetted area of exposed wing $(S_{wet})_e$= 987.8 m^2

Location of aerodynamic center from nose = 31.41m (see section 5.3).

According to Ref.3 the location of c.g. is at 0.25 of $\bar{c}_R$. When measured from the figure it is almost at the same location as the a.c. (aerodynamic center).

Hence, c.g. location is at 31.41 m from nose.

Note: The estimated areas are obtained by the method of counting the squares.

2.2. Fuselage geometry

Remark:

Following Ref.2, the quantities referring to fuselage are denoted by suffix 'B'.

Length (l_B) = 68.63 m

Maximum height of fuselage (h) = 8.4 m

Maximum width of fuselage (w) = 6.41 m

Diameter of the cylindrical portion = 6.41m

Fuselage height at $l_B / 4$ from nose (h_1) = 7.8 m

Fuselage height at $3l_B / 4$ from nose (h_2) = 5.8 m

c.g. location from nose (x_m) = 31.41m

Estimated maximum cross-sectional area (S_B) = 32.92 m^2

Estimated wetted surface area (S_s) = 1125.6 m^2

Estimated fuselage side area (S_{BS}) = 416 m^2

Figure 3 gives the widths of fuselage at different locations

2.3. Nacelle geometry

The nacelle geometry is approximated as shown in Fig.4.

Number of nacelles = 4

Estimated maximum cross-sectional area = 5.31 m^2

Estimated wetted area = 36.97 m^2

Length of nacelle without exhaust cone = 5.52 m

2.4. Horizontal tail geometry

Remark:

Following Ref.2, the quantities referring to horizontal tail are denoted by suffix 'H'.

Span (b_H) = 22.17 m

Root chord $(c_r)_H$ = 9.62 m

Tip chord $(c_t)_H$ = 2.56 m

Taper ratio = 0.266

Mean aerodynamic chord $(\bar{c})_H = (2 / 3) \times 9.62\ (1 + 0.266 + 0.266^2) /(1 + 0.266) = 6.77$m

Plan form area (S_H) = 135.07 m^2

Aspect ratio = 3.642

Leading edge sweep = 41^0

Quarter chord sweep = 35.3^0

Half-chord sweep = 28.5^0

Wetted area of exposed tail surface = 270.15 m^2

Dihedral angle = 7^0

Location of tail aerodynamic center from nose = 61.75 m (see section 5.3)

See Fig.5 for some additional dimensions.

2.5. Vertical tail geometry

Figure 7.4 of Ref.2 gives guide-lines as to what should be taken as vertical tail area. From this description it appears that the root chord of the vertical tail should lie along the center line of the rear end of fuselage. Accordingly, the trapezoidal shape shown in Fig.3 is taken as vertical tail for the purpose of calculation of stability derivatives.

Root chord $(c_r)_V$ = 13.0 m

Tip chord $(c_t)_V$ = 3.85 m

Height (b_V) = 11.6 m

Vertical tail area (S_V) = 11.6 × (13.0 + 3.85) / 2 = 97.73 m^2

Aspect ratio $(A_V) = b_V^2 / S_V = 11.6^2 / 97.73$ = 1.38

Taper ratio (λ_V) = 0.296

Leading edge sweep angle = 46.26^0

Quarter chord sweep angle = 39.8^0

Half chord sweep angle = 33.7^0

Wetted area of exposed tail surface = 156.16 m^2

Fuselage depth at quarter chord of vertical tail = $2r_1$ = 5 m

Hence, $b_V / 2r_1$ = 2.32

Location of the leading edge of vertical tail root chord from nose = 52 m

Height of the a.c of the horizontal tail from center line (z_H) is taken as zero because a.c. of horizontal tail is below the center line. Location of the horizontal tail a.c. from leading edge of the vertical tail root chord (X) = distance of horizontal tail a.c. from nose minus the distance of leading edge of vertical tail from nose = 61.75 - 52.0 = 9.75 m.

Mean aerodynamic chord of vertical tail

$$= (2 / 3) \times 13 \times (1 + 0.296 + 0.296^2) / (1 + 0.296) = 9.253 \text{ m.}$$

Z_H = distance from body centerline to the quarter chord point of exposed wing root chord = 3.5 m

d = maximum body diameter at wing body intersection = 7.2 m

Location of the a.c of vertical tail from root chord of v.tail = 0.597 $(c_r)_V$ (see section 6.2)

Location of a.c. of v.tail from nose = 52 + 0.597 × 13 = 59.76 m

l_v = distance between c.g. and a.c. of vertical tail 59.76 - 31.41 = 28.35 m

Z_v = distance of vertical tail a.c. above c.g. = 6.35 m (assuming c.g. to lie on the center line of the cylindrical portion of fuselage).

3. Flight condition

In Ref.3 the estimated values are given for ten different flight conditions. The condition No.9 which corresponds to a flight Mach number of 0.8 at 40000 feet altitude is chosen for present calculations. This is one of the cases where compressibility effects are significant but, the flight Mach number is below critical Mach number.

Altitude = 40,000 ft = 12,200 m.

Mach number = 0.8

Kinematic viscosity = 4.7096×10^{-5} m^2/s

Speed of sound = 295.2 m/s.

Flight speed = 236.16 m/s.

Density ratio (σ) = 0.2460

Weight of the airplane = 636600 1bs = 2,852,129 N

c.g. location at 0.25 of reference chord i.e. 31.41 m behind nose.

Moments of Inertia:

I_{XX} = 0.182×10^8 slug ft^2 = 24726520 kg m^2

I_{YY} = 0.331×10^8 slug ft^2 = 44969660 kg m^2

I_{ZZ} = 0.497×10^8 slug ft^2 = 67522420 kg m^2

I_{XZ} = 970056 slug ft^2 = 1317918 kg m^2

Note: As per Ref.2 the drag polar is initially obtained at M = 0.6 and then the correction for increment in drag at transonic speed is applied.

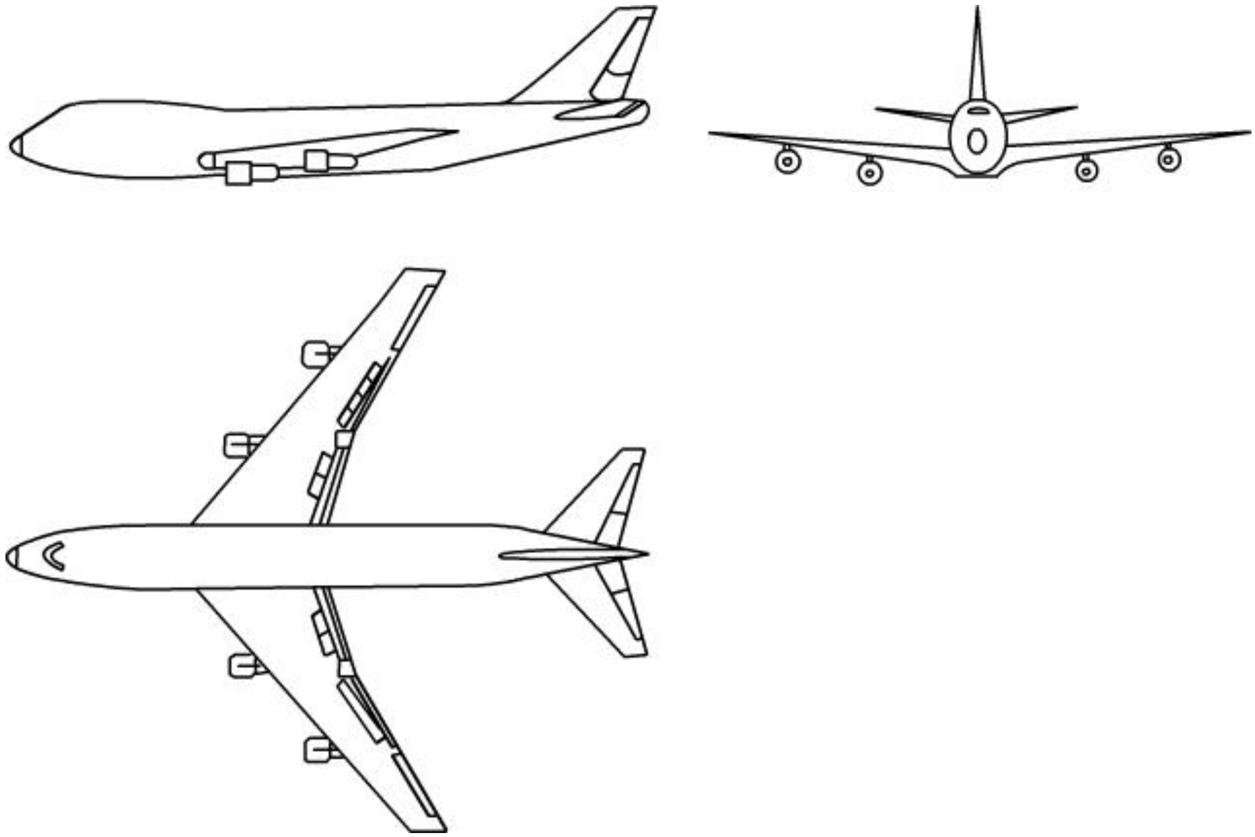


Fig.1. B-747 General arrangement

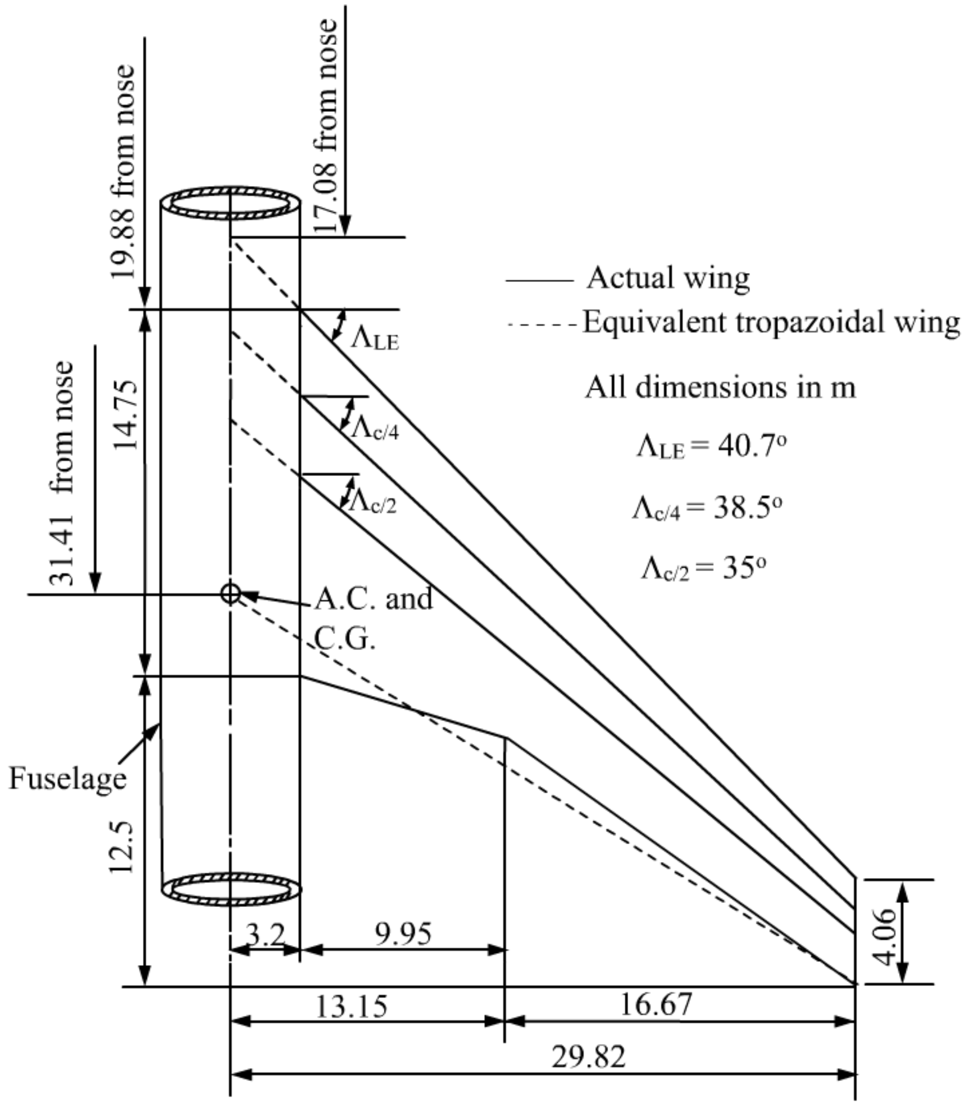
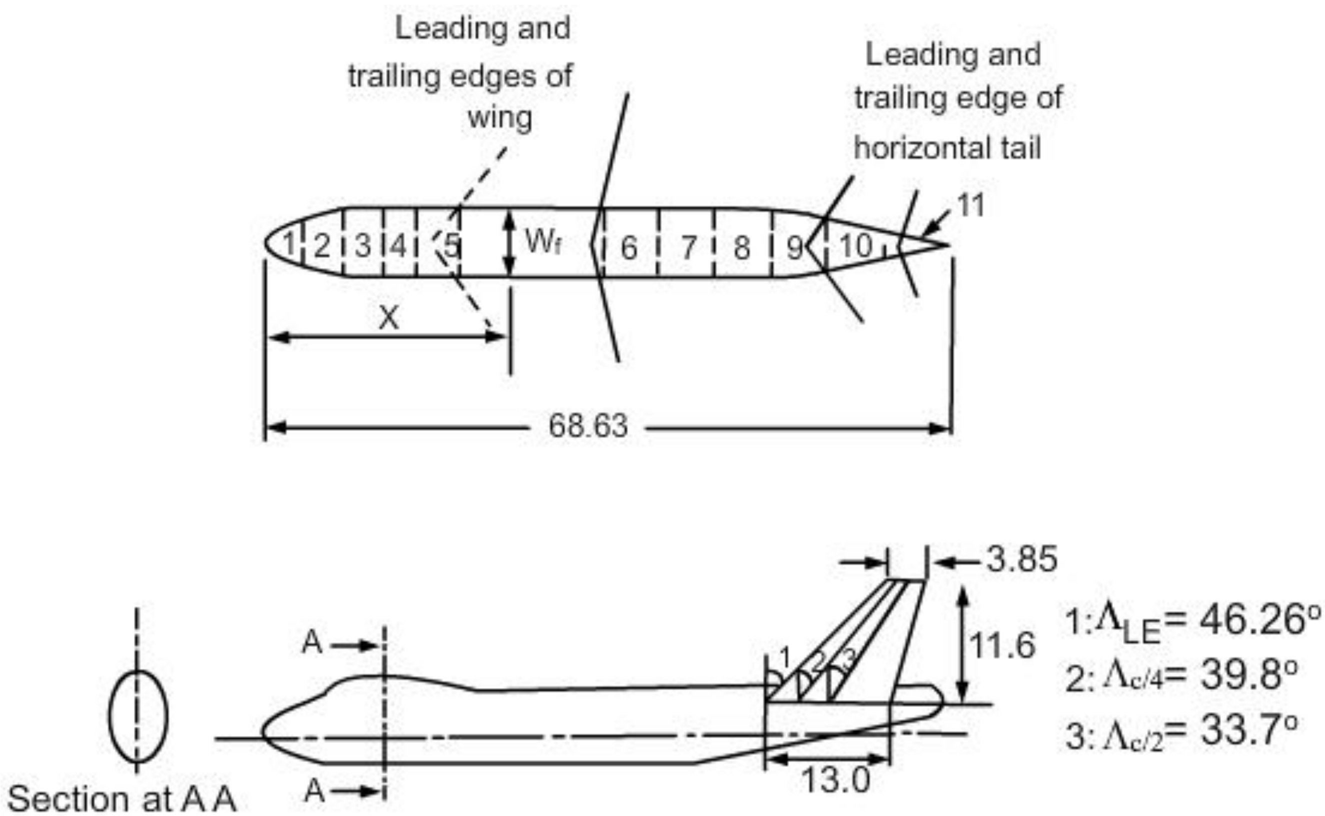


Fig.2 Wing geometry



X	0	3.21	7.05	10.9	13.5	49.38	55.79	62.20	68.63
W_f	0	3.85	5.43	6.10	6.41	6.41	5.48	2.56	0

All dimensions in m.

Fig.3 Fuselage and vertical tail geometry

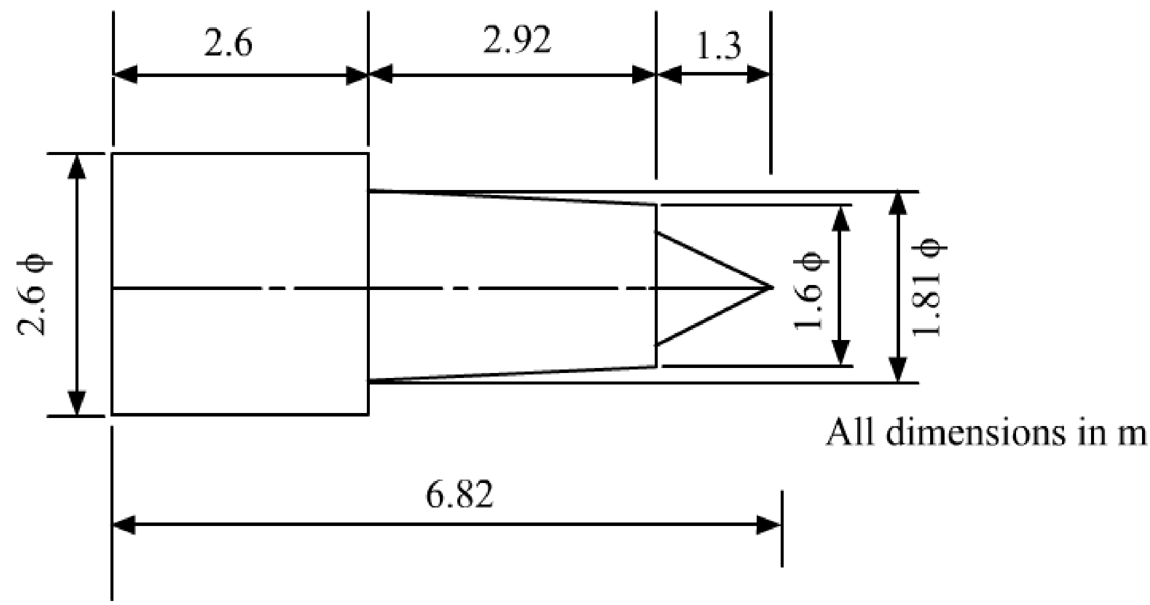
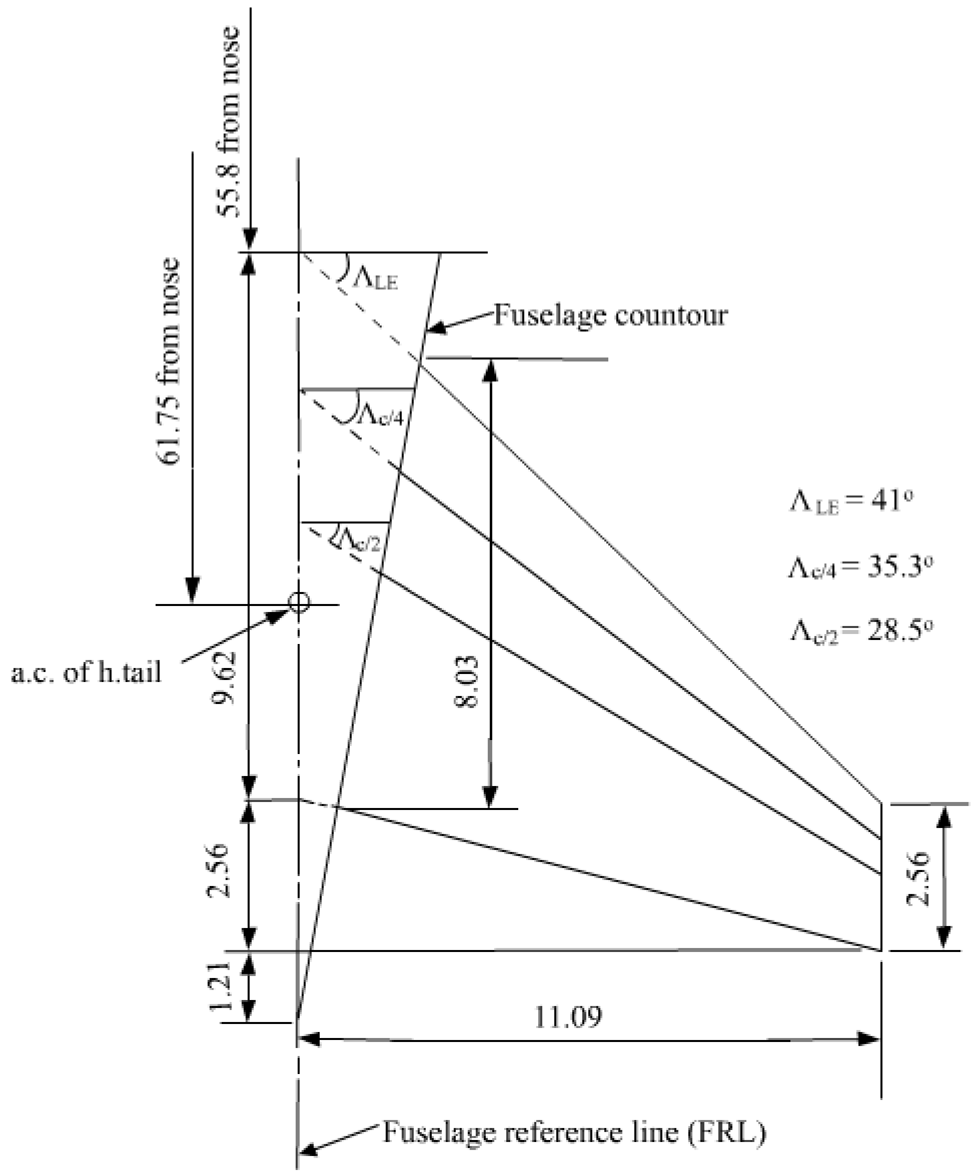


Fig.4 Nacelle geometry



All dimensions in m.

Fig.5 Horizontal tail geometry

4. Estimation of drag polar

The drag polar is assumed to be of the form:

$$C_D = C_D + (1/\pi Ae)\, C_L^2$$

The quantity C_{Do} is assumed to be given by:

$$C_{D0} = (C_{D0})_{WB} + (C_{D0})_H + (C_{D0})_V + (C_{D0})_N + (C_{D0})_{MISC}$$

Where suffices WB, H, V, N and MISC denote wing-body combination, horizontal tail , vertical tail, nacelle and miscellaneous respectively.

Note: During the calculations of drag polar and stability derivatives the equivalent wing and tail areas are used. However, for the sake of comparison, with the values given in Ref.3, the final values are recalculated using reference wing area of 511m^2 and reference chord of 8.33 m. In the case of lateral derivatives the reference length is the wing span, which is same in the present calculation and in Ref.3.

4.1 Estimation of $(C_{D0})_{WB}$

For a flight Mach number of 0.6, $(C_{D0})_{WB}$ is given by Eq.(3.6) of Ref.1 as :

$$(C_{D0})_{WB} = (C_{D0})_W + (C_{D0})_B \times (S_B / S_{REF})$$

The suffix B denotes fuselage and S_B is the maximum frontal area of fuselage. $(C_{D0})_W$ is given by Eq.(3.7) of Ref.1 as :

$$(C_{Do})_W = (C_{Df})_W + (C_{Dw})_W$$

Where, $(C_{Df})_W$ is the skin friction drag obtained by first determining the skin friction of a flat plate and then correcting it for the shape of airfoil. From Eq.(3.8) of Ref.1

$$(C_{Df})_W = C_{fw}\,[1 + L(t/c)]\,\frac{(S_{wet})_e}{S}$$

Where, C_{fW} is the skin friction drag of a rough flat plate over which a turbulent boundary layer is developing. The Reynolds number is generally calculated assuming flight Mach number to be 0.6 and reference length as mean aerodynamic chord of the exposed wing ($\bar{c}_e$).

Root chord of exposed wing = 12.6 m

Tip chord = 4.06 m, exposed span = 2 × 26.62 = 53.24m

Hence, taper ratio of exposed wing = 0.322 . Consequently, reference length (*l*) for wing is:

$l = \bar{c}_e = (2/3) \times 12.6 \times (1 + 0.322 + 0.322^2) / (1 + 0.322) = 9.06$ m

$R_e = 0.6 \times 295.2 \times 9.06 / (4.7096 \times 10^{-5}) = 34.07 \times 10^6$

To calculate the roughness parameter (l / k) the value of k, the roughness height, is taken as that corresponding to mass production spray paint. From table 3.1 of Ref.1, k = 1.2 $\times 10^{-3}$ inch or 3.05×10^{-5} m.

Hence, $l / k = 9.06 / (3.05 \times 10^{-5}) = 2.97 \times 10^5$

The roughness parameter decides the cut-off Reynolds number above which the skin friction drag coefficient remains constant.

From Fig.3.2 of Ref.1, $(R_e)_{cutoff} = 30 \times 10^6$

Then from Fig.3.1 of Ref.1, $C_{fw} = 0.00245$.

To choose L in the equation for $(C_{Df})_W$, it is assumed that $(t/c)_{max}$ occurs at x / c > 0.3 as would be the case with a supercritical airfoil. In this case L = 1.2. The thickness ratio (t / c) is 9.26 %,

$(S_{wet})_e$ is the wetted area of the exposed wing = 987.8 m^2

Hence, $(C_{Df})_W = 0.00245\ [1 + 1.2 \times 0.0926]\ (987.8 / 550.5) = 0.00488$

$(C_{Dw})_W$ is the wave drag at transonic speeds. It depends on (t /c) and aspect ratio. It requires construction of the $(C_{Dw})_W$ vs Mach number curve for the wing. This can be done by using Fig.3.7 of Ref.1 and the procedure given on pages 3.5 and 3.6 of Ref.1. The calculations are presented in Table 1.

Note:

$(t/c)^{\frac{1}{3}} = 0.0926^{\frac{1}{3}} = 0.4524$

$(t/c)^{\frac{5}{3}} = 0.0926^{\frac{5}{3}} = 0.01895$

$A(t/c)^{1/3} = 6.46 \times 0.4524 = 2.923$

M	$\frac{\lvert(M^2-1)\rvert^{1/2}}{(t/c)^{1/3}}$	$\frac{(C_{Dw})_W}{(t/c)^{5/3}}$ from Fig.3.7 of Ref.1	$(C_{Dw})_W$ for unswept wing
0.8	1.326	0.2	0.0038
0.84	1.2	0.5	0.0095
0.88	1.05	1.3	0.0246
0.92	1.87	2.5	0.0476
0.96	0.62	3.35	0.0625
1.0	0.0	3.5	0.0663
1.02	0.4442	3.5	0.0663

Table 1 Calculation of wave drag coefficient

The curve of $(C_{Dw})_W$ vs. M for the unswept wing is shown in Fig.6. From this curve the peak value of wave drag coefficient $(C_{Dw})_{peak}$, is 0.0665 and the corresponding Mach number, M_{CDpeak} , is 1.0. The critical Mach number $(M_D)_{\Lambda=0}$ is defined as the Mach number at which $\partial C_D / \partial M$ = 0.10. In the present case, from Fig.6 $(M_D)_{\Lambda=0}$ = 0.816.

The following values are obtained for a swept wing with $\Lambda_{c/4}$ = 38.5^0

$$(M_D)_{\Lambda=38.5} = \frac{0.816}{(\cos 38.5)^{1/2}} = 0.922$$

$$(M_{CD})_{peak} = \frac{1.0}{(\cos 38.5)^{1/2}} = 1.13$$

$$(C_{Dw})_{peak} = 0.0663 \times (\cos 38.5)^{2.5} = 0.036$$

Based on these values, the C_{Dw} vs. M curve for the swept wing is constructed and shown in Fig.6. It shows that the wave drag at chosen Mach number of 0.8 is negligible.

Hence, at M = 0.8,

$C_{DoW\,=}$ 0.00488 + 0 = 0.00488

$(C_{D0})_B$:

From Eq.(3.12) of Ref.1

$$(C_{Do})_B = (C_{Df})_B + (C_{Dp})_B + C_{Db}$$

$(C_{Df})_B$ is the skin friction drag given by:

$$(C_{Df})_B = C_{fB} \frac{S_S}{S_B}$$

C_{fB} is calculated in a manner similar to that for the wing and is based on (a) the fuselage length taken as reference length (*l*) and (b) the roughness height.

$$R_{eB} = \frac{0.6 \times 295.2 \times 68.63}{4.7096 \times 10^{-5}} = 258.1 \times 10^6$$

$$l / k = 68.63 / (3.05 \times 10^{-5}) = 2.25 \times 10^6$$

From Fig.3.2 of Ref.1, cut-off Reynolds number = 200×10^6

From Fig.3.1 of Ref.1, $C_{fB} = 0.0019$

Ss = wetted surface area of fuselage = 1125.6 m^2

S_B = maximum cross-sectional area = 32.96 m^2

Hence , $(C_{Df})_B = 0.0019 \times \frac{1125.6}{32.96} = 0.0649$

$\mathbf{(C_{Dp})_B}$: From Eq.(3.14) for Ref.1 it is given by as :

$$(C_{Dp})_B = (C_{fB})_{M=0.6} \left[\frac{60}{(l_B/d)^3} + 0.0025\,(l_B/d)\right] \frac{S_s}{S_B}$$

$$= 0.0019 \left[\frac{60}{(68.63/6.41)^3} + 0.0025\,(68.63/6.41)\right] \frac{1125.6}{32.96}$$

$$= 0.004908$$

C_{DB} is taken zero as there is no appreciable base area at the end of the fuselage. Hence

$(C_{D0})_B$ = 0.0649 + 0.004908 + 0 = 0.0698

From Fig.3.34 of R.1, $(\Delta C_D)_B$ for canopy = 0.005

Hence,

$(C_{D0})_B$ = 0.0698 + 0.0050 = 0.0748

Consequently, $(C_{Do})_{WB} = 0.00488 + 0.0748 \times \frac{32.96}{550.5} = 0.00936$

4.2 Estimation of $(C_{D0})_V$ and $(C_{D0})_H$

Since, the critical Mach numbers of the tail surfaces are likely to be higher than that of the wing, the procedure given in article 3.2.1 of Ref.1 is used to calculate the drag co-efficients.

From Eq.(3.15) of Ref.1 ,

$$(C_{Do})_P = C_{fP}\ [1+L(t/c) + 100\,(t/c)^4](R_{L.S})_p[\frac{[(S_{wet})_p]_e}{S_{REF}}]$$

C_{fP} is the skin friction drag for vertical or horizontal tail.

Calculations for horizontal tail

Exposed root chord = 8.1 m , Exposed tip chord = 2.56m

Taper ratio of exposed portion of h.tail = 2.56 / 8.1 = 0.316

Hence, the reference length (l) for h.tail, which is the mean aerodynamic chord for the exposed h.tail is :

$$(\bar{c}_e)_H = \frac{2}{3}\times 8.1(1+0.316+0.316^2)/(1+0.316) = 5.81\ \text{m};$$

$$R_{eH} = 0.6\times 295.2\times 5.81/(4.7096\times 10^{-5}) = 2.18\times 10^7$$

$$\frac{l}{k} = \frac{5.81}{3.05\times 10^{-5}} = 1.9\times 10^5$$

$$(R_e)_{cutoff} = 1.6\times 10^7$$

Hence, $c_{fH} = 0.0026$

Calculations for vertical tail

Exposed root chord = 11.86m , Exposed tip chord = 3.85m

Taper ratio of exposed portion of v.tail = 3.85 / 11.86 = 0.325

Hence, the reference length (l) for v.tail, which is the mean aerodynamic chord for the exposed v.tail is :

$$(\bar{c}_e)_V = \frac{2}{3}\times 11.86(1+0.325+0.325^2)/(1+0.325) = 8.54;$$

$$R_{eV} = 0.6\times 295.2\times 8.54/(4.7096\times 10^{-5}) = 4.28\times 10^7$$

$$l/k = 8.54/3.05\times 10^{-5} = 2.8\times 10^5$$

$$(R_e)_{cutoff} = 2.8\times 10^7$$

Hence, C_{fV} = 0.0025

The thickness ratios for both horizontal and vertical tails are taken as 8% from the following consideration. The critical Mach number of the tail surfaces is generally higher than that of the wing. Since, the quarter chord sweeps of wing and tails are almost than that the same for this airplane, it is assumed that t/c of tails is smaller than that.

$(R_{LS})_p$ = 1.24 for horizontal tail with $\Lambda_{0.3c} = 31.0^0$

= 1.23 for vertical tail with $\Lambda_{0.3c} = 32.5^0$

Hence,

$$(C_{D0})_H = 0.0026 \; [1 + (1.2 \times 0.08) + (100 \times 0.08^4)] \times 1.24 \times \frac{270.15}{550.5} = 0.00174$$

$$(C_{D0})_V = 0.0025 \; [1 + (1.2 \times 0.08) + (100 \times 0.08^4)] \times 1.23 \times \frac{156.16}{550.15} = 0.00096$$