Chapter 8

Performance analysis IV – Accelerated level flight and climb

(Lecture 27)

Keywords: Accelerated level flight; accelerated climb; energy height.

Topics

8.1   Introduction

8.2   Accelerated level flight

   8.2.1 Equations of motion in accelerated level flight

   8.2.2 Time taken and distance covered in accelerated level flight

8.3   Accelerated climb

   8.3.1   Equations of motion in accelerated climb

   8.3.2   Effect of acceleration on rate of climb

   8.3.3   Performance in accelerated climb from energy point of view

   8.3.4   Energy height

Exercise

Chapter 8

Lecture 27

Performance analysis IV – Accelerated level flight and climb

Topics

8.1 Introduction

8.2 Accelerated level flight

    8.2.1 Equations of motion in accelerated level flight

    8.2.2 Time taken and distance covered in accelerated level flight

8.3 Accelerated climb

    8.3.1 Equations of motion in accelerated climb

    8.3.2 Effect of acceleration on rate of climb

    8.3.3 Performance in accelerated climb from energy point of view

    8.3.4 Energy height

8.1 Introduction

The last three chapters dealt with the performance airplane in steady flights. The flights with acceleration are considered in this and the next two chapters. The accelerated flights could be along a straight line e.g. accelerated level flight and accelerated climb or along curved paths like loops and turn. In this chapter the accelerated level flight and climb are discussed.

8.2 Accelerated level flight

When an airplane moves along a straight line at a constant altitude but its velocity changes with time, then it is said to execute an accelerated level flight. This type of flight occurs in the following situations.

(i) The take-off speed of an airplane is about 1.15 to 1.3 times the stalling speed. However, the speed corresponding to the best rate of climb is generally much higher than this speed (see Figs.6.3a and c). Hence the airplane may accelerate from the take-off speed to the speed corresponding to the desired rate of climb. Similarly, the speed , at the end of the climb to the cruising altitude, is lower than

the cruising speed (Figs.6.3a and c) and an airplane would accelerate at the cruising altitude to attain the desired cruising speed.

(ii) The airplane may also accelerate in the transonic flight range to quickly pass-over to the supersonic speeds (see Fig.5.11)

(iii) The airplane may decelerate during a combat or when the pilot notices the possibility of over-shooting a target.

8.2.1 Equation of motion in accelerated level flight

The forces acting on an airplane in an accelerated level flight are shown in Fig.8.1. It may be recalled that the equations of motion are obtained by applying Newton's second law. For this purpose, the forces acting on the airplane are resolved along and the perpendicular to the flight path. Sum of the components of the forces in each of these directions, is equated to the product of the mass of the airplane and the component of the acceleration in that direction.

The flight path in this case is a horizontal line. Hence, the equations of motion are :

$$T - D = m \, a = \frac{W}{g} a \qquad\qquad (8.1)$$

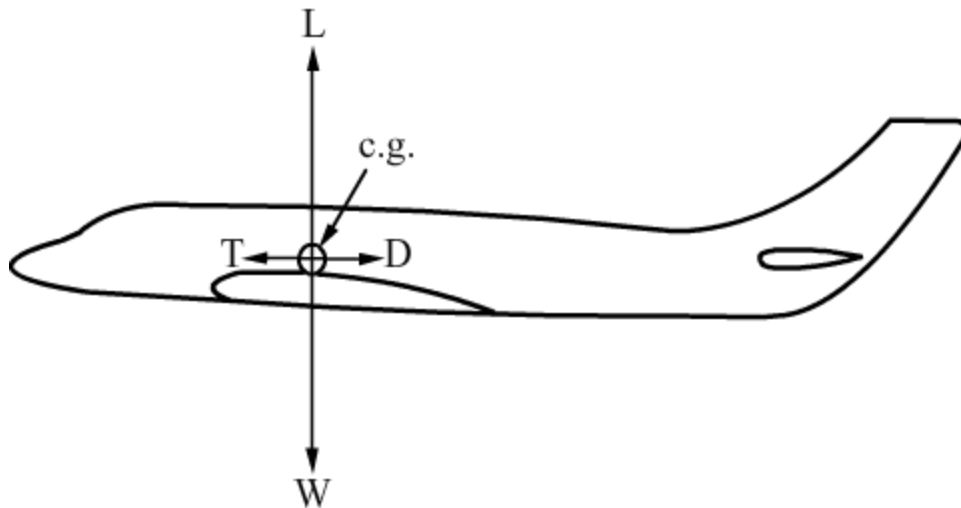$$L - W = 0 \qquad\qquad (8.2)$$

where 'a' is the acceleration.

Fig.8.1 Forces in accelerated level flight

8.2.2 Time taken and distance covered in an accelerated level flight

As regards the analysis of performance in an accelerated flight, it is of interest to obtain the time taken and the distance covered for a given change in velocity. The accelerated or decelerated flights last only for a short duration and the weight of the airplane can be assumed to remain constant during such flights. However, in a level flight $L = W = (1/2) \rho V^2 S C_L$, should be satisfied. Hence, the value of C_L and consequently of C_D change continuously as the flight velocity changes. From Eq.(8.1), the acceleration 'a' is given by:

$a = g (T-D) / W$

Substituting for D as $(1/2)\rho V^2 S C_D$, gives:

$$a = \frac{g}{W} (T - \frac{1}{2} \rho V^2 S C_D) \tag{8.3}$$

Note that : $V = \dfrac{ds}{dt}$ and $a = \dfrac{dV}{dt} = \dfrac{dV}{ds}\dfrac{ds}{dt} = V\dfrac{dV}{ds}$

Consequently, $dt = \dfrac{dV}{a}$ and $ds = \dfrac{V\,dV}{a}$ $\tag{8.3a}$

Let the distance covered and the time taken for velocity to change from V_1 to V_2 be denoted by 's' and 't' respectively, Integrating expressions in Eq.(8.3a) gives:

$$s = \int_{V_1}^{V_2} \frac{VdV}{a} \text{ and } t = \int_{V_1}^{V_2} \frac{dV}{a} \tag{8.4}$$

Substituting for 'a' from Eq.(8.3) yields:

$$s = \int_{V_1}^{V_2} \frac{W\,V\,dV}{g\,(T - \frac{1}{2} \rho V^2 S C_D)} \text{ and } t = \int_{V_1}^{V_2} \frac{W\,dV}{g\,(T - \frac{1}{2} \rho V^2 S C_D)} \tag{8.5}$$

The expressions in Eq.(8.5) can be directly integrated if T and D are simple functions of velocity. Otherwise a numerical integration as illustrated in the following example can be carried out.

Example 8.1

An airplane with a weight of 156,960 N and a wing area of 49 m^2 has a drag polar given by $C_D = 0.017+0.06C_L{}^2$. It accelerates under standard sea level conditions from a velocity of 100 m/s to 220 m/s. Obtain the distance covered and the time taken during the acceleration, assuming the thrust output to remain roughly constant at 53,950 N.

Solution:

$$L = W = \frac{1}{2}\rho \, V^2 S C_L$$

$$D = \frac{1}{2}\rho \, V^2 S C_D = \frac{1}{2}\rho \, V^2 S \; C_{D0} + \frac{2\,KW^2}{\rho \, SV^2}$$

$$\text{Or } D = \frac{1}{2} \times 1.225 \times 49 \times 0.017 \times V^2 + \frac{2 \times 0.06 \times 156960^2}{1.225 \times 49 \times V^2}$$

$$\text{Or } D = 0.5102 \; V^2 + \frac{4.9225 \times 10^7}{V^2}$$

To carry out the numerical integration, the integrands in Eq.(8.5) are evaluated for several values of V and the methods like trapezoidal rule or Simpson's rule are used. Books on numerical analysis be consulted for further details of these methods. Simpson's rule gives accurate results with a small number of points and is used here. For this purpose the range between V_1 and V_2 is divided into six intervals, each of 20 m/s. The values are tabulated below:

V (m/s)	100	120	140	160	180	200	220
D (N)	10042	10771	12518	14999	18050	21660	25731
$\dfrac{W}{g(T-D)}$	0.3644	0.3705	0.3861	0.4107	0.4456	0.4954	0.5669
$\dfrac{W\,V}{g(T-D)}$	36.44	44.46	54.06	65.72	80.21	99.09	124.72

Using Simpson's rule,

s = (20/3) {36.44 + 4 (44.46 + 65.72 + 99.09) + 2(54.06 + 80.21) + 124.72}

   = 8445 m = 8.445 km

t = (20/3) {0.3644 + 4 (0.3705 + 0.4107+0.4954) + 2 (0.3861 + 0.4456) + 0.5669}

   = 51.34 s.

Answers:

Distance covered = 8.445 km ; time taken = 51.39 s.

8.3 Accelerated Climb

     In this case, the flight takes place along a straight line inclined to the horizontal at an angle γ as shown in Fig.8.2. The flight velocity increases or decreases along the flight path. Figure 8.2 also shows the forces acting on the airplane.
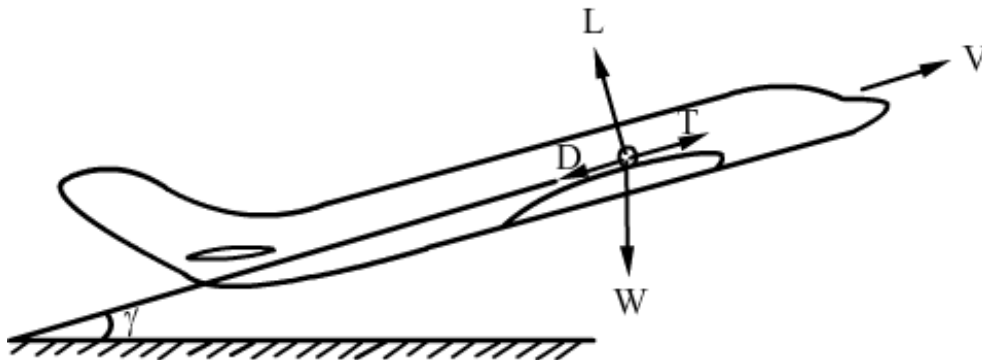

Fig.8.2 Accelerated climb

8.3.1 Equations of motion in accelerated climb

The equations of motion are:

$$T - D - W\sin \gamma = \frac{W}{g}a \tag{8.6}$$

$$L - W \cos \gamma = 0 \tag{8.7}$$

8.3.2 Effect of acceleration on rate of climb

From Eq.(8.6), the acceleration can be expressed as:

$$a = \frac{g (T - D - W \sin \gamma)}{W} \tag{8.8}$$

Note that : $a = \dfrac{dV}{dt} = \dfrac{dV}{dh} \cdot \dfrac{dh}{dt}$; but $\dfrac{dh}{dt} = V_C = R/C$

Consequently, $a = V_C \dfrac{dV}{dh}$ \hfill (8.9)

Substituting for 'a' in Eq.(8.6) and noting that sin $\gamma = V_C / V$,

$$T - D - W \dfrac{V_C}{V} - \dfrac{W}{g} V_C \dfrac{dV}{dh} = 0 \quad \text{or} \quad V_C = \dfrac{(T-D)V}{W\left(1+\dfrac{V}{g}\dfrac{dV}{dh}\right)} \tag{8.10}$$

From Eq.(6.4), (T-D) V / W is the rate if climb in steady flight. Denoting it by V_{co},

Eq.(8.10) reduces to:

$$V_C = \dfrac{V_{co}}{\left(1+\dfrac{V}{g}\dfrac{dV}{dh}\right)} \tag{8.11}$$

Remark:

The term (dV/dh) in Eq.(8.11) represents the rate of change of velocity with altitude. This quantity would be positive if the flight velocity increases with altitude. Thus, in an accelerated climb, the rate of climb, for given values of thrust, speed and altitude, will be lower than that in a steady climb. This has relevance to the flight with shortest time to climb, i.e., to calculate the shortest time required to achieve desired altitude.

From Fig.6.3c it is observed that the flight speed for maximum rate of climb ($V_{R/Cmax}$) increases with altitude. Thus, in a climb which attempts to fly the airplane at speeds corresponding to the maximum rate of climb ($V_{(R/C)max}$) at different altitudes, would not be a steady climb but an accelerated climb. Consequently, the values of $(R/C)_{max}$ given in Fig.6.3e may need to be corrected for the effect of acceleration.

8.3.3 Performance in accelerated climb from energy point of view

The performance of an airplane in an accelerated flight can also be viewed from the energy point of view. Multiplying Eq.(8.6) by V gives:

$$T V - DV - W V \sin\gamma = \dfrac{W}{g} V \dfrac{dV}{dt}$$

Or $TV = DV + W\dfrac{dh}{dt} + \dfrac{W}{g}\dfrac{d}{dt}\left(\dfrac{V^2}{2}\right)$ \hfill (8.12)

In Eq.(8.12) the term 'TV' represents the available energy provided by the propulsive system. The term 'DV' represents the energy dissipated in overcoming the drag. The term 'W (dh / dt)' represents the rate of change of potential energy and $(W/g)\{d(V^2 / 2) / dt\}$ represents the rate of change of kinetic energy. Thus, the total available energy can be utilized in three ways viz. overcoming drag, change of potential energy and change of kinetic energy. If the flight takes place at V_{max} or $(V_{min})_e$ in level flight, then entire energy is used in overcoming the drag and no energy is available for climb or acceleration. Only at speeds in between $(V_{min})_e$ and V_{max}, can an airplane climb or accelerate and the excess power (T-D)V has to be shared for increase of potential energy or kinetic energy or both. If climb takes at $V_{(R/C)max}$ then no acceleration is possible.

8.3.4 Energy height

Equation (8.12) can be rewritten as:

$\dfrac{(T\text{-}D)V}{W} = \dfrac{d}{dt}\left(h + \dfrac{V^2}{2g}\right)$ \hfill (8.13)

The term $(h + V^2/2g)$ is denoted by h_e and is called 'Specific energy or Energy height'. It is called specific energy because it is equal to the sum of potential energy and kinetic energy divided by the weight. It is called energy height because this term has the dimensions of height. It may be noted that

$(dh_e / dt) = (T\text{-}D)V/ W$ \hfill (8.14)

The energy height concept is used in optimization of climb performance.
Reference 1.9 chapter 7 and Ref.1.12 chapter 2 may be referred to for details.
The quantity (dh_e/ dt) is called specific excess power and denoted by 'P_s'.

Example 8.2

An airplane climbs at constant equivalent air speed in troposphere. Obtain an expression for the correction to be applied to the value of rate of climb calculated with the assumption of the steady climb(the denominator in Eq.8.11).

Solution:

In a climb with V_e as constant, the true air speed (V) is given by:

$$V = V_e / \sigma^{\frac{1}{2}},$$

Consequently, $\dfrac{dV}{dh} = -\dfrac{1}{2} V_e\, \sigma^{-3/2}\, \dfrac{d\sigma}{dh}$

In troposphere the variation of σ with h is given as follows (Eq.2.7):

$$\sigma = \left(\frac{T_O - \lambda h}{T_O}\right)^{\frac{g}{\lambda R} - 1}$$

where, T_O = Temperature at sea level,

λ = Temperature lapse rate and

R = gas constant.

Hence, $\dfrac{dV}{dh} = \dfrac{1}{2} V_e\, \dfrac{\lambda}{T_O} \left(\dfrac{g - \lambda R}{\lambda R}\right) \sigma^{\frac{-(g + \lambda R)}{2(g - \lambda R)}}$ \hfill (8.15)

In I.S.A., $\lambda = 0.0065$ K/m. Using g = 9.81 m/s and R = 287.05 m/s^2 K, the correction factor in Eq.(8.11) is:

$$1 + \frac{V}{g} \frac{dV}{dh} = 1 + 4.894 \times 10^{-6}\, V_e^2\, \sigma^{-1.235} \tag{8.16}$$

It is seen that the correction required depends on V_e and σ. Typical values of the correction factor at sea level ($\sigma = 1$) and at 11 km altitude ($\sigma = 0.2971$) are given in Table E8.1.

V_e (m/s)	50	100	200
$\left(1+\dfrac{V}{g}\dfrac{dV}{dh}\right)$ at s.l	1.01224	1.0489	1.1958
$\left(1+\dfrac{V}{g}\dfrac{dV}{dh}\right)$ at 11 km	1.0548	1.2191	1.8766

Table E8.1 Correction factor in climb at constant equivalent air speed in troposphere.

It is worth noting that at 11 km altitude the actual rate of climb, in constant V_e flight at 200 m/s, is reduced to about half of its value in a steady climb.

Remark:

In a constant Mach number flight in troposphere, the flight velocity decreases with altitude. Hence, the term (dV / dh) is negative and the rate of climb in constant Mach number flight is more than that in a steady climb. See exercise 8.1.