Chapter 6

Lateral static stability and control

(Lectures 19,20 and 21)

Keywords : Dihedral effect ; criterion for stable directional effect ($C'_{l\beta}$); contributions of wing, fuselage, vertical tail and power to $C'_{l\beta}$; choice of dihedral angle; aileron, differential aileron and spoiler aileron ; rolling moment due to aileron deflection ; damping moment ; aerodynamic balancing ; trim tab, balance tab and servo tab.

Topics

Chapter 6

Lateral static stability and control - 1

Lecture 19

Topics

6.1 Introduction

Chapters 2 to 4 dealt with the static stability and control of motion about the y-axis. Subsequently, chapter 5 dealt with the static stability and control of motion about the z-axis. In this chapter, the static stability and control of motion about the x-axis are discussed. However, as mentioned in section 5.1, the lateral and directional motions are interlinked and this aspect is highlighted when needed.

6.2 Static stability of motion about x-axis – dihedral effect

The lateral stability analysis deals with the motion about x-axis. In this context the following three points may be noted.

(a) The rotation about x-axis leads to the bank angle Φ (Fig.1.12).

(b) A disturbance would change the bank angle from Φ to $(\Phi + \Delta\Phi)$.

(c) For static stability about x-axis, an airplane should develop, a rolling moment to bring the airplane to the original bank angle.

To examine the lateral static stability,consider an airplane in a steady, level flight. In this flight the x-axis coincides with the velocity vector (**V**) and $\Phi = 0$. Let the airplane be given a bank angle Φ gently so that the rate of roll is negligible. It is noticed that even in the banked position the aerodynamic field remains symmetric about the plane of symmetry. Hence, no restoring rolling moment is produced. Thus, the airplane is neutrally stable about x-axis. The restoring moment is brought about in the following manner.

(1) when an airplane acquires a bank angle a component of the weight,
W sinΦ, acts in the y-direction and the airplane begins to sideslip. Consider a roll to right, i.e. right wing down. Due to W sinΦ, the airplane begins to sideslip to right or experiences a relative wind from right to left. This produces a positive β.

(2) If the airplane is rolled to right with an angular velocity 'p' then, as pointed out in subsection 5.8.1 (on adverse yaw), the airplane develops yaw to left. Which results again in a positive β.

(3) When an airplane has a sideslip it produces both rolling moment and yawing moment. If the rolling moment so produced, tends to restore the airplane to the original attitude of $\Phi = 0$, then it can be considered as a stabilizing effect.

Rolling moment due to sideslip is called dihedral effect.

Remark:

Some books (e.g. Ref.1.5) do not discuss static stability about x-axis. However, continuing with Ref.1.1, chapter 2, Ref.1.7, chapter 8 and Ref.1.12, chapter 3, this topic is treated in this chapter.

6.3. Rolling moment and its convention

Rolling moment is denoted as L′ to distinguish it from lift which is denoted by L. This notation is also used in Ref.3.1.

Rolling moment coefficient is denoted as C'_l i.e.

$$C'_l = \frac{L'}{\frac{1}{2}\rho V^2 S b} \tag{6.1}$$

Convention for rolling moment

A rolling moment which causes roll to right or right wing down, is taken as positive. It may be recalled that a moment is positive in clockwise direction when looking along the positive direction of the axis.

6.4 Criterion for stabilizing dihedral effect

As mentioned earlier, a bank to right produces positive β. This β should produce a negative L′ to bring the airplane back to zero roll. Hence, for stabilizing effect, $C'_{l\beta}$ should be negative.

The contribution to $C'_{l\beta}$ can be expressed as:

$$C'_{l\beta} = (C'_{l\beta})_w + (C'_{l\beta})_{f,n,p} + (C'_{l\beta})_{vt} \tag{6.2}$$

6.5 Contribution of wing to $C'_{l\beta}$

The contributions of wing to $C'_{l\beta}$ are due to the dihedral angle (Γ) and the sweep (Λ).

6.5.1 Contribution of wing dihedral angle to $C'_{l\beta}$

A wing is said to have a dihedral, when the tips of the wing are at a higher level than the root of the wing (Fig.6.1). The contribution to $C'_{l\beta}$ due to dihedral angle (Γ) can be calculated with the following steps.

(a) Assume that the airplane rolls to right.

(b) It develops positive β.

(c) Vsin β is the sideward component of the relative velocity (side wind).

(d) The component of the side wind (V sin β) perpendicular to the wing is V sin β sin Γ. But, it is upward on the right wing and downward on the left wing (Fig.6.1).

(e) Δα, the magnitude of the change in the angle of attack on the two wing halves, is:

$$\Delta\alpha = \frac{V \sin\beta \sin\Gamma}{V} \approx \Gamma\,\beta = \Gamma\,\frac{v}{V}$$

v = Vsin β is the sideward velocity.

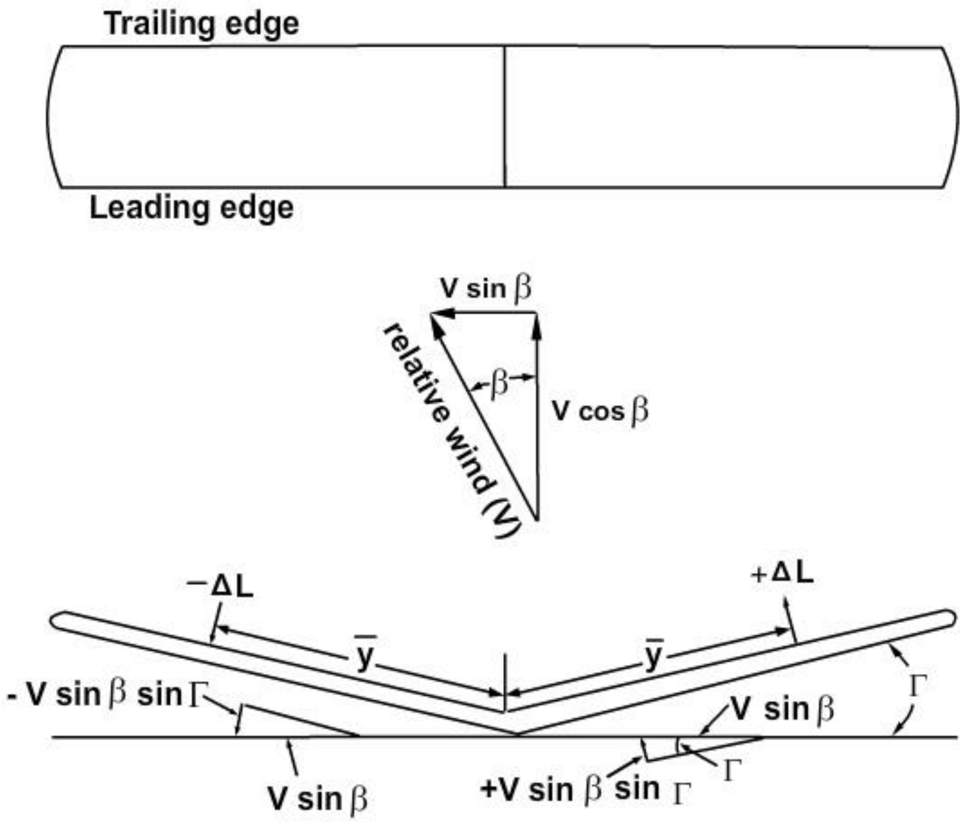


Fig 6.1 Contribution of dihedral angle to C'_l

(f) However, Δα on the right wing = βΓ and Δα on the left wing = -βΓ. Hence, the lifts on the two wing halves are unequal and a rolling moment is produced.

Rolling moment due to Δα on the right wing is:

$$L'_{wr} = -\frac{1}{2}\rho V^2 \frac{dC_L}{d\alpha}\Delta\alpha\int_0^{b/2} c\,y\,dy \qquad \text{Note: } \Delta C_L = \Delta\alpha\,(dC_L/d\alpha)$$

Rolling moment due to Δα on the left wing is:

$$L'_{wl} = -\frac{1}{2}\rho V^2 \frac{dC_L}{d\alpha}\Delta\alpha\int_0^{b/2} c\,y\,dy \qquad \text{Note: } \Delta\alpha \text{ is negative on left wing.}$$

Finally ,

$$(L'_w)_\Gamma = -2\,\frac{1}{2}\rho V^2 \frac{dC_L}{d\alpha}\Delta\alpha\int_0^{b/2} c\,y\,dy$$

Substituting $\Delta\alpha = \beta\,\Gamma$ and $\bar{y} = \frac{2}{S}\int_0^{b/2} c\,y\,dy$, gives :

$$(L'_w)_\Gamma = -2\,\frac{1}{2}\rho\,V^2\,\frac{dC_L}{d\alpha}\,\Gamma\,\beta\,\frac{S}{2}\,\bar{y} \qquad (6.3)$$

$$\text{Or }\ (C'_{l\,w})_\Gamma = (L'_w)_\Gamma\,/\,(\frac{1}{2}\rho\,V^2\,S\,b) = -\,\Gamma\,\beta\,\frac{dC_L}{d\alpha}\,\frac{\bar{y}}{b} \qquad (6.4)$$

$$\text{Hence, }\ (C'_{l\beta})_\Gamma = -\,\Gamma\,\frac{dC_L}{d\alpha}\,\frac{\bar{y}}{b} \qquad (6.4a)$$

Remarks:

i)It may be noted that the contribution of dihedral to $C'_{l\beta}$ is negative.Since, $C'_{l\beta}$ should be negative for static stability, the contribution of dihedral to $C'_{l\beta}$ is called a stabilizing contribution.

ii)For a wing with taper ratio λ, Eq.(6.4a) gives:

$$(C'_{l\beta})_\Gamma = -0.25\,\Gamma\,\frac{dC_L}{d\alpha}\left[\frac{2(1+2\lambda)}{3(1+\lambda)}\right] \text{ per radian} \qquad (6.5)$$

iii) Reference 1.12 chapter 3 and Ref.2.2, section 5.1.2 give refined estimates of $(C'_{l\beta})_\Gamma$.

6.5.2 Contribution of wing to sweep to $C'_{l\beta}$

While discussing the contribution of wing sweep to $C_{n\beta}$, it was pointed out in section 5.3 that for a wing with sideslip, the normal components of free stream velocity (V) are different on the two wing halves (Fig.5.2). This gives rise to different drags on the two wing halves and contributes to yawing moment. Similarly, the difference in normal velocity components would result in the lift on the two wing halves being different in this case. This would give rise to a rolling moment. The contribution of sweep to $C'_{l\beta}$ is obtained by the following steps.

The contributions of the right wing and left wing to the rolling moment are:

$$(L'_{wr})_\Lambda = -\,C_L\,\frac{S}{2}\,\frac{1}{2}\rho\,V_\infty^2\,\bar{y}\,\cos^2(\Lambda - \beta) \qquad (6.6)$$

$$(L'_{wl})_\Lambda = C_L\,\frac{S}{2}\,\frac{1}{2}\rho\,V_\infty^2\,\bar{y}\,\cos^2(\Lambda + \beta) \qquad (6.7)$$

$$\text{Hence,}(L'_w)_\Lambda = -\,C_L\,\frac{S}{2}\,\frac{1}{2}\rho\,V_\infty^2\,\bar{y}\,\{\cos^2(\Lambda - \beta) - \cos^2(\Lambda + \beta)\}$$

$$= - C_L \frac{S}{2}\frac{1}{2}\rho V_\infty^2 \bar{y}\ \{4\cos\Lambda\cos\beta\sin\Lambda\sin\beta\} \qquad (6.8)$$

where, $\bar{y}$ is the location of the resultant lift on the wing half.

Since, β is small,

$$(L'_w)_\Lambda = - C_L \frac{1}{2}\rho V_\infty^2 S\bar{y}\beta \sin 2\Lambda \qquad (6.9)$$

$$(C'_{l\beta w})_\Lambda = - C_L \frac{\bar{y}}{b}\beta \sin 2\Lambda \qquad (6.10)$$

$$(C'_{l\beta w})_\Lambda = - C_L \frac{\overline{y}}{b}\sin 2\Lambda \qquad (6.11)$$

Remarks:

i) The contribution due to sweep back is negative and hence stabilizing. Note that it is proportional to C_L.

ii) Reference 1.12 gives an improved estimate of $(C'_{l\beta w})_\Lambda$. The value of $(C'_{l\beta w})_\Lambda$ given by this reference, increases monotonically with Λ.

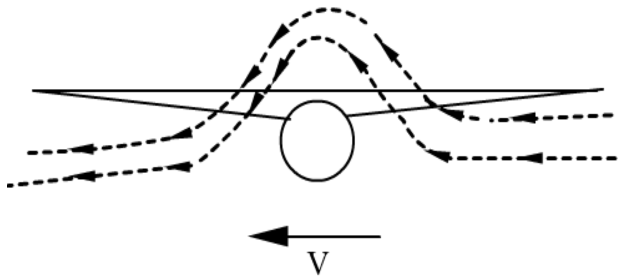
6.6 Contribution of fuselage to $C'_{l\beta}$

The contribution of fuselage to $C'_{l\beta}$ arises due to the interference effect. Consider an airplane having positive β or the sideward velocity component (v = V sinβ) from right to left. Figure 6.2 shows the displacement of streamlines for high wing and low wing configurations. It is observed that for the high wing case the change in angle of attack, Δα, is positive on the right wing and negative on the left wing. This would result in a negative rolling moment which is a stabilizing contribution. For a low wing configuration the effect would be opposite and a destabilizing contribution. References 1.8b and 1.12, give elaborate methods of estimating effect of wing fuselage interference. Reference 1.7 chapter 9 gives the following approximate estimates for high wing, mid-wing and low wing cases.

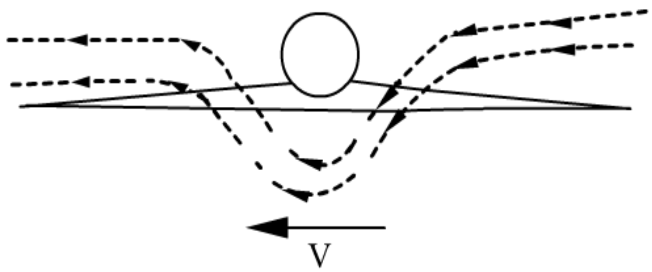
$\Delta C'_{l\beta}$ = - 0.0006 deg^{-1} for high wing,

= 0.00 for mid-wing and

= +0.0006 deg^{-1} for low wing configuration.



High wing configuration



Low wing configuration

Fig 6.2 Effect of wing location on $C'_{l\beta}$

6.7 Contribution of vertical tail to $C'_{l\beta}$

It is shown in section 5.6 that a vertical tail with a positive β causes a negative side force. This side force generally acts above the c.g and would contribute a negative rolling moment (Fig.6.3).

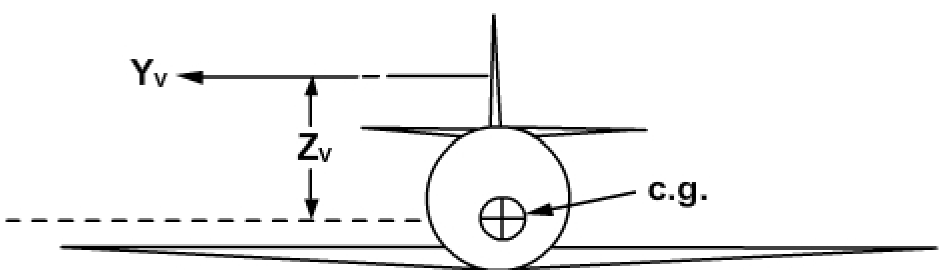


Fig.6.3 Contribution of vertical tail to $C'_{l\beta}$

The contribution of vertical tail can be expressed as:

$$L' = - Y_v \, Z_v \quad (6.12)$$

$$= -\frac{1}{2}\rho V^2 \, \eta_v \, S_v \, \left(\frac{dC_L}{d\alpha}\right)_v \beta Z_v$$

Hence, $(C'_{l\beta})_v = -\,\eta_v \dfrac{S_v}{S}\dfrac{Z_v}{b} C_{L\alpha v}$ (6.13)

Remarks:

i) When the airplane has an angle of attack (α) the height Z_v will depend on the tail length l_v and the angle of attack α. The example presented in appendix 'C' takes into account this correction.

ii) Generally $(C'_{l\beta})_v$ is small.

6.8 Contributions due to propeller and flaps to $C'_{l\beta}$

Figure 5.6 shows a propeller in positive sideslip. As pointed out in section 5.5 the slipstream is rendered asymmetric due to sideslip. A larger portion of the left wing is influenced by the slipstream as compared to that on the right wing. Hence, the left wing will experience a higher dynamic pressure and consequently will produce more lift as compared to the right wing. This causes a positive rolling moment. Hence, $C'_{l\beta}$ is positive, i.e. a destabilizing contribution. This influence worsens when the flaps are deflected (Fig.5.6). However, the contributions due to propeller and flap to $C'_{l\beta}$ are small.

6.9 Selection of dihedral angle

As noted earlier (Exercise 2.6 and Example 5.2), the levels of longitudinal and directional static stability ($C_{m\alpha}$ and $C_{n\beta}$) can be adjusted by changing the areas of the horizontal tail (S_t) and the vertical tail (S_v) respectively. The level of $C'_{l\beta}$, can be adjusted by choosing an appropriate dihedral angle. To arrive at the dihedral angle needed for an airplane, the contributions due to wing sweep, fuselage, power plant and vertical tail are first calculated. Then, the difference between the sum of these contributions and the desirable level of $C'_{l\beta}$ is provided by choosing an appropriate dihedral angle.

Reference 1.7 chapter 9, provides a rough guideline as:

$(C'_{l\beta})_{desirable} = -\,(C_{n\beta}/2)$ (6.14)

However, the data on seven airplanes given in appendix 'B' of reference 1.1 indicates that Eq.(6.14) may be approximately valid for military airplanes. For other airplanes, $C'_{l\beta}$ could be equal to or higher than $C_{n\beta}$. Actual values of $C'_{l\beta}$ and

$C_{n\beta}$ are arrived at after carrying out the lateral dynamic stability analysis (refer to chapter 9).

6.9.1 Wing with anhedral

As mentioned earlier, $C'_{l\beta}$ should not be too high. When an airplane has highly swept wings and in addition has high wing configuration then the contributions due to these factors may be large. Sometimes, such airplanes have negative dihedral which is called anhedral (Fig.6.4).

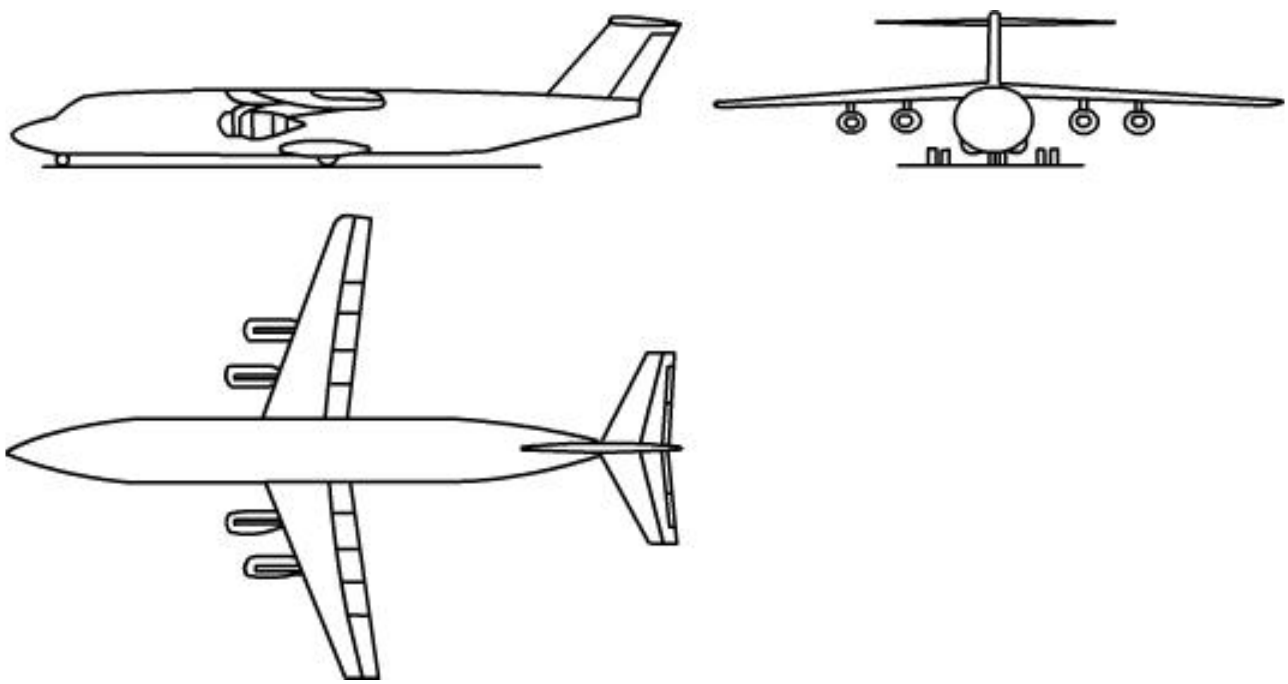


Fig.6.4 An airplane with anehedral

(Note: swept and high wing configuration)