Chapter 6

Lecture 22

Performance analysis II – Steady climb, descent and glide – 2

Topics

6.5 Maximum rate of climb and maximum angle of climb

6.5.1 Parameters influencing $(R/C)_{max}$ of a jet airplane

6.5.2 Parameters influencing $(R/C)_{max}$ of an airplane with engine-propeller combination

6.5 Maximum rate of climb and maximum angle of climb

Using the procedure outlined above, the rate of climb and the angle of climb can be calculated at various speeds and altitudes. Figures 6.3a to 6.3f present typical climb performance of a jet transport. Figure 6.4a to 6.4d present the climb performance of a piston engined airplane. Details of the calculations for these two cases are presented in Appendices B and A respectively.

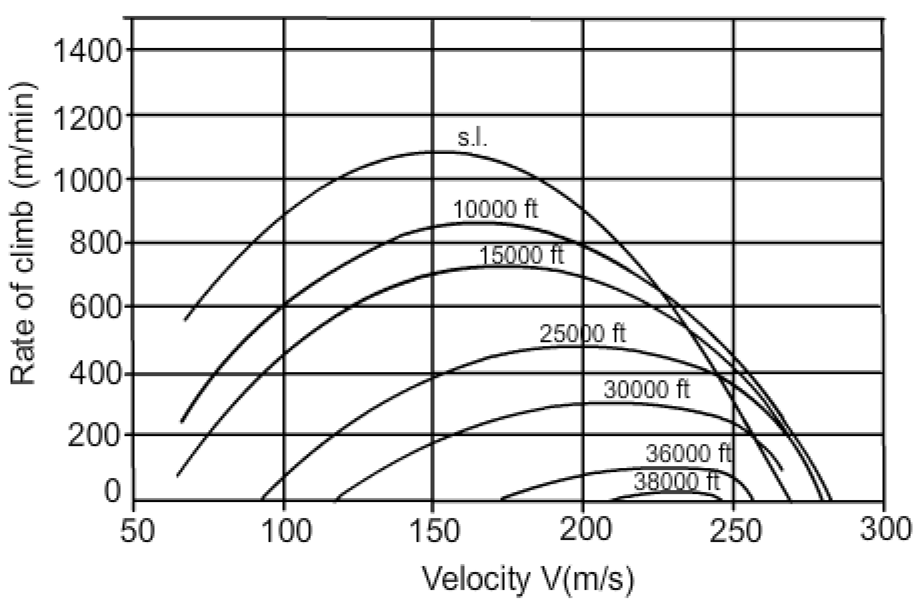

Fig.6.3a Climb performance of a jet transport - rate of climb

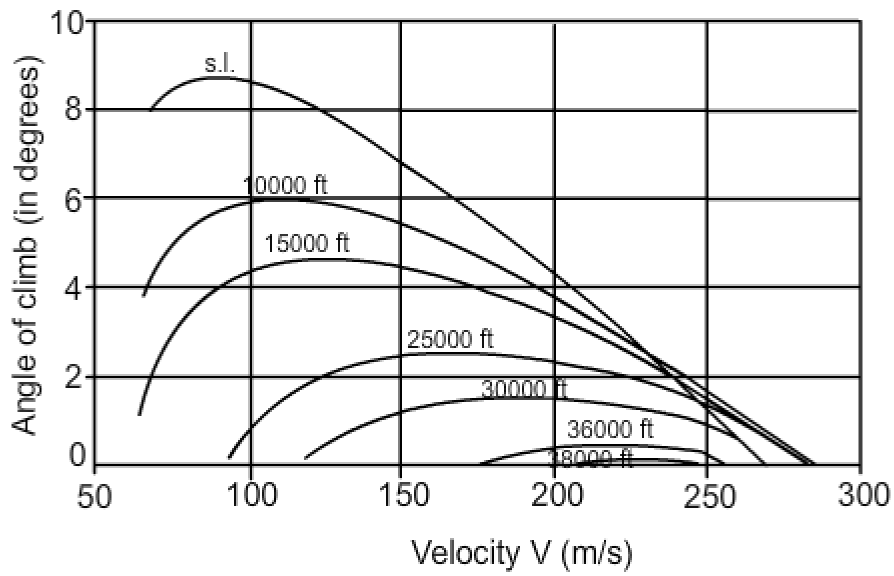

Fig.6.3b Climb performance of a jet transport - angle of climb

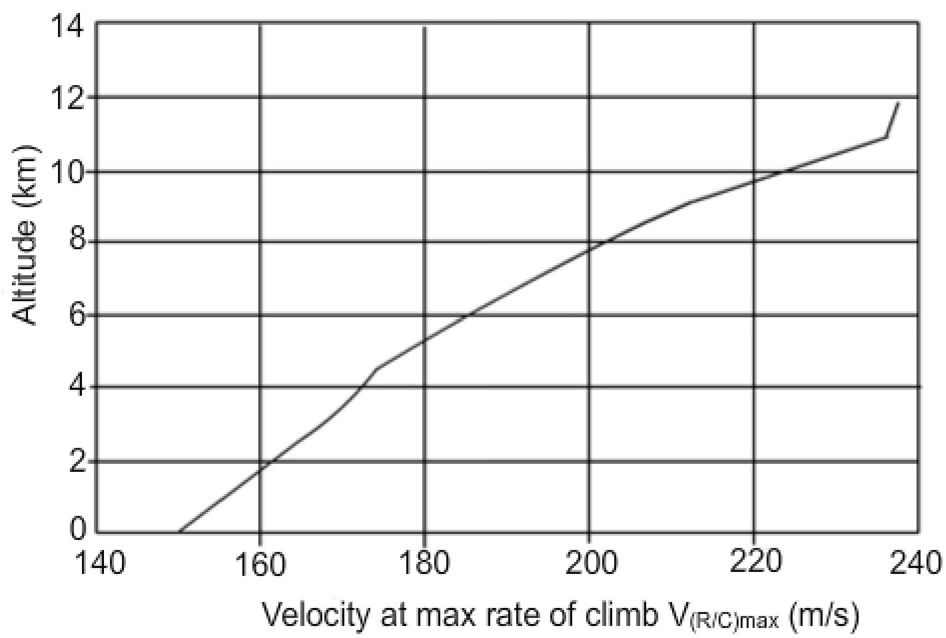

Fig.6.3c Climb performance of a jet transport - $V_{(R/C)\ max}$

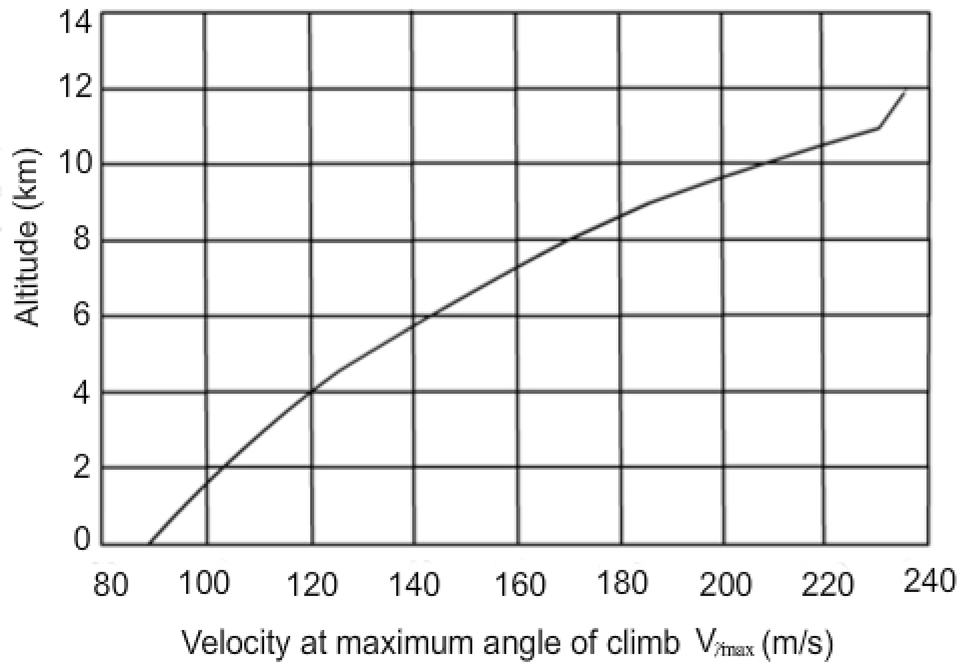

Fig.6.3d Climb performance of a jet transport-$V_{\gamma max}$

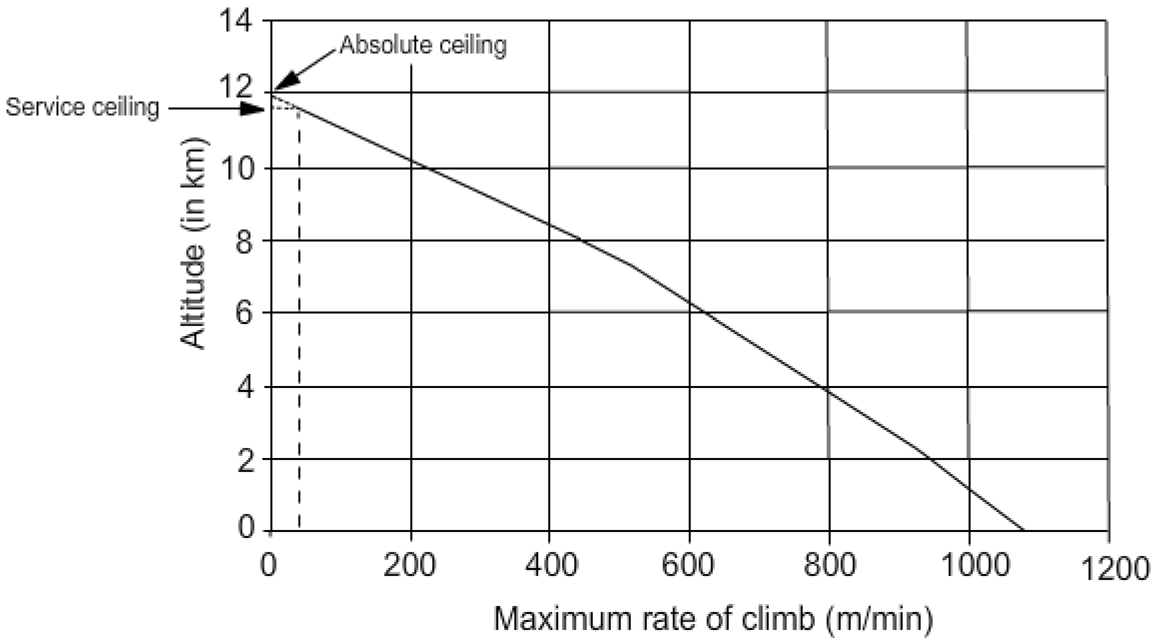

Fig.6.3e Climb performance of a jet transport - variation of $(R/C)_{max}$ with altitude

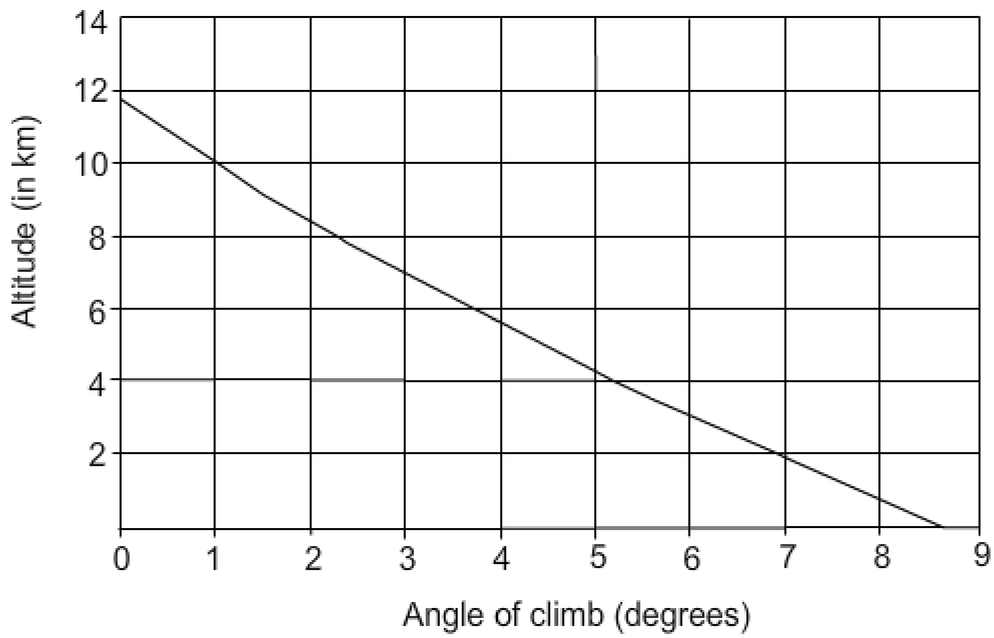

Fig.6.3f Climb performance of a jet transport - variation of γ_{max} with altitude

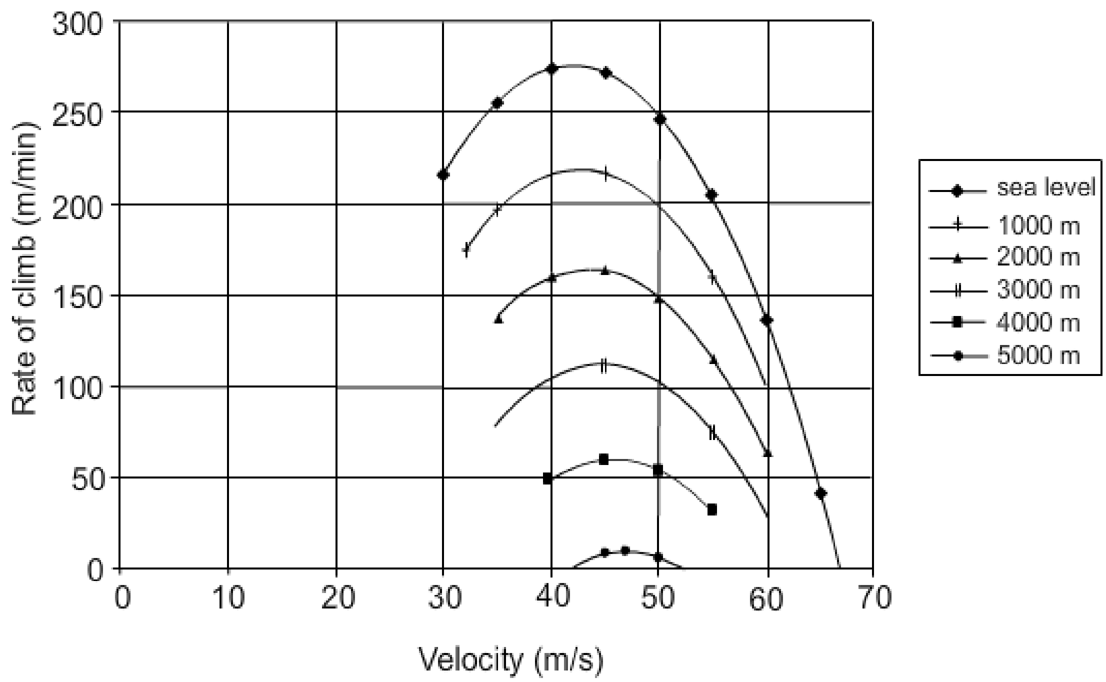

Fig.6.4a Climb performance of a piston engined airplane- rate of climb

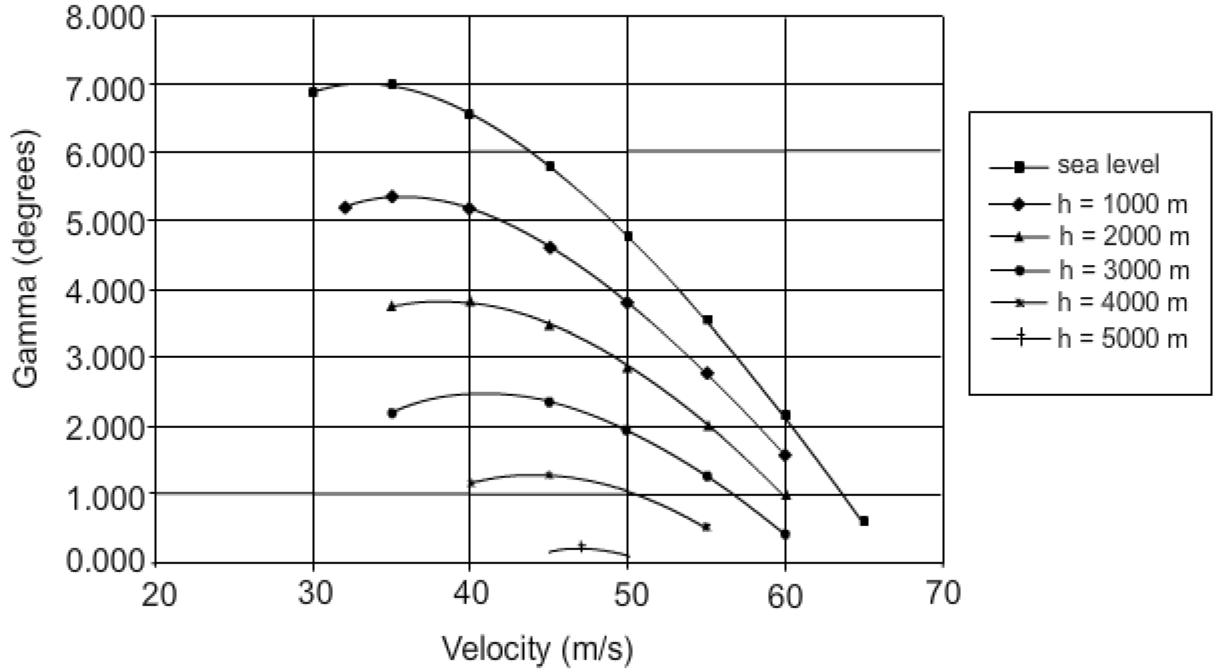

Fig.6.4b Climb performance of a piston engined airplane- angle of climb

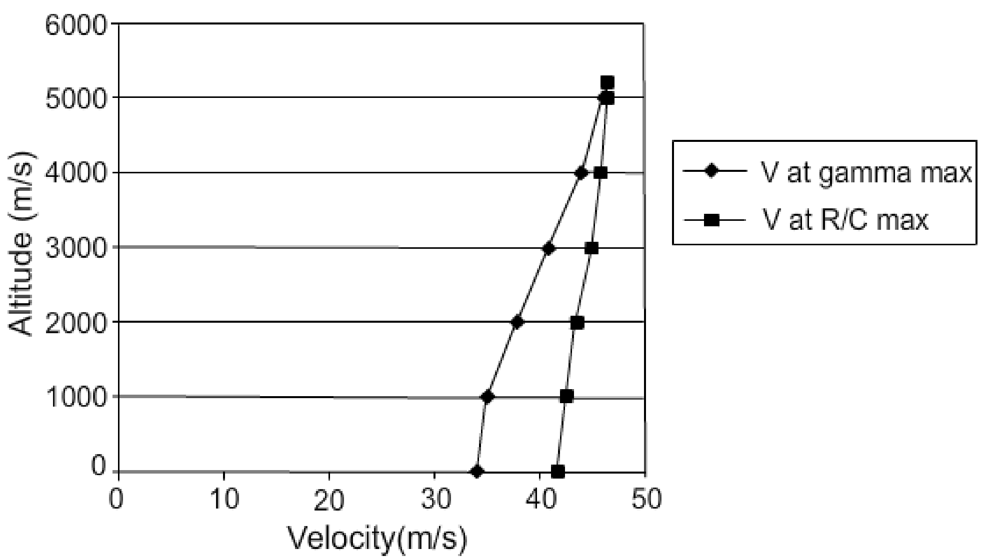

Fig.6.4c Climb performance of a piston engined airplane - $V_{(R/C)max}$, and $V_{\gamma max}$

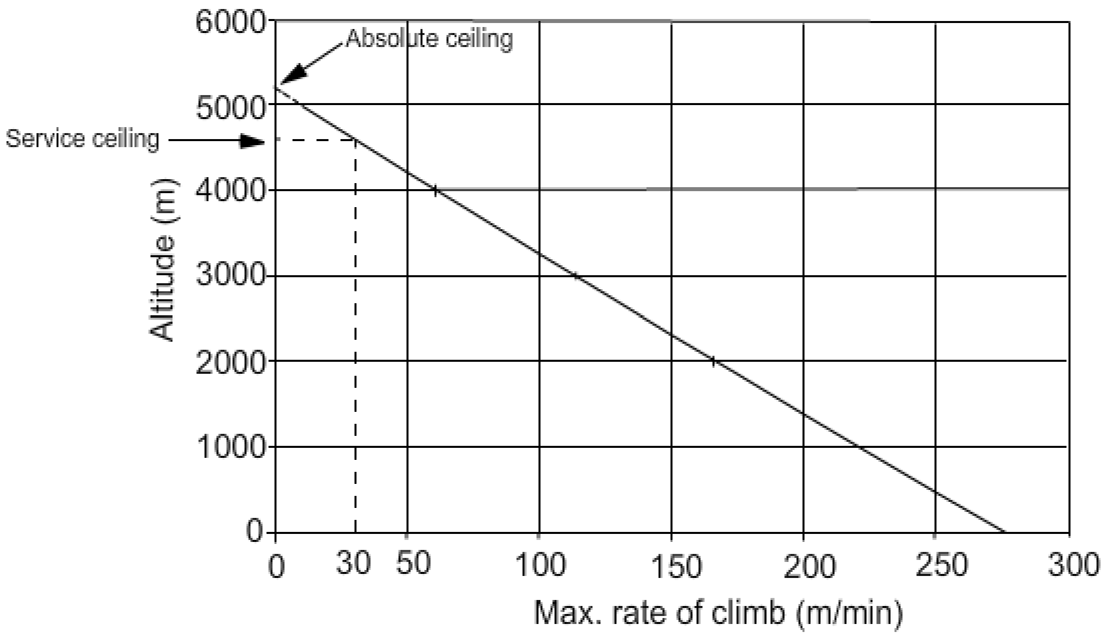

Fig.6.4d Climb performance of a piston engined airplane - Variation of $(R/C)_{max}$ with altitude

Remarks:

i) At $V = V_{max}$ the available thrust or THP is equal to the thrust required or power required in level flight. Hence climb is not possible at this speed. Similar is the case at $(V_{min})_e$ limited by engine output (Figs.6.3a and 6.4a). For the same reasons, at V_{max} and $(V_{min})_e$ the angle of climb (γ) is also zero (Figs.6.3b and 6.4b). It may be recalled from subsection 5.9, that at low altitudes the minimum speed is decided by stalling and hence the calculations regarding the rate of climb and the angle of climb are restricted to flight speeds between V_{min} and V_{max}.

ii) The speed at which R/C is maximum is denoted by $V_{(R/C)max}$, and the speed at which γ is maximum is called $V_{\gamma max}$. Figures 6.3c and d and Fig.6.4c show the variation of these speeds with altitudes for a jet transport and a piston engined airplane respectively. It may be noted that $V_{(R/c)max}$ and $V_{\gamma max}$ are different from each other. For a jet airplane $V_{(R/c)max}$ is higher than $V_{\gamma max}$ at low altitudes . The two velocities approach each other as the altitude increases. For a piston engined airplane $V_{(R/C)max}$ is lower than $V_{\gamma max}$ at low altitudes . The two velocities approach each other as the altitude increases. These trends can be explained as follows.

From Eqs.(6.3) and (6.4), it is observed that γ is proportional to the excess thrust i.e. $(T_a - D_c)$ and the rate of climb is proportional to the excess power i.e. $(T_aV - D_cV)$. It may be recalled that for a piston engined airplane the power available remains roughly constant with velocity and hence, the thrust available $(T_a = P_a / V)$ will decrease with velocity. On the other hand, for a jet airplane the thrust available is roughly constant with velocity and consequently the power available increases linearly with velocity (see exercise 6.3). The differences in the variations of T_a and P_a with velocity, in the cases of jet engine and piston engine, decide the aforesaid trends.

iii) As the excess power and the excess thrust decrease with altitude, $(R/C)_{max}$ and γ_{max} also decrease with altitude.

6.5.1 Parameters influencing $(R/C)_{max}$ of a jet airplane

In subsection 5.10.2, the parameters influencing V_{max} were identified by simplifying the analysis with certain assumptions. In this subsection the parameters influencing $(R/C)_{max}$ are identified in a similar manner. The limitations of the simplified analysis are pointed out at the end of this subsection.

The case of a jet airplane is considered in this subsection.

From Eq.(6.5) it is noted that :

$$V_C = R/C = V\left(\frac{T - D}{W}\right)$$

Following simplifying assumptions are made to identify the parameters influencing $(R/C)_{max}$.

(a)Drag polar is parabolic with constant coefficients i.e. C_{DO} and K are constants.

(b) Though the angle of climb (γ) is not small, for the purpose of estimating the induced drag, the lift (L) is taken equal to weight. See comments at the end of section 6.4.1 for justification of this approximation.

(c) At a chosen altitude, the thrust available (T_a) is constant with flight speed.

With these assumptions, the expression for drag simplifies to that in the level flight i.e.

$$D = \frac{1}{2}\rho V^2 S C_D = \frac{1}{2}\rho V^2 S\left\{C_{DO} + K\left(\frac{2W}{\rho S V^2}\right)^2\right\}$$

Hence,

$$V_c = V\left\{\frac{T_a - D}{W}\right\}$$

$$\text{Or } V_c = V\left\{\frac{T_a}{W} - \frac{\frac{1}{2}\rho V^2}{W/S} C_{DO} - \frac{2K}{\rho V^2}\frac{W}{S}\right\}$$

$$\text{Or } V_c = \frac{T_a}{W}V - \frac{1}{2}\rho V^3 (W/S)^{-1} C_{DO} - \frac{2K}{\rho V}\left(\frac{W}{S}\right) \qquad (6.20)$$

To obtain the flight speed corresponding to $(V_c)_{max}$, Eq.(6.20) is differentiated with respect to V and equated to zero i.e.

$$\frac{dV_c}{dV} = \frac{T_a}{W} - \frac{3}{2}\rho V^2 (W/S)^{-1} C_{DO} + \frac{2K}{\rho V^2}\left(\frac{W}{S}\right) = 0 \qquad (6.21)$$

Simplifying Eq.(6.21) yields:

$$V^4_{(R/C)max} - \frac{2(T_a/W)(W/S)}{3\rho C_{DO}} V^2_{(R/C)max} - \frac{4K(W/S)^2}{3\rho^2 C_{DO}} = 0 \qquad (6.22)$$

Noting from Eq.(3.56) that $(L/D)^2_{max} = \frac{1}{4 C_{DO} K}$, yields:

$$V^4_{(R/C)max} - \frac{2(T_a/W)(W/S)}{3\rho C_{DO}} V^2_{(R/C)max} - \frac{(W/S)^2}{3\rho^2 C^2_{DO} (L/D)^2_{max}} = 0$$

Thus,

$$V_{(R/C)max} = \left\{\frac{(T_a/W)(W/S)}{3\rho C_{DO}}\left[1 \pm \sqrt{1 + \frac{3}{\left(\frac{L}{D}\right)^2_{max}(T_a/W)^2}}\right]\right\}^{\frac{1}{2}}$$

The negative sign in the above equation, would give an imaginary value for $V_{(R/C)max}$ and is ignored.

Hence,

$$V_{(R/C)max} = \left\{\frac{(T_a/W)(W/S)}{3\rho C_{DO}}\left[1+\sqrt{1+\frac{3}{(L/D)_{max}^2(T_a/W)^2}}\right]\right\}^{\frac{1}{2}}$$

$$= \left\{\frac{(T_a/W)(W/S)Z}{3\rho C_{DO}}\right\}^{\frac{1}{2}}, \qquad (6.23)$$

$$\text{where, } Z = 1+\sqrt{1+\frac{3}{(L/D)_{max}^2(T_a/W)^2}} \qquad (6.24)$$

Substituting $V_{(R/C)max}$ from Eq.(6.23) in Eq.(6.5) yields:

$$(R/C)_{max} = \left[\frac{(T_a/W)(W/S)Z}{3\rho C_{DO}}\right]^{\frac{1}{2}} \times \left[\frac{T_a}{W} - \frac{1}{2}\rho\frac{(T_a/W)(W/S)ZC_{DO}}{3\rho C_{DO}(W/S)} - \frac{2(W/S)K(3\rho C_{DO})}{\rho(T_a/W)(W/S)Z}\right]$$

$$\text{Or } (R/C)_{max} = \left[\frac{(T_a/W)(W/S)Z}{3\rho C_{DO}}\right]^{\frac{1}{2}}\left[\frac{T_a}{W} - \frac{Z}{6}(T_a/W) - \frac{6KC_{DO}}{(T_a/W)Z}\right]$$

$$= \left[\frac{(W/S)Z}{3\rho C_{DO}}\right]^{1/2}\left(\frac{T_a}{W}\right)^{3/2}\left[1-\frac{Z}{6}-\frac{3}{2(T_a/W)^2(L/D)_{max}^2 Z}\right] \qquad (6.25)$$

Remarks:

The following observations can be made based on Eqs.(6.23) to (6.25)

(i) In Eq.(6.24), the quantity Z appears to depend on $(L/D)_{max}$ and (T_a/W). In this context it may be noted that for jet airplanes (a) the value of $(L/D)_{max}$ would be around 20 and (b) the value of (T_a/W) would be around 0.25 at sea level and around 0.06 at tropopause. With these values of $(L/D)_{max}$ and (T_a/W), Z would be around 2.1 at sea level and 2.7 at tropopause.

However, in Eqs.(6.23) and (6.25) the terms involving Z, appear as $Z^{1/2}$ or Z/6. Hence, the dependence of $V_{(R/C)max}$ and $(R/C)_{max}$ on Z does not appear to be of primary importance. The important parameters however, are (T_a/W), (W/S), ρ and C_{DO}. It may be recalled from section 4.5 that for a turbofan engine, T_a decreases with altitude in proportion to $\sigma^{0.7}$; σ being the density ratio.

(ii) From Eq.(6.25) it is observed that for given values of W/S and C_{DO}, $(R/C)_{max}$ decreases with altitude. Hence, suitable value of (T_a/W) is required to achieve the specified rates of climb at different altitudes.

The same equation also indicates that the rate of climb increases when wing loading increases and C_{DO} decreases. However, the performance during cruise and landing generally place a limit on the value of (W/S).

(iii) From Eq.(6.23) it is observed that the flight speed for maximum rate of climb($V_{(R/Cmax)}$), increases with (T_a/W), (W/S) and altitude. In this context it may be pointed out that the Mach number corresponding to $V_{(R/C)max}$, should always be worked out and corrections to the values of C_{DO} and K be applied when this Mach number exceeds M_{cruise}. Without these corrections, the values of $(R/C)_{max}$ obtained may be unrealistic.

(iv) Figure 4.12 shows typical variations of thrust vs Mach number with altitude as parameter. It is observed, that the thrust decreases significantly with Mach number for altitudes equal to or less than 25000′ (7620 m). Thus, the assumption of thrust being constant with flight speed is not a good approximation for h ≤ 25000′ .

6.5.2 Parameters influencing $(R/C)_{max}$ of an airplane with engine-propeller combination

In this subsection the simplified analysis is carried out for climb performance of an airplane with engine-propeller combination.

From Eq.(6.5)

$$V_C = R/C = \frac{V(T\text{-}D)}{W} = \frac{TV - DV}{W}$$

$TV = 1000\eta_p P_a$; P_a = power available in kW

DV = Power required to overcome drag = $\frac{1}{2}\rho V^3 S C_D$

Following assumptions are made to simplify the analysis and obtain parameters which influence $(R/C)_{max}$ are $V_{(R/C)max}$ in this case.

(a) Drag polar is parabolic with C_{DO} and K as constants.

(b) L = W for estimation of induced drag.

(c) Power available is constant with flight speed (V).

Consequently,

$$DV = \frac{1}{2}\rho V^3 S\left\{C_{DO} + K\left(\frac{2W}{\rho S V^2}\right)^2\right\} \qquad (6.26)$$

Since P_a is assumed to be constant, the maximum rate of climb would be obtained when DV is minimum. This occurs at the flight speed corresponding to minimum power V_{mp}.

Hence, in this case : $V_{(R/C)max} = V_{mp}$

The expression for V_{mp} is given in Eq.(5.24b), consequently :

$$V_{(R/C)max} = V_{mp} = \left\{\frac{2W}{\rho S}\right\}^{\frac{1}{2}}\left(\frac{K}{3C_{DO}}\right)^{\frac{1}{4}} \qquad (6.27)$$

Substituting $V_{(R/C)max}$ from Eq.(6.27) in Eq.(6.5) gives:

$$(R/C)_{max} = \frac{1000\eta_p P_a}{W} - \frac{V_{(R/C)max}}{W}\left\{\frac{1}{2}\rho V^2_{(R/C)max} S C_{DO} + K\left(\frac{2W}{\rho S V^2_{(R/C)max}}\right)^2\right\}$$

$$= \frac{1000\eta_p P_a}{W} - V_{(R/C)max}\left\{\frac{1}{2}\rho (W/S)^{-1} C_{DO}\frac{2}{\rho}\sqrt{\frac{K}{3C_{DO}}}\frac{W}{S} + \frac{2K(W/S)}{\rho\frac{2}{\rho}\sqrt{K/(3C_{DO})}(W/S)}\right\}$$

(6.28)

Noting that $(L/D)_{max} = \frac{1}{2\sqrt{C_{DO}K}}$, and

$$\frac{\left(1/\sqrt{3}\right)+\sqrt{3}}{2} = 1.155, \text{ yields:}$$

$$(R/C)_{max} = \frac{1000\eta_p P_a}{W} - \frac{1.155}{(L/D)_{max}} V_{(R/C)max} \tag{6.29}$$

Substituting for $V_{(R/C)max}$ from Eq.(6.27) yields :

$$(R/C)_{max} = \frac{1000\eta_p P_a}{W} - \frac{2}{\rho}\sqrt{\frac{K}{3C_{DO}}}(W/S)^{1/2}\frac{1.155}{(L/D)_{max}} \tag{6.30}$$

Remarks:

(i) From Eq.(6.27) it is observed that $V_{(R/C)max}$ increases with wing loading (W/S).

(ii) From Eq.(6.30) it is observed that $(R/C)_{max}$ increases as η_p , P_a and $(L/D)_{max}$ increase. However, the second term on the right hand side of this equation indicates that $(R/C)_{max}$ decreases with increase of wing loading. This trend is opposite to that in the case of jet airplanes. Thus, for a specified $(R/C)_{max}$, the wing loading for an airplane with engine-propeller combination should be rather low, to decrease the power required.

(iii) The first term in Eq.(6.30) involves η_p and P_a. From subsection 4.2.2 it is noted that P_a is nearly constant with flight speed (V). However, the assumption of η_p being constant with V is roughly valid only when the airplane has a variable pitch propeller. For a fixed pitch propeller η_p varies significantly with V (Fig.4.5a) and the assumption of P_a being constant with V is not appropriate in this case.