Chapter 6

Lateral static stability and control - 2

Lecture 20

Topics

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6.10 Roll control

       A control in roll is needed to give a desired rate of roll (p).It may be recalled from section 5.8.1 that pb/2V = 0.07 and 0.09 are desirable for cargo and military airplanes respectively. The desired rate of roll is provided by the ailerons.

6.10.1 Aileron, differential aileron and spoiler aileron

Roll control is achieved in the following ways:

(a) The deflections of the left and right ailerons are same in magnitude but opposite in direction ($\pm\delta_a$).

(b) Differential aileron: In this case the deflections of the right and the left ailerons are unequal. This avoids adverse yaw. The up going aileron moves through a larger angle than the down going aileron.

(c) The spoiler ailerons, on the two wing halves, are shown in Fig.1.16. When a spoiler is deployed, it disturbs the flow on the upper surface of that wing half.

This causes loss of lift on that wing half and a rolling moment is produced. The spoilers are generally used at high speeds on large airplanes to counter the loss of effectiveness due to aileron reversal. For further discussion on spoiler aileron see Ref.1.13, chapter 6.The phenomenon of aileron reversal can be briefly explained as follows.

Though the airplane is assumed to be rigid in the present discussion, the structure of an actual airplane is elastic. It deflects and twists under loads. When an aileron is deflected down it increases the lift on that wing half but, it also makes M_{ac} more negative. Consequently, the wing twists and the angle of attack decreases. The twist increases with flight speed. There is a speed, called aileron reversal speed, at which the reduction in the angle of attack due to twist will nullify the increase in the lift due to deflection of aileron. Beyond this speed a downward deployment of aileron would actually decrease the lift. This is called aileron reversal. It may be added that the interaction between aerodynamic and elastic forces is discussed under the topic "Aeroelasticity".

Remarks:

i) See section 6.11.5 for frise aileron.

ii) To calculate the rate of roll when the ailerons are deflected, the following two aspects need to be discussed.

(a) The rolling moment due to aileron deflection.

(b) The damping in roll i.e. opposite rolling moment caused due to rolling motion. The final rate of roll will be the balance between these two effects which are estimated in the next two subsections.

6.10.2 Rolling moment due to aileron

The deflections of ailerons cause changes in the lift distributions on the two wing halves which produce a rolling moment. The changes in the lift distributions can be calculated using wing theory. However, this is a complicated task and a simple method called 'Strip theory' is used for preliminary estimates. In this theory, it is assumed that the change in the lift distribution is confined to the portion of the wing span over which the aileron extends. Further, the change in local lift coefficients (ΔC_l) is given by:

$\Delta C_l = (\partial C_l / \partial \delta_a) \delta_a$

Where, δ_a is the aileron deflection and

$\partial C_l / \partial \delta_a = \tau_{ail} \dfrac{\partial C_l}{\partial \alpha}$; τ_{ail} = aileron effectiveness parameter.
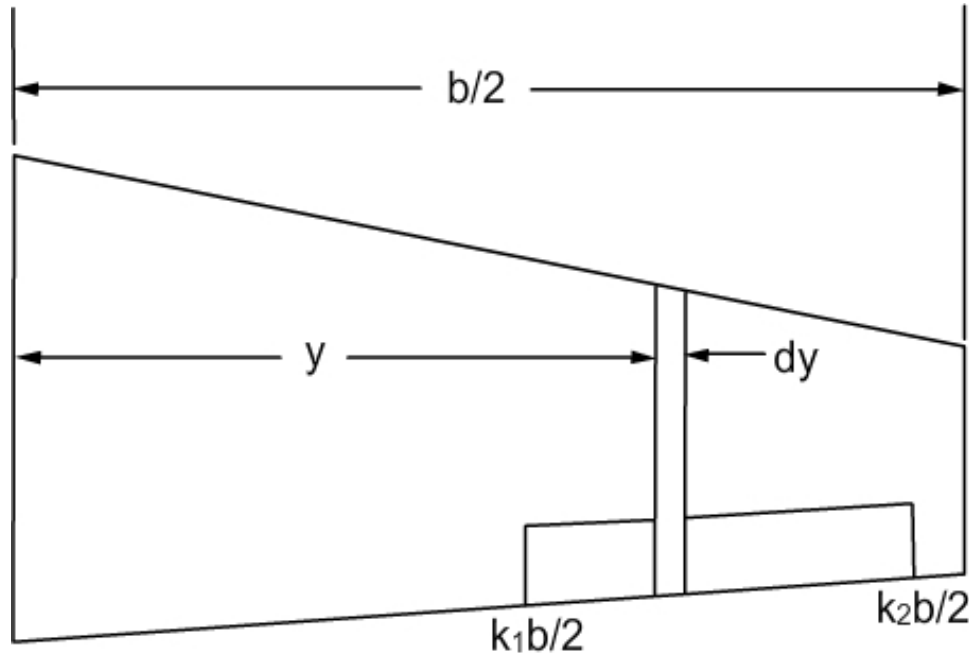

Fig.6.5 Strip theory

Rolling moment due to aileron from a strip of width Δy (Fig.6.5) is:

$$\Delta L' = \frac{1}{2} \rho V^2 c\, dy\; y\, \Delta C_l \quad ; \text{c being the local chord} \tag{6.15}$$

$$\Delta C'_l = \frac{\frac{1}{2} \rho V^2 c\, dy\; y\, \Delta C_l}{\frac{1}{2} \rho V^2\, Sb} ;$$

$$= \frac{c\, y}{Sb} a_0\, \tau_{ail}\, \delta_a\, dy; \quad a_0 = \frac{\partial C_l}{\partial \alpha} \text{ of the airfoil} \tag{6.16}$$

Hence, integrating over the portion of span where aileron extends, gives the rolling moment coefficient due to both ailerons as:

$$(C'_l)_{aileron} = \frac{2 a_0\, \tau_{ail}\, \delta_a}{Sb} \int_{k_1 \frac{b}{2}}^{k_2 \frac{b}{2}} c\, y\, dy \tag{6.17}$$

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Note that the aileron extends from $(k_1 b/2)$ to $(k_2 b/2)$.

To apply correction for the effect of finite aspect ratio of the wing, the slope of the lift curve of the aerofoil (a_0) in Eq.(6.17) is replaced by the slope of the lift curve of wing 'a' (Ref.1.7, chapter 9). Note: $a = C_{L\alpha w}$

Hence, $(C'_l)_{aileron} = \dfrac{2a\ \tau_{ail}\ \delta_a}{Sb} \displaystyle\int_{k_1 \frac{b}{2}}^{k_2 \frac{b}{2}} c\ y\ dy$ $\qquad\qquad\qquad$ (6.18)

6.10.3 Damping moment

As the airplane rolls in flight, it produces an opposite rolling moment or a damping moment. The explanation for this is as follows.

Consider an airplane rolled to right i.e. positive rolling motion with the right wing going down and the left wing going up. Let, the angular velocity be 'p'. Now, a wing section at a distance 'y' from the c.g. experiences, on the right wing, a downward velocity of magnitude 'py' or a relative wind of 'py' in the upward direction (Fig 5.9). Similarly, a section at a distance 'y' on the up going left wing experiences a downward relative wind of 'py' (Fig.5.9). Thus, the section on down going wing experiences an increase in angle of attack of $\Delta\alpha = py\ /\ V$. The section on the up going wing experiences a decrease in angle of attack $\Delta\alpha = -py\ /\ V$. These changes in angles of attack would produce changes in the lift on the two wing halves and hence a rolling moment of positive sign is produced. However, this positive moment in present only when there is a rolling velocity 'p'. Hence, it is called damping moment. It should be noted that the change in angle of attack, though dependent on 'py', takes place over the entire wing span. Again, using strip theory, the rolling moment due to a strip of length Δy is:

$(\Delta L')_{damp} = \frac{1}{2}\ \rho V^2\ c\ dy\ (\Delta C_l)_{damp}\ y$

Or $(\Delta C'_l)_{damp} = \dfrac{\frac{1}{2}\rho V^2\ c\ dy\ (\Delta C_l)_{damp}\ y}{\frac{1}{2}\rho\ V^2\ Sb}$ $\qquad\qquad\qquad$ (6.19)

Noting $\quad (\Delta C_l)_{damp} = a_0\ \Delta\alpha = a_0\ \dfrac{py}{V}$,

$$(\Delta C'_I)_{damp} = \frac{c\,(\Delta C_I)_{damp}\,y\,dy}{Sb} = \frac{a_0\,p\,c\,y^2\,dy}{VSb} \tag{6.20}$$

Integrating over both the wing halves and noting that the rolling moment on both the two wing halves reinforce each other, yields:

$$(C'_I)_{damp} = \frac{2a_0\,p}{V\,Sb} \int\limits_{0}^{b/2} c\,y^2\,dy \tag{6.21}$$

To apply correction for the effect of finite aspect ratio of the wing, the slope of the lift curve of the aerofoil (a_0) is replaced again by the slope of the lift curve of wing (a) i.e

$$(C'_I)_{damp} = \frac{2a\,p}{V\,Sb} \int\limits_{0}^{b/2} c\,y^2\,dy \tag{6.22}$$

6.10.4 Rate of roll achieved

The airplane would attain a steady rate of roll when the moment due to the aileron deflection equals the moment due to damping i.e.

$$\frac{2a\,\tau_{ail}\,\delta_a}{Sb} \int\limits_{k_1\frac{b}{2}}^{k_2\frac{b}{2}} c\,y\,dy = \frac{2a\,p}{V\,Sb} \int\limits_{0}^{b/2} c\,y^2\,dy \tag{6.23}$$

Simplifying,

$$p = \tau_{ail}\,V\,\delta_a \frac{\displaystyle\int\limits_{k_1\frac{b}{2}}^{k_2\frac{b}{2}} c\,y\,dy}{\displaystyle\int\limits_{0}^{b/2} c\,y^2\,dy} \tag{6.24}$$

Hence, $$\frac{pb}{2V} = \frac{\tau_{ail}\,b\,\delta_a}{2} \frac{\displaystyle\int\limits_{k_1\frac{b}{2}}^{k_2\frac{b}{2}} c\,y\,dy}{\displaystyle\int\limits_{0}^{b/2} c\,y^2\,dy} \tag{6.25}$$

Some times the deflections of aileron on the up going wing (δ_{aup}) and on the down going wing (δ_{adown}) may not be equal. In this case δ_a is taken as :

$$\delta_a = \frac{\left|\delta_{aup}\right| + \left|\delta_{adown}\right|}{2} = \frac{\delta_{atotal}}{2} \ , \text{ in this case,}$$

$$\frac{pb}{2V} = \frac{\tau_{ail}\, b\, (\delta_{atotal})}{4} \ \frac{\displaystyle\int_{k_1\frac{b}{2}}^{k_2\frac{b}{2}} c\, y\, dy}{\displaystyle\int_{0}^{b/2} c\, y^2\, dy} \tag{6.26}$$

6.10.5 Aileron power

Taking derivative of Eq.(6.18) with respect to δ_a gives the aileron power as:

$$\frac{\partial C_l'}{\partial \delta_a} = C'_{l\delta a} = \frac{2 C_{L\alpha w}\, \tau_{ail}}{Sb} \int_{k_1\frac{b}{2}}^{k_2\frac{b}{2}} c\, y\, dy \tag{6.27}$$

6.10.6 Control force due to aileron

Ailerons are operated by sideward movement of the control stick. An analysis of the control force required can be done in a manner similar to that for rudder and elevator. However, in practice it is more complex as the ailerons on the two wing halves move in opposite direction. See Ref. 1.7 Chapter 9 for some information.

Example 6.1

The lift curve of a light airplane wing of rectangular planform is almost straight between angle of zero lift (-3^0) and the incidence of 10^0 at which $C_L=1.066$. The wing chord is 2.14 m, the aspect ratio is 8.3 and the dihedral angle is 5^0. Assuming that the level flight speed is 41.15 m/s, calculate rolling moment set up by a sudden yaw of 5^0 (Adapted from Ref.1.4, chapter 14 with permission of author).

Solution:

The data supplied are as follows.

$\alpha_{0L} = -3^0$, $C_L=1.066$ at $\alpha = 10^0$.

Hence, $(dC_L/d\alpha)_{wing} = C_{L\alpha w} = (1.066/13) = 0.082 \text{ deg}^{-1}$.

$c = \bar{c} = 2.14$ m, $A = 8.3$, $\Gamma = 5^0$, $V = 41.15$ m/s,

The rolling moment due to wing dihedral $(L'_w)_\Gamma$ is to be calculated when $\beta = 5^0$.

From Eq.(6.3)

$$(L'_w)_\Gamma = -\beta\Gamma \ \frac{dC_L}{d\alpha} \ \frac{1}{2} \ \rho V_\infty^2 \ S \bar{y}$$

$$\bar{y} = \frac{2}{S} \int\limits_0^{b/2} c \ y \ dy$$

In this case c is constant, hence

$$\bar{y} = \frac{2c}{S} \left[\frac{y^2}{2} \right]_0^{b/2} = \frac{2c}{S} \frac{1}{2} \frac{b^2}{4} = \frac{c}{S} \frac{b^2}{4}$$

$c = 2.14$ m, $S = b \times c$, $A = b^2 / S = b^2 / b \times c$

Hence, $b = 8.3 \times 2.14 = 17.762$ m ; $b/2 = 8.881$ m

$S = b \times c = 17.762 \times 2.14 = 38.01$ m^2

Hence,

$$\bar{y} = \frac{2.14}{17.762 \times 2.14} \times \frac{(17.762)^2}{4} = 4.44 \text{ m}$$

And

$$\left(L'_W\right)_{\Gamma=5^0, \beta=5^0} = -\frac{5}{57.3} \times \frac{5}{57.3} \times 0.082 \times 57.3 \times \frac{1}{2} \times 1.225 \ (41.15)^2 \times 38.01 \times 4.44$$

$$= -6262.3 \text{ Nm}$$

Remarks:

i) From the available data we can obtain $(C'_{l\beta})_\Gamma$. From Eq.(6.4a)

$$(C'_{l\beta})_\Gamma = -\Gamma \ \frac{dC_L}{d\alpha} \ \frac{\bar{y}}{b} = -\frac{5}{57.3} \times 0.082 \times 57.3 \times \frac{4.44}{17.762} = -0.102 \text{ rad}^{-1}$$

$$= -0.00179 \text{ deg}^{-1}$$

Hence, $\dfrac{(C'_{l\beta})_\Gamma}{\Gamma} = -\dfrac{0.00179}{5} = -0.000358$

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ii)The procedure to estimate $(C'_{l\beta})_\Gamma$ given in Ref.1.8b, which is based on the one given in Ref.2.2, gives for this case with $\Lambda = 0$, $\lambda = 1$ and $A = 8.3$:

$$\frac{(C'_{l\beta})_\Gamma}{\Gamma} = -0.00027 .$$

Example 6.2

A light airplane has a wing of rectangular planform 12.8 m span, 2.14 m chord and C_{Lmax} of 1.5. The wing loading is 850 N/m^2. The airplane is rolled through 45^0 in one second when flying at three times its stalling speed. Estimate the rolling moment created by the ailerons assuming steady motion (Adapted from Ref.1.4, chapter 14 with permission of author).

Solution:

The prescribed data are as follow.

b = 12.8 m, c = 2.14 m, C_{Lmax} =1.5, W/S = 850 N m^{-2}

The rate of roll (p) = 45^0 s^{-1} = 0.785 rad^{-1}.

Flight velocity = V = 3 V_{stall}

$$V_{stall} = \sqrt{\frac{2W}{\rho\ S\ C_{Lmax}}} = \sqrt{\frac{2\times850}{1.225\ \times1.5}} = 30.41 \text{m/s}$$

V = 3 V_{stall} = 91.23 m/s.

To determine L′ due to aileron we assume that the rolling moment due to aileron equals the damping moment.

From Eq.(6.22)

$$(C'_l)_{damp} = \frac{2ap}{VSb}\int\limits_0^{b/2} c\ y^2 dy$$

S = 12.8 x 2.14 = 27.392 m^2, A = b^2/S = (12.8)^2 / 27.395 = 5.981

Using $C_{L\alpha} = \dfrac{2\pi A}{2+\sqrt{A^2+4}}$ which is approximately valid for unswept wing at low Mach number (see example 5.2),

$$C_{L\alpha} = \frac{2\pi\times5.981}{2+\sqrt{5.981^2+4}} = 4.552\ \text{rad}^{-1}$$

Further, $c\int\limits_0^{b/2} y^2\ dy = c\,[\dfrac{y3}{3}]_0^{b/2} = \dfrac{c}{3}\dfrac{b^3}{8} = \dfrac{2.14}{3}\ \dfrac{(12.8)^3}{8} = 187\,\text{m}^4$

$$(C'_l)_{damp} = \frac{2 \times 4.552 \times 0.785}{91.23 \times 27.392 \times 12.8} \times 187 = 0.0415$$

$$L'_{damp} = \frac{1}{2}\rho V^2 \, Sb \, (C'_l)_{damp}$$

$$= \frac{1}{2} \times 1.225 \times 91.23^2 \times 27.392 \times 12.8 \times 0.0415 = 74173 \text{ Nm}$$

Example 6.3

An airplane has a straight tapered wing with taper ratio (λ) of 0.4 and aspect ratio of 8. It has 0.20 c ailerons extending from 0.55 semi span to 0.90 semi span. If aileron defects up 18^0 and down 12^0 at full deflection, estimate pb/2V for the airplane. If the wing span is 13.64 m, obtain the rate of roll in degrees per second at sea level for air speeds between 150 to 500 kmph.

Solution:

The data supplied are as follows.

λ = 0.5, A = 8, b = 13.64 m

Hence, S = b^2 / A = $(13.64)^2$ / 8 = 23.26 m^2.

Ailerons of 0.2c extends from 0.55 b/2 to 0.9 b/2.

$(\delta_a)_{up} = 18^0$, $(\delta_a)_{down} = 12^0$ hence $(\delta_a)_{total} = 30^0$.

From Eq.(6.26),

$$\frac{pb}{2V} = \frac{\tau_{ail}\,b\,(\delta_{a\,total})}{4} \cdot \frac{\displaystyle\int_{k_1\frac{b}{2}}^{k_2\frac{b}{2}} c\,y\,dy}{\displaystyle\int_{0}^{b/2} c\;y^2 dy}$$

To evaluate the integrals an expression is needed for 'c' as function of 'y'. The root chord (c_r) and the tip chord (c_t) are obtained as :

$$S = \frac{b}{2}(c_r + c_t) = \frac{13.64}{2}\,(c_r + 0.4\,c_r)$$

$$\text{Or} \quad c_r = \frac{23.26 \times 2}{13.64 \times 1.4} = 2.436 \text{ m}$$

c_t = 0.974 m ; (b/2) = 6.82 m

For a straight tapered wing :

$$c = c_r - \frac{y}{b/2}(c_r - c_t)$$

Hence, $c = 2.436 - \dfrac{y}{6.82}(2.436 - 0.974) = 2.436 - 0.2144\,y$

Consequently, $cy = 2.436\,y - 0.2144\,y^2$

The aileron extends for 0.55 b/2 to 0.9 b/2 or from y = 3.751 to 6.138 m.

Hence,

$$\int_{3.751}^{6.138} (2.436\,y - 0.2144\,y^2)\,dy = \left[2.436\frac{y^2}{2} - 0.2144\frac{y^3}{3}\right]_{3.751}^{6.138} = 15.996\,m^3$$

$$\int_{0}^{b/2} c\,y^2\,dy = \int_{0}^{b/2} (2.436 - 0.2144\,y)\,y^2\,dy$$

$$= \left[2.436\frac{y^3}{3} - \frac{0.2144}{4}y^4\right]_{0}^{6.86}$$

$$= \frac{2.436}{3} \times 6.82^3 - \frac{0.2144}{4} \times 6.82^4 = 141.620\,m^4$$

The quantity τ_{ail} can be roughly estimated using Fig. 2.32. For $c_a/c = 0.2$, $\tau = 0.4$

Hence,

$$\frac{pb}{2V} = 0.40 \times 13.64 \times \frac{30}{4\times57.3} \times \frac{15.996}{141.620} = 0.08067$$

The quantity $\dfrac{pb}{2V}\dfrac{1}{\delta_a}$ is a measure of aileron effectiveness. In the present case:

$$\frac{pb}{2V}\frac{1}{\delta_a} = 0.08067/15 = 0.005378\ deg^{-1}.$$

The variation of p with V is given in the table below.

$$p = \frac{pb}{2V} \times \frac{2V}{b} = 0.08067 \times \frac{2}{13.64}\,V = 0.01183\,V$$

V (kmph)	150	200	300	400	500
V (m/s)	41.67	55.55	83.33	111.11	138.89
p (rad/sec)	0.4930	0.6573	0.9858	1.314	1.643

Remark:

The quantity pb/2V will remain constant upto certain speed, then decrease due to reduction in aileron effectiveness owing to flexibility of the structure. It (pb/2V) would be zero at aileron reversal speed.