

## Chapter 9

### Lecture 31

#### Performance analysis V – Manoeuvres – 4

#### Topics

#### 9.4 Miscellaneous topics – flight limitations, operating envelop and V-n diagram

9.4.1 Flight limitations

9.4.2 Operating envelop

9.4.3 V-n diagram

#### 9.4 Miscellaneous topics

A flight is called free flight when the airplane is away from the influence of the ground i.e. it is at a height more than a few wing spans above the ground. The performance in level flight, climb, turn etc. come under this category. In contrast, the analysis of take-off and landing requires consideration of the influence of proximity of ground. The discussion of performance in free flight is concluded in this section by describing aspects like flight limitations, operating envelop and V-n diagram. Chapter 10 describes the performance in take-off and landing.

#### 9.4.1 Flight Limitations:

In chapters 5 to 8 and the previous subsections of this chapter, the performance of an airplane in free flight has been discussed under categories of level flight, climb, accelerated flights and manoeuvres. The important aspects of these analyses are generally brought out in a diagram which is called here as 'Variations of characteristics velocities'. Figure 9.7a shows the variations of  $V_{\max}$ ,  $(V_{\min})_e$ ,  $V_S$ ,  $V_{(R/C)\max}$ ,  $V_{\gamma\max}$ ,  $V_{rmin}$  and  $V_{\psi\max}$  with altitude for a typical subsonic jet airplane. Figure 9.7b presents similar plots for a piston engined airplane.

Flight dynamics-I  
Chapter-9

It has been pointed out earlier that (a) the maximum lift coefficient limits the minimum speed in level flight ( $V_S$ ), the minimum radius of turn ( $r_{\min}$ ) and the maximum rate of turn ( $\dot{\psi}_{\max}$ ), (b) the power output limits the maximum speed ( $V_{\max}$ ), the minimum speed ( $V_{\min}e$ ), the maximum angle of climb ( $\gamma_{\max}$ ), the maximum rate of climb ( $(R/C)_{\max}$ ),  $r_{\min}$  and  $\dot{\psi}_{\max}$ , (c) the maximum allowable load factor,  $(n_{\max})_{\text{str}}$ , limits  $r_{\min}$  and  $\dot{\psi}_{\max}$ . In addition to these, the performance of the airplane may also be limited by considerations like buffeting, sonic boom, maximum dynamic pressure ( $q$ ) limit and aerodynamic heating. These limitations are briefly described below.

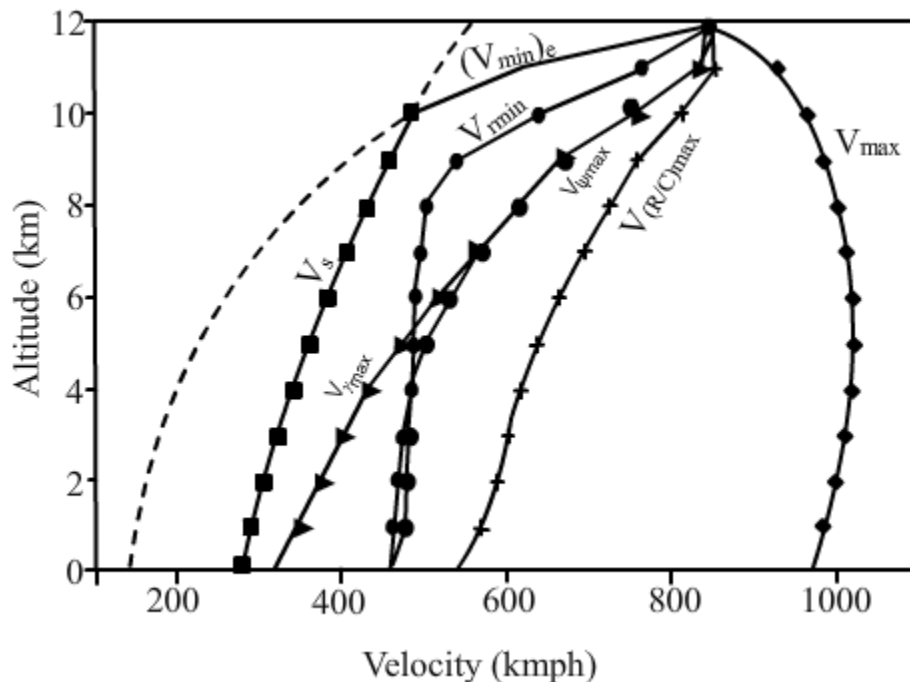


Fig.9.7a Variations of characteristic velocities – Jet transport

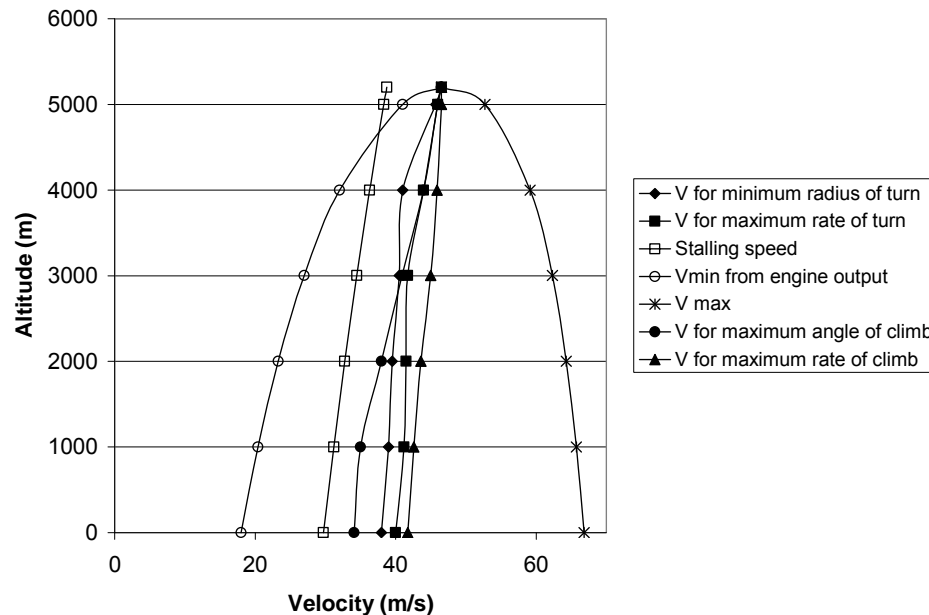


Fig.9.7b Variations of characteristic velocities – Piston engine airplane

(i) Buffeting is an irregular oscillation of a part of an airplane, caused by the passing of separated flow from another component. For example, the horizontal tail experiences buffeting when the separated flow from the wing passes over it (horizontal tail). This happens when the wing is at a high angle of attack or the shock stall takes place on it in the transonic flow regime. To prevent buffeting, the permissible value of  $C_{L_{max}}$  may be limited. This in turn would affect  $V_S$ ,  $r_{min}$  and  $\psi_{max}$ .

(ii) The sonic boom problem is encountered when an airplane flies at supersonic speed at low altitudes. The shock waves created by an airplane, when it is flying at a supersonic speed, coalesce and form two waves across which there is a finite pressure rise (overpressure). When these waves reach the ground each of them is perceived as an explosive like sound called sonic boom or sonic bang. The intensity of the boom depends on the size and shape of the airplane, its flight altitude and the atmospheric conditions. It increases with the increase in the size of the airplane and decreases with the increase of the altitude of the flight. An overpressure in excess of about  $100 \text{ N/m}^2$  is quite annoying and may cause

vibrations of buildings and rattling of window panes. To keep the overpressure on the ground within socially acceptable limits, the supersonic transport airplanes are generally not permitted to cross Mach number of one below tropopause and they cruise at altitudes of 16 to 20 km.

(iii) The airplanes are generally designed for a dynamic pressure ( $q = \frac{1}{2}\rho V^2$ ) of  $100,000 \text{ N/m}^2$ . This limit would not permit attainment of high supersonic Mach number at low altitudes.

(iv) As the flight Mach number increases, the stagnation temperature ( $T_s$ ) on the surface increases. It is given by:

$$T_s = T_{amb} \left(1 + \epsilon \frac{\gamma - 1}{2} M^2\right) \quad (9.32)$$

where,  $T_{amb}$  is the ambient temperature and  $\epsilon$  is the recovery factor which has a value of around 0.9 for turbulent boundary layer on the surface. The maximum stagnation temperature ( $T_s$ ) may be limited from the consideration of material used for the fabrication of the airplane. This would limit the maximum permissible Mach number.

(v) Reference 3.9, chapter 17, mentions about other limits like engine relight limit, pilot ejection altitude and duct pressure limit. The minimum speed from engine relight limit is encountered in some cases at high altitudes where enough air may not be available to restart the engine in the event of flame-out. The highest altitude may be limited to about 15 kms which is the the highest altitude at which ejection by the pilot is permitted.

#### 9.4.2 Operating envelop

The maximum speed and the minimum speed of the airplane can be calculated from the level flight analysis. However, the attainment of maximum speed may be limited by the considerations mentioned in the previous subsection. A diagram which indicates the range of flight speeds permissible for an airplane at different altitudes is called 'Operating envelope'. Typical operating

Flight dynamics-I  
Chapter-9

envelope for a military airplane is shown in Fig.9.8. Explanation of the curves in this figure is as follows.

(i)The curve ABCDE is the level flight boundary based on the engine output. The portion ABC is the  $V_{max}$  or  $(M_{max})$  boundary. The portion CDE is the  $(V_{min})_e$  or  $(M_{min})$  boundary, limited by the engine output. It may be mentioned that for these

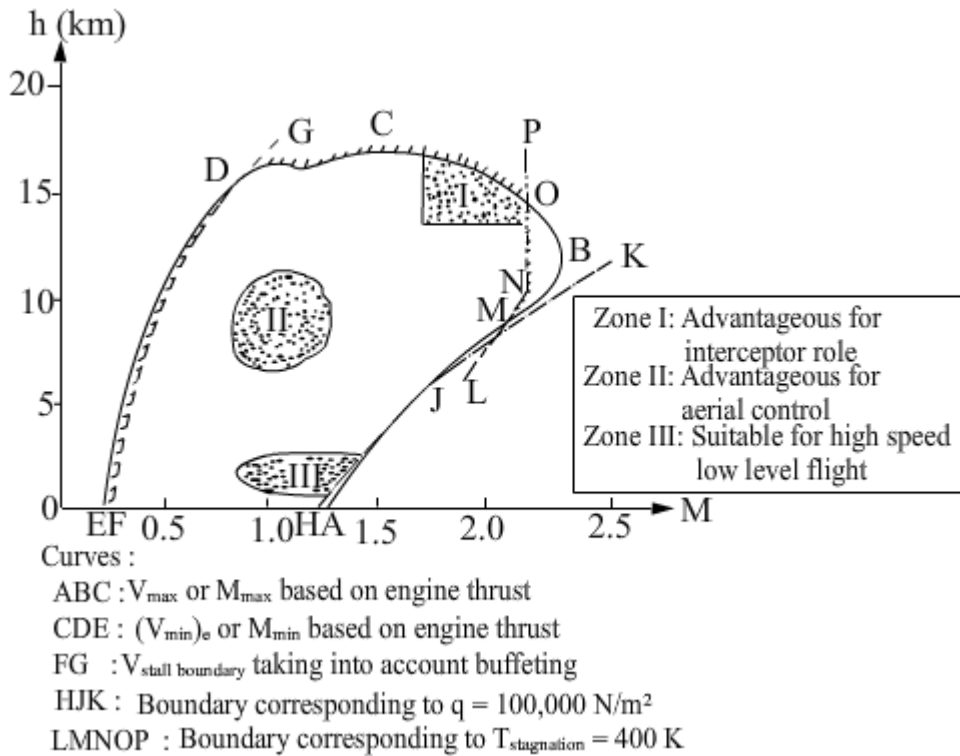


Fig.9.8 Operating envelope of a military airplane - Schematic

curves (ABC and CDE) the engine output is with the afterburner on. On this boundary (ABCDE) the specific excess power ( $P_S$ ) is zero.

(ii)The curve FG is the line representing stalling speed ( $V_S$ ).

$$V_S = \sqrt{\frac{2W}{\rho S C_{Lmax}}} ; C_{Lmax} \text{ without flap}$$

Recalling that when Mach number exceeds 0.5, the maximum lift coefficient ( $C_{Lmax}$ ) decreases due to shock stall or buffeting. The line FG includes this effect when Mach number corresponding to  $V_S$  is more than 0.5.

(iii) The line HJK represents the dynamic pressure ( $q$ ) limit corresponding to  $q$  of  $100,000 \text{ N/m}^2$ .

(iv) The line LMNOP represents the boundary corresponding to stagnation temperature ( $T_s$ ) of  $400\text{K}$ . It may be pointed out that  $T_{amb}$  and hence the speed of sound change with altitude in troposphere. They are constant in lower stratosphere. Hence, the allowable flight Mach number, for stagnation temperature to be below allowable value, changes with altitude.

The flight envelope taking into account the above limits is the curve FDCONMJH.

**Remark:**

Figure 9.8 also shows zones marked as : (I) advantageous for interceptor role, (II) advantageous for aerial combat and (III) suitable for high speed low altitude flight.

It may be added that for the interceptor role, it is advantageous if the airplane flies at high altitude and high speed (zone I in Fig.9.8).

For aerial combat the manoeuvrability, which is measured mainly by the rate of turn, is important. It may be recalled from subsection 9.3.3 that the rate of turn is low at (a) altitudes near the ceiling and (b) flight speeds close to  $V_{max}$  and  $V_{min}$ . Further, the aerial combat cannot take place at very low altitudes. Hence, the aerial combat zone is the region marked as (II) in Fig.9.8.

For airplanes used as ground attack fighter, the ability to fly at high speed and at low altitude is important. Zone (III) in Fig.9.8 is appropriate for these airplanes.

### 9.4.3 V-n diagram

The load factor ( $n$ ) has already been defined as the ratio of lift and weight i.e.  $n = L / W$ . In level flight  $n = 1$ . However, as pointed out in subsections 9.2.3 and 9.3.3 the value of 'n' during a manoeuvre is greater than one. Hence, the structure of the airplane must be designed to withstand the permissible load factor. Further, when an airplane encounters a gust of velocity  $V_{gu}$  (see Fig.7.1b) the angle of attack of the airplane would increase by  $\Delta\alpha = V_{gu} / V$ . This increase in angle of attack, would increase the lift by  $\Delta L$ , given by :

Flight dynamics-I  
Chapter-9

$$\begin{aligned}\Delta L &= \frac{1}{2} \rho V^2 S C_{L\alpha} \Delta\alpha = \frac{1}{2} \rho V^2 S C_{L\alpha} V_{gu} / V \\ &= \frac{1}{2} \rho V S C_{L\alpha} V_{gu}\end{aligned}\quad (9.33)$$

$$\text{Hence, } \Delta n = \Delta L / W = \frac{1}{2} \rho V S C_{L\alpha} V_{gu} / W \quad (9.34)$$

From Eqs.(9.33) and (9.34),  $\Delta L$  increases with  $V_{gu}$ . Further, for a given  $V_{gu}$ , the values of  $\Delta L$  &  $\Delta n$  increase with flight velocity. An airplane must be designed to withstand the gust loads also.

In aeronautical engineering practice, the load factors due to manoeuver and gust are indicated by a diagram called 'Velocity-load factor diagram or V-n diagram'. A typical V-n diagram is shown in Fig.9.9. This diagram can be explained as follows.

(i) Curves OIA and OHG : The lift (L) produced by an airplane is given by

$L = \frac{1}{2} \rho V^2 S C_L$ . It should be noted that (i)  $C_L \leq C_{L_{max}}$  and (ii) at stalling speed ( $V_S$ ),  $L = W$  and  $n = 1$ . However, if the airplane is flown with  $C_L = C_{L_{max}}$  at

speeds higher than  $V_S$ , then (a) L will be more than W and (b) L or n would be

proportional to  $V^2$ . This variation is a parabola and is shown by curve OIA in Fig.9.9. In an inverted flight the load factor will be negative and the V vs n curve in such a flight is indicated by the curve OHG in Fig.9.9. It may be mentioned that an airplane can fly only at  $V \geq V_S$  and hence the portions OI and OH in Fig.9.9 are shown by chain lines.

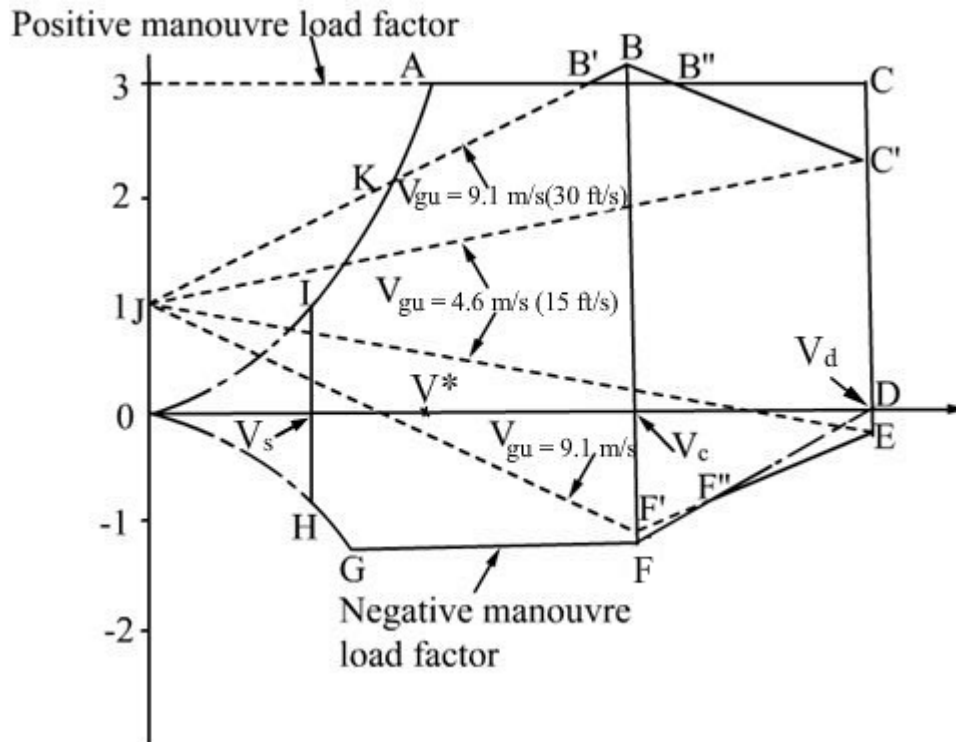


Fig.9.9 A typical V-n diagram

(ii) Positive and negative manoeuvre load factors : An airplane is designed to withstand a certain permissible load factor. Higher the permissible load factor, heavier will be the weight of airplane structure. Hence, for actual airplanes the manoeuvre load factor is limited depending on its intended use. Federal Aviation Administration (FAA) in USA and similar agencies in other countries, prescribe the values of permissible manoeuvre limit load factors ( $n_{\text{positive}}$  and  $n_{\text{negative}}$ ) for different categories of airplanes. Table 9.1 gives typical values. A limit load is obtained by multiplying the limit load factor with the weight ( $W$ ). The airplane structure is designed such that it can withstand the limit load without yielding. The ultimate load factor, in aeronautical practice, is 1.5 times the limit load factor. The ultimate load is obtained by multiplying the ultimate load factor with the weight ( $W$ ). The airplane structure is designed such that it can withstand the ultimate load without failing, though there may be permanent damage to the structure.



Type of airplane	$n_{\text{positive}}$	$n_{\text{negative}}$
General aviation-non aerobatic	2.5 to 3.8	-1
Transport	3 to 4	-1
Fighter	6 to 9	-3

Table 9.1 Typical limit load factors

In Fig.9.9,  $n_{\text{positive}} = 3$  and  $n_{\text{negative}} = -1.2$  have been chosen; the actual values depend on the weight of the airplane and its category. Reference 3.18 part V, chapter 4 may be consulted for details.

(iii) Lines AC, GF and FD : The positive manoeuvre load factor is prescribed to be constant upto the design diving speed ( $V_d$ ); line AC in Fig.9.9. According to Ref.3.9, chapter 14, the design diving speed could be 40 to 50% higher than the cruising speed ( $V_c$ ) for subsonic airplanes. For supersonic airplanes, the Mach number corresponding to  $V_d$  could be 0.2 faster than the maximum level flight Mach number. The negative manoeuvre load factor is prescribed to be constant upto design cruising speed (line GF in Fig.9.9) and then increases linearly to zero at  $V = V_d$  (line FD in Fig.9.9).

(iv) Manoeuvre load diagram : The diagram obtained by joining the points OACDFGO is called 'Manoeuvre load diagram'.

(v) Manoeuvre point and Corner speed : The point 'A' in Fig.9.9 is called 'Manoeuvre point'. The flight speed at this point is denoted by  $V^*$  and is called 'Corner speed'. At point 'A' the lift coefficient equals  $C_{L_{\text{max}}}$  and the load factor equals  $n_{\text{positive}}$ . This combination would result in the maximum instantaneous turn rate at the speed  $V^*$ . See subsection 9.3.6 for definition of instantaneous turn rate.

(vi) Positive and negative gust load factors : From Eq.(9.33) it is observed that the gust load factor varies linearly with velocity. The regulating agencies like FAA

Flight dynamics-I  
Chapter-9

prescribe that an airplane should be able to withstand load factors corresponding to  $V_{GU} = 30$  ft/s (9.1 m/s) upto cruising speed ( $V_C$ ) and  $V_{GU} = 15$  ft/s (4.6 m/s) upto design diving speed ( $V_D$ ). Lines JB and JF' in Fig.9.9 show the gust lines corresponding to  $V_{GU} = 30$  ft/s (9.1 m/s) and Lines JC' and JE in the same figure show the gust lines corresponding to  $V_{GU} = 15$  ft/s (4.6 m/s).

It may be pointed out that a gust in real situation, is not a sharp edged gust as shown in Fig.7.1b and the velocity  $V_{GU}$  is attained in a gradual manner. This causes reduction in the gust load factor. To take care of this reduction the gust load factor is multiplied by a quantity called 'Gust alleviation factor'. Reference 3.9 chapter 14 may be referred for details.

(vii) Gust load diagram : The diagram obtained by joining the points JBC'EF'J is called 'Gust load diagram'.

(vi) Final V-n diagram : For its safe operation, an airplane must be designed to withstand load factors occurring at all points of the gust and manoeuvre load diagrams. Hence, the final V-n diagram is obtained by joining the parts of these two diagrams representing the higher of the manoeuvre and gust load factors. The final V-n diagram in the case presented in Fig.9.9, is given by the solid curve obtained by joining the points IAB''BB''CEF''FGHI.

It may be pointed out that the gust load line JB' is above the curve IA in the region IK. However, along the curve IK the airplane is already operating at  $C_{Lmax}$  and any increase in angle of attack due to gust cannot increase  $C_L$  beyond  $C_{Lmax}$ . Hence, the portion JK of the line JB is not included in the final V-n diagram.

It may also be pointed out that the angles of attack of the airplane are different at various points of the V-n diagram. Consequently, the components of the resultant aerodynamic force along and perpendicular to the chord of the wing (N and C in Fig.3.7) would be different at different angles of attack. The structural analysis needs to take this into account. For example, the angle of attack is positive and high at point A and it is positive and low at point C. At points G and E the angles of attack are negative. Books on Airplane structures may be consulted for details.