Chapter 10

Lecture 33

Performance analysis VI – Take-off and landing –2

Topics

10.4.3 Distance covered and time taken during transition phase

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10.4.3 Distance covered and time taken during transition phase

From Fig.10.1a it is observed that during the transition phase the airplane changes the direction of flight and its speed would increase from V_1 to V_2. The height attained during this phase and the horizontal distance traversed can be obtained by treating the flight path as part of a circle. However, according to the procedure given in Royal Aeronautical Data sheets (now called Engineering Sciences Data Unit, ESDU for short), the increase in height during the transition phase is small and the horizontal distance (s_2) can be obtained by assuming that the work done by the engine is used in overcoming the drag and in increasing the kinetic energy of the airplane i.e.

$$T\,s_2 = D\,s_2 + \frac{W}{2g}\left(V_2^2 - V_1^2\right)$$

$$\text{Or}\ \ s_2 = \frac{W}{2g}\,\frac{\left(V_2^2 - V_1^2\right)}{T - D} \qquad (10.10)$$

T and D in Eq.(10.10) are evaluated at the mean speed during this phase i.e., at $(V_2 + V_1) / 2$. The time taken (t_2) in transition is given by:

$$t_2 = \frac{s_2}{0.5\,(V_2 + V_1)} \qquad (10.11)$$

V_1 generally lies between (1.15 to 1.2) V_S and V_2 is (1.05 to 1.1) V_1.

10.4.4 Distance covered and time taken during climb phase

Since the vertical height covered during the transition has been ignored, the horizontal distance covered in climb phase (s_3) is the distance covered while climbing to screen height i.e.

$$s_3 = (\text{Screen height}) / \tan\gamma \qquad (10.12)$$

where, γ is the angle of climb at velocity V_2 :

$$\sin\gamma = \frac{T-D}{W}$$

where, T and D are evaluated at V_2.

The time taken in climb phase (t_3) is:

$$t_3 = (\text{Screen height}) / V_2 \sin\gamma \qquad (10.13)$$

Hence, the take-off distance (s) and the time taken for it (t) are given by :

$$s = s_1 + s_2 + s_3 \qquad (10.14)$$

$$t = t_1 + t_2 + t_3 \qquad (10.15)$$

Example 10.1

A jet airplane with a weight of 441, 450 N and wing area of 110 m^2 has a tricycle type landing gear. Its C_{Lmax} with flaps is 2.7. Obtain the take-off distance to 15 m screen height and the time taken for it. Given that:

(i) $V_1 = 1.16\ V_S$

(ii) $V_2 = 1.086\ V_1$

(iii) C_L during ground run is 1.15

(iv) Drag polar with landing gear and flaps deployed is $C_D = 0.044 + 0.05C_L^2$

(v) Thrust variation during take-off can be approximated as :

$T = 128{,}500 - 0.0929\ V^2$; where V is in kmph and T is in Newton (vi)Take-off takes place from a level, dry concrete runway (μ = 0.02) at sea level.

Solution:

$$\left(C_{Lmax}\right)_{T0} = 0.8\, C_{Lmax} = 0.8 \times 2.7 = 2.16$$

$$V_S = \left(\frac{2W}{\rho S\, C_{Lmax}}\right)^{1/2} = \left(\frac{2 \times 441450}{1.225 \times 110 \times 2.16}\right)^{1/2} = 55.08 \text{ m/s}$$

Hence,

V_1 = 1.16 x 55.08 = 63.89 m/s

and V_2 = 1.086 x 63.89 = 69.38 m/s.

For C_L = 1.15,

$C_D = 0.044 + 0.05 \times 1.15^2 = 0.1101$

Hence, $T - D - \mu\,(W-L) = T - \mu W - \frac{1}{2}\rho V^2 S\,\{C_D - \mu C_L\}$

$= 128500 - 0.0929\,(3.6V)^2 - 0.02 \times 441450$

$\quad - 0.5 \times 1.225 \times V^2 \times 110\,(0.1101 - 0.02 \times 1.15)$

$= 119671 - 7.0752\, V^2$

Using Eqs.(10.6) and (10.7) the ground run (s_1) and time taken for it (t_1) are:

$$s_1 = \frac{441450}{2 \times 9.81 \times 7.0752} \ln\,[119671/(119671 - 7.0752 \times 63.89 \times 63.89)] = 878.32 \text{ m}$$

$$t_1 = \frac{441450}{2 \times 9.81\,(119671 \times 7.0752)^{\frac{1}{2}}} \ln\left[\frac{(119671)^{\frac{1}{2}} + (7.0752)^{\frac{1}{2}} \times 63.89}{(119671)^{\frac{1}{2}} - (7.0752)^{\frac{1}{2}} \times 63.89}\right] = 26.34 \text{ s.}$$

The distance covered during transition (s_2) is obtained as follows.

$$\text{Average speed during transition} = \frac{63.89 + 69.38}{2} \text{ m/s} = 66.635 \text{ m/s} = 239.9 \text{ kmph}$$

Hence, thrust during this phase = $128500 - 0.0929 \times 239.9^2$ = 123,153 N

To get the drag during this phase it is assumed that C_L equals C_{LTO} and it is given by :

$$C_{LTO} = C_{Lmax}\,(V_S / V_1)^2 = 2.16 / (1.16)^2 = 1.605$$

Assuming the same drag polar as in the ground run gives:

$C_D = 0.044 + 0.05 \times 1.605^2 = 0.1728$

Hence, $D = 0.5 \times 1.225 \times (66.635)^2 \times 110 \times 0.1728 = 51695$ N

Using Eqs.(10.10) and (10.11) gives :

$$s_2 = \frac{441450}{2 \times 9.81}\left(\frac{69.38^2 - 63.89^2}{123159 - 51695}\right) = 230.4 \text{ m}$$

and $t_2 = 230.4 / 66.635 = 3.46$ s.

During the climb phase, V = 69.38 m/s = 249.77 kmph.

Hence, $T = 128500 - 0.0929 \times 249.77^2 = 122704$ N

To get the drag in the climb phase the lift coefficient should be known. For this purpose L is taken roughly equal to W.

$$\text{Hence, } C_L = W/\left(\frac{1}{2}\rho V^2 S\right) = \frac{441450}{\frac{1}{2} \times 1.225 \times 110 \times 69.38^2} = 1.36$$

Consequently, $C_D = 0.044 + 0.05 \times 1.36^2 = 0.1365$

and $D = 0.5 \times 1.225 \times (69.38)^2 \times 110 \times 0.1365 = 44269$ N

$$\text{Hence, } \sin\gamma = \frac{122704 - 44269}{441450} = 0.1777$$

Consequently, $\tan\gamma = 0.1805$.

Using Eqs.(10.12) and (10.13) gives:

$s_3 = 15/0.1805 = 83.1$ m

and $t_3 = 15/(69.38 \times 0.1805) = 1.20$ s.

Finally, $s = s_1 + s_2 + s_3 = 878.32 + 230.4 + 83.1 = 1192$ m

and $t = t_1 + t_2 + t_3 = 26.34 + 3.46 + 1.20 = 31.0$ s.

Answers:

Take off distance = 1192 m ; Time taken for take-off = 31 s.

10.4.5 Parameters influencing take-off run

The major portion of the take-off distance is the ground run. Hence if ground run is reduced, the take-off distance is also reduced. From Eq.(10.8), it is observed that the distance s_1 is given by :

$$s_1 = \frac{W}{g}\frac{V_1^2}{\left[T-D-\mu(W-L)\right]_{avg}} \qquad (10.16)$$

Let V_1 = 1.1 V_S. Recalling, $V_S = \sqrt{\frac{2W}{\rho S C_{Lmax}}}$, Eq.(10.16) can be rewritten as :

$$s_1 = \frac{1.21 \times 2W^2}{2g\rho S C_{Lmax}\left[T-D-\mu(W-L)\right]_{avg}}$$

$$= \frac{1.21(W/S)}{g\rho C_{Lmax}\left[(T/W)-(D/W)-\mu(1-L/W)\right]_{avg}} \qquad (10.17)$$

The following observations can be made from Eq.(10.17).

i) The ground run increases when the wing loading (W / S) increases.

ii)The ground run also increases when ρ decreases. Since ρ decreases with altitude, the take-off distance will be more when the altitude of the airport increases.

iii)The ground run decreases as C_{Lmax} increases. Hence, the high speed airplanes which have high wing loading from consideration of cruise, employ elaborate high lift devices to increase C_{Lmax}.

iv)The take-off run decreases by increasing the accelerating force which mainly depends on (T/W). It may be recalled from subsection 4.3.5 that the thrust of a jet engine can be increased temporarily by using an afterburner. The thrust can also be augmented by using an auxiliary rocket fired during the take-off run. In shipboard airplanes a catapult is used to augment the accelerating force.

10.4.6 Effect of wind on take-off run

While discussing the range performance it was shown, with the help of a derivation in section 7.8, that the distance covered with respect to the ground decreases when the flight takes place in the presence of head wind. Same effect occurs during the take-off and the take-off distance reduces in the presence of head wind. In a hypothetical case of head wind being equal to the stalling speed (V_S), the airplane can get airborne without having to accelerate along the ground. A quantitative estimate of the effect of wind velocity (V_w) on s_1, can be obtained from Eq.(10.4), by replacing the limits of integration from (0 to V_1) by (V_w to V_1) i.e. :

$$(s_1)_{\text{with wind}} = \frac{W}{g}\int_{V_w}^{V_1} \frac{V\,dV}{T - D - \mu(W - L)}$$

Thus, the head wind, though bad for range, is beneficial during take-off as it reduces the take-off distance.

Airports have a device to indicate the direction of wind. The take-off flight takes place in such a manner that the airplane experiences a head wind. This is referred to as 'Take-off into the wind'.

10.4.7 Guidelines for estimation of take-off distance

In subsections 10.4.1, 10.4.3 and 10.4.4, a procedure to estimate the take-off distance has been presented. However, it is based on several assumptions and consequently has significant amount of uncertainty. In actual practice, there would be further uncertainty due to factors like condition of the runway surface (wet or dry), and piloting technique. Hence, for the purpose of preliminary design of airplane, the following guidelines can be used.

For airplanes with engine-propeller combination, the Federal Aviation Regulations designated as FAR-23 (Ref.10.1) are used. Under these regulations, the take-off distance to attain 50 feet (or 15 m) is obtained under certain prescribed conditions. This distance is denoted here by 's_{to23}'. Reference 10.2 has estimated s_{to23} for several airplanes. It is observed (Ref.10.2) that s_{to23} is related to the following parameter.

$$\frac{(\frac{W}{S})_{T0}\ (\frac{W}{P})_{T0}}{\sigma\ C_{LT0}}$$

where,

$(W/S)_{T0}$ = wing loading based on take-off weight.

$(W/P)_{T0}$ = power loading based on take-off weight and sea level static power output.

σ = density ratio = ρ/ρ_0

C_{LT0} = Lift coefficient in take off configuration (about 80% of C_{Lmax} in landing configuration)

The above quantity is called take-off parameter for FAR-23 and denoted by 'TOP_{23}' i.e.

$$TOP_{23} = \frac{(\frac{W}{S})_{T0}\ (\frac{W}{P})_{T0}}{\sigma\ C_{LT0}} \quad (10.18)$$

Based on the data of Ref.10.2, the following relationship has been deduced in Ref.3.18, pt.I, chapter 3.

$$s_{to23}\ \text{(in ft)} = 8.134\ TOP_{23} + 0.0149\ TOP_{23}^2 \quad (10.19)$$

where, W/S is in lbs / ft^2, W in lbs and P in hp.

When SI units are used this relationship takes the following form.

$$s_{to23}\ \text{(in m)} = 8.681\text{x}10^{-3}\ TOP_{23} + 5.566\text{x}10^{-8}\ TOP_{23}^2 \quad (10.20)$$

where W / S is in N / m^2 , W in N and P in kW.

Example 10.2

Consider an airplane with the following features.

W/S = 2400 N / m^2 , W/P = 24 N /kW , C_{LTO} = 1.6 and σ = 1.

Estimate the take-off distance for this airplane.

Solution :

The parameter 'TOP_{23}' in this case is :

TOP_{23} = 2400 x 24/(1 x 1.6) = 36000 $N^2/(m^2\ kW)$

Using Eq.(10.20) gives,

$s_{to23} = 8.681\times10^{-3} \times 36000 + 5.566\times10^{-8} \times 36000^2$ = 385.9 m or 1260 ft.

Answer : Take-off distance = 385.9 m or 1260 ft.

As regards the airplanes with jet engines, the take-off parameter (TOP) is defined as :

$$TOP = \frac{(\frac{W}{S})_{T0}}{\sigma C_{LT0}(\frac{T}{W})_{T0}} \qquad (10.21)$$

where, T = sea level static thrust.

Reference 3.9 chapter 5, gives a curve as guideline for s_{to} in feet and TOP in lbs / ft^2. However, when a second order equation is fitted to that curve, the relationship can be expressed in SI units in the following form.

$$s_{to}\text{ (in m)} = 0.1127\ TOP + 1.531 \times 10^{-6}\ TOP^2 \qquad (10.22)$$

Example 10.3

Consider a jet airplane with the following features.

$W/S = 5195\ N/m^2$, $T / W = 0.3$, $C_{LT0} = 2.16$ and $\sigma = 1$.

Estimate the take-off distance.

Solution :

In this case TOP is :

$$TOP = \frac{5195}{1\times2.16\times0.3} = 8017\ N/m^2$$

From Eq.(10.22) :

$s_{to} = 0.1127\times8017 + 1.531 \times 10^{-6} \times 8017^2$ = 1002 m.

Answer : Take-off distance = 1002 m.