Chapter 5

Performance analysis I – Steady level flight

(Lectures 17 to 20)

Keywords: Steady level flight – equations of motion, minimum power required, minimum thrust required, minimum speed, maximum speed; stalling speed; equivalent airspeed.

Topics

Chapter 5

Lecture 17

Performance analysis I – Steady level flight – 1

Topics

5.1 Introduction:

During its normal operation an airplane takes –off, climbs to the cruising altitude, cruises at almost constant altitude, descends and lands. It may also fly along curved paths like turns, loops etc. The flights along curved paths are also called manoeuvres. Analyses of various flights are the topics under the performance analysis. A revision of section 1.6 would be helpful at this stage.

5.1.1 Subdivisions of performance analysis

Performance analysis covers the following aspects.

I) Unaccelerated flights:

(a) In a steady level flight an airplane moves with constant velocity at a constant altitude. This analysis would give information on the maximum level speed and minimum level speed at different altitudes.

(b) In a steady climb an airplane climbs at constant velocity. This analysis would provide information on the maximum rate of climb, maximum angle of climb and maximum attainable altitude (ceiling).

(c) In a steady descent an airplane descends with constant velocity. A glide is a descent with zero thrust. This analysis would give the minimum rate of sink and time to descend from an altitude.

(d) Range is the horizontal distance covered, with respect to a given point on the ground, with a given amount of fuel. Endurance is the time for which an airplane can remain in air with a given amount of fuel.

II) Accelerated flights:

(a) In an accelerated level flight an airplane moves along a straight line at constant altitude and undergoes change in flight speed. This analysis provides information about the time required and distance covered during acceleration over a specified velocity range.

(b) In an accelerated climb, an airplane climbs along a straight line accompanied by a change in flight speed. This analysis gives information about the change in the rate of climb in an accelerated flight as compared to that in a steady climb.

(c) Loop is a flight along a curved path in a vertical plane whereas a turn is a flight along a curved path in a horizontal plane. This analysis would give information about the maximum rate of turn and minimum radius of turn. These items indicate the maneuverability of an airplane.

(d) During a take-off flight an airplane starts from rest and attains a specified height above the ground.This analysis would give information about the take-off distance required.

(e) During a landing operation the airplane descends from a specified height above the airport, lands and comes to rest. This analysis would provide information about the distance required for landing.

5.1.2 Importance of performance analysis

The performance analysis is important to asses the capabilities of an airplane as indicated in the previous subsection. Moreover, from the point of view of an airplane designer, this analysis would give the thrust or power required, maximum lift coefficient required etc. to achieve a desired performance. This analysis would also point out the new developments required, in airplane aerodynamics and engine performance, to achieve better airplane performance.

5.1.3 Approach in performance analysis

As mentioned in subsection 1.1.3 the approach here is to apply the Newton's laws and arrive at the equations of motion. The analysis of these equations would give the performance.

Remarks:

i) References 1.1, 1.5 to 1.13 may be referred to supplement the analysis described in this and the subsequent five chapters.

ii) It would be helpful to recapitulate the following points.

(a) A 'Flight path' is the line along which the centre of gravity (c.g.) of the airplane moves. Tangent to the flight path gives the direction of the 'Flight velocity' (see Fig.5.1).

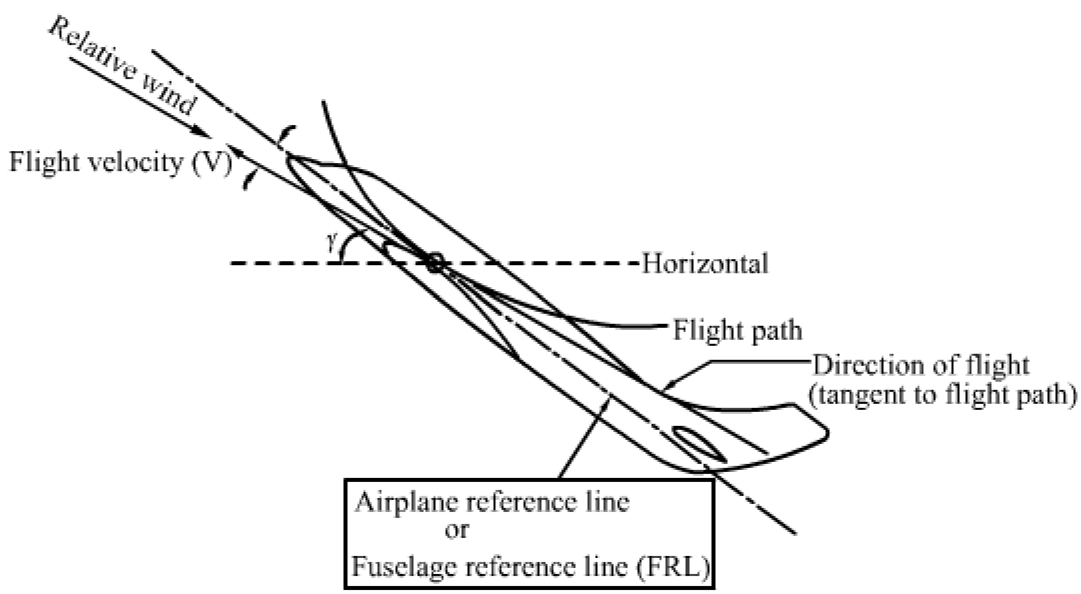

Fig.5.1 Flight path

(b) The external forces acting on a rigid airplane are:

(I) Aerodynamic forces (lift and drag)

(II) Gravitational force

(III) Propulsive force (thrust)

(c) The forces produced due to control deflection, needed to balance the moments, are assumed to be small as compared to the other forces. With this assumption all the forces acting on the airplane are located at the centre of gravity (c.g.) of the airplane (Fig.5.2) and its motion is simplified to that of a point mass moving under the influence of aerodynamic, propulsive and gravitational forces.

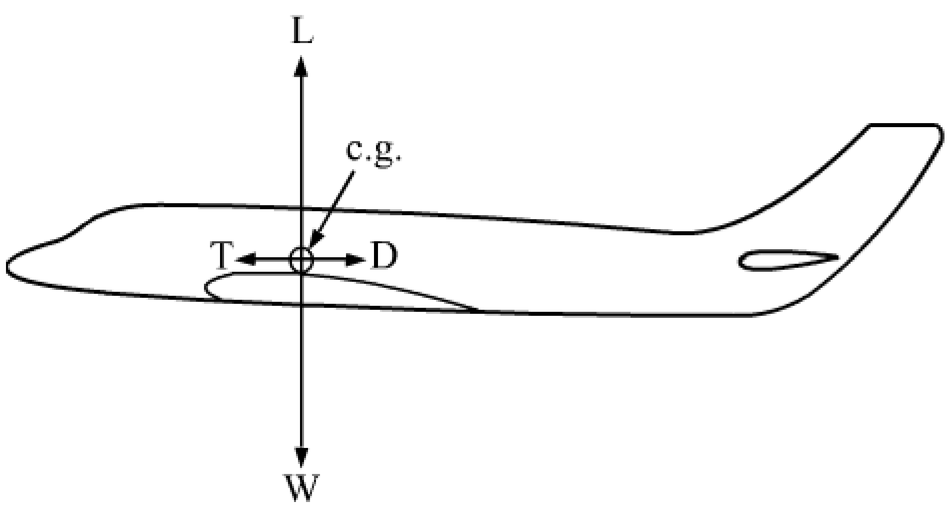

Fig.5.2 Steady level flight

5.2 Equations of motion for steady level flight

In this flight the c.g. of the airplane moves along a straight line at a constant velocity and at a given altitude. The flight path, in this case, is a horizontal line. The forces acting on the airplane are shown in Fig.5.2. 'T' is Thrust, 'D' is Drag, 'L' is lift and 'W' is the weight of the airplane.The equations of motion are obtained by resolving, along and perpendicular to the flight direction, the forces acting on the airplane. In the present case, the following equations are obtained.

$$T - D = m\, a_x$$

$$L - W = m\, a_z$$

where, m is the mass of airplane and a_x, and a_z, are the components of the acceleration along and perpendicular to the flight path respectively.

As the flight is steady i.e. no acceleration along the tangent to the flight path, implies that $a_x = 0$. Further, the flight is straight and at constant altitude, hence, $a_z = 0$.

Consequently, the equations of motion reduce to:

$$T - D = 0, \; L - W = 0 \tag{5.1}$$

Noting that, $L = (1/2)\rho V^2 S C_L$ and L = W in level flight, gives :

$$W = (1/2)\rho V^2 S C_L$$

Or

$$V = (2W / \rho S C_L)^{1/2} \tag{5.2}$$

Further, $(1/2)\rho V^2 S = W / C_L$

Noting that, $D = (1/2)\rho V^2 S C_D$ and T = D in level flight, gives the thrust required (T_r) as :

$$T_r = D = (1/2)\,\rho V^2 S C_D$$

Substituting for $(1/2)\,\rho V^2 S$ as W / C_L, yields:

$$T_r = W\,(C_D / C_L) \tag{5.3}$$

The power required (P_r), in kiloWatts, is given by:

$$P_r = T_r V/1000 \tag{5.3a}$$

where T_r is in Newton and V in m/s.

Substituting for V and T_r from Eqs. (5.2) and (5.3) in Eq.(5.3a) yields:

$$P_r = \frac{W}{1000} \times \frac{C_D}{C_L} \times \sqrt{\frac{2W}{\rho S\, C_L}}$$

$$\text{Or } P_r = \frac{1}{1000}\sqrt{\frac{2W^3}{\rho\, S}}\;\frac{C_D}{C_L^{3/2}} \tag{5.4}$$

Remarks:

i) Equations (5.1) to (5.4) are the basic equations for steady level flight and would be used in subsequent analysis of this flight.

ii) To fly in a steady level flight at chosen values of h and V, the pilot should adjust the following settings.

(a) The angle of attack of the airplane to get the desired lift coefficient so that the lift(L) equals the weight(W).

(b) The throttle setting of the engine, so that thrust equals drag at the desired angle of attack. He (pilot) will also have to adjust the elevator so that the airplane is held in equilibrium and the pitching moment about c.g. is zero at the required angle of attack. As noted earlier, the forces (lift and drag) produced due to the elevator deflection are neglected.

5.3 Stalling speed:

Consider that an airplane which has weight (W) and wing area (S), is flying at an altitude (h). From Eq.(5.2) it is observed that, the flight velocity (V) is proportional to $1/C_L^{1/2}$. Thus, the value of C_L required would increase as the flight speed decreases. Since C_L cannot exceed C_{Lmax}, there is a flight speed below which level flight is not possible. The flight speed at which C_L equals C_{Lmax} is called 'Stalling speed' and is denoted by V_S. Consequently ,

$$V_S = (2W / \rho SC_{Lmax})^{1/2} \qquad (5.5)$$

It is evident from Eq.(5.5) that V_S increases with altitude since the density (ρ) decreases with height.The variations of V_S with h for a typical piston engined airplane and a typical jet airplane are presented in Figs.5.3a and b respectively. Appendices A & B give the details of calculations.

Remark:

The maximum lift coefficient (C_{Lmax}) depends on the flap deflection (δ_f). Hence, V_S will be different for the cases with (a) no flap (b) flap with take-off setting (c) flap with setting for landing. Figure 5.3a presents the variations of stalling speed, with altitude, for four cases viz. with no flap and with three different flap settings.

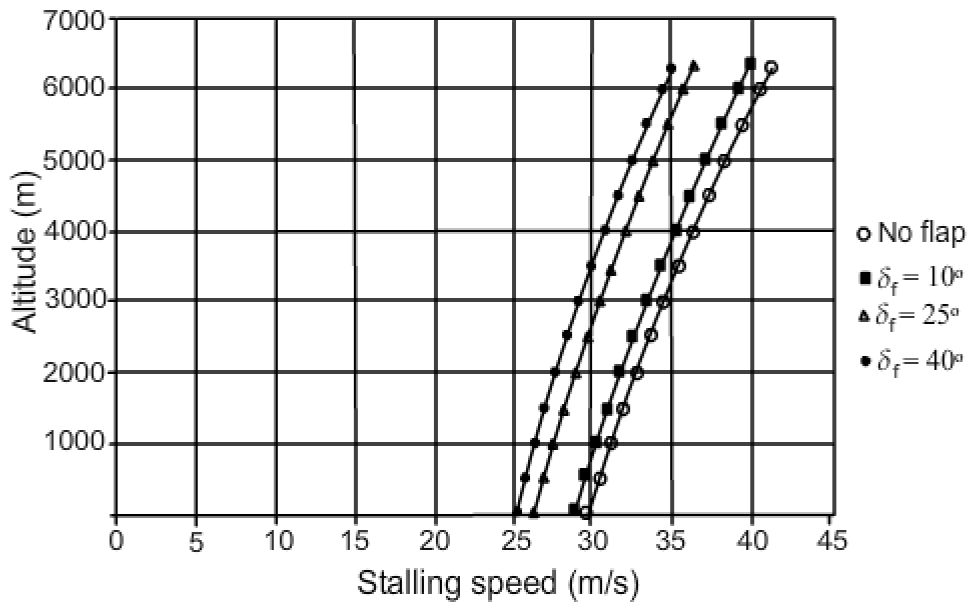

Fig.5.3a Variations of stalling speed with altitude for a low speed airplane

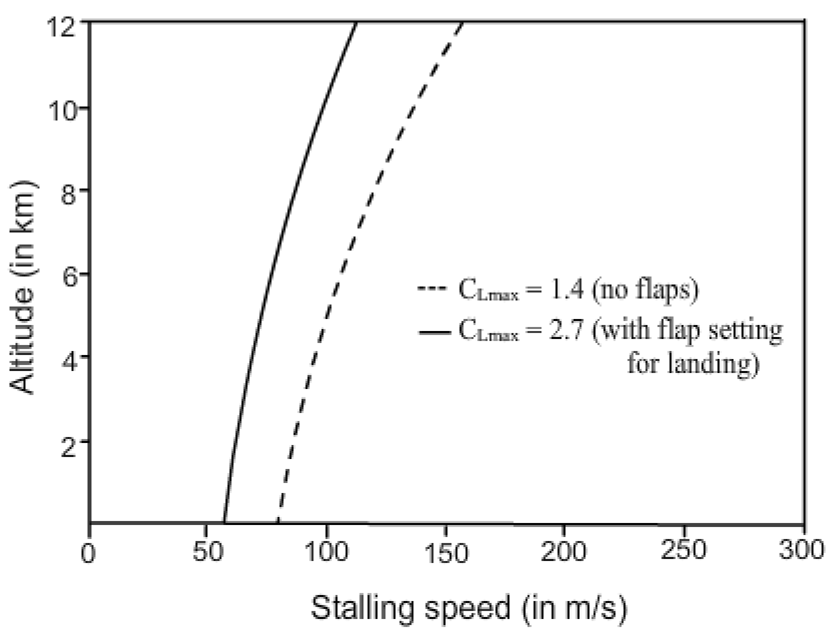

Fig.5.3b Variations of stalling speed with altitude for a jet transport

5.4 Equivalent airspeed

Equivalent airspeed (V_e) is defined by the following equation.

$$\frac{1}{2}\rho V^2 = \frac{1}{2}\rho_o V_e^2$$

Noting $\sigma = \rho/\rho_o$, V_e can be expressed as :

$$V_e = V\sigma^{1/2} = \sqrt{\frac{2W}{\rho_o S C_L}} \qquad (5.6)$$

Remarks:

i) From Eq.(5.6) it is evident that for a given wing loading (W/S), the equivalent airspeed in steady level flight is proportional to $1/C_L^{1/2}$ and is independent of altitude. Thus the stalling speed, for a given airplane configuration, when expressed as equivalent airspeed is independent of altitude.

ii) To avoid confusion between equivalent airspeed (V_e) and the actual speed of the airplane relative to the free stream (V), the latter is generally referred to as true airspeed.

5.4.1 Airspeed indicator

The equivalent airspeed is also significant from the point of view of measurement of speed of the airplane using Pitot-static system. It may be recalled from the topics studied in fluid machanics that a Pitot-static tube senses the Pitot (or total) pressure (p_t) and the static pressure (p_s). The difference between p_t and p_s is related to the velocity of the stream (V_∞) by the following equation.

$$p_t - p_s = \frac{1}{2}\rho V_\infty^2\left(1+\frac{M^2}{4}+\frac{M^4}{40}+....\right); M = V_\infty/a,\ a = \text{speed of sound} \qquad (5.6a)$$

Thus, at low speeds (M < 0.2),

$$p_t - p_s \approx \frac{1}{2}\rho V_\infty^2$$

It may be pointed out that, in the case of an airplane, the air is stationary and the airplane is moving. Hence, the quantity V_∞ in the above expressions, equals the speed of the airplane(V). Hence, at low speeds:

$$p_t - p_s \approx \frac{1}{2}\rho V^2 = \frac{1}{2}\rho_0 V_e^2 \qquad (5.6b)$$

In an airplane the Pitot pressure is sensed by a Pitot tube mounted on the airplane and static pressure is sensed by a hole located at a suitable point on the airplane.These two pressures are supplied to the airspeed indicator mounted in the cockpit of the airplane. The mechanism of the airspeed indicator in low speed airplanes is such that it senses $\left(p_t - p_s\right)$ and indicates V_e. Note that ρ_0 is a constant value.

At subsonic speeds, when the compressibility effects become significant, the airspeed indicator mechanism is calibrated to indicate 'Calibrated airspeed (V_{cal})', based on the following equation which is a simplified form of Eq.(5.6a).

$$p_t - p_s = \frac{1}{2}\rho_0 V_{cal}^2 \left\{1 + \frac{1}{4}\frac{V_{cal}^2}{a_0^2}\right\} \qquad (5.6c)$$

where, a_0 = speed of sound under sea level standard conditions.

For further details like construction of airspeed indicators and measurement of airspeed at supersonic Mach numbers, Refs. 5.1, 5.2 and 5.3 may be consulted. Information is also available on the internet. However, it may be added that the static pressure sensed by the static pressure hole may be influenced by the flow past the airplane. It may be slightly different from the free stream static pressure and hence the speed indicated by the airspeed indicator may be slightly different from V_e or V_{cal}. The speed indicated by the airspeed indicator is called 'Indicated airspeed' and denoted by V_i.

Remark:

On high speed airplanes the speed with respect to ground called 'Ground speed' is deduced from the coordinates given by the global positioning system (GPS). However, airspeed indicator based on Pitot static system is one of the mandatory instruments on the airplane.

5.5 Thrust and power required in steady level flight – general case

From Eqs.(5.3) and (5.4) it is noted that :

$$T_r = W\frac{C_D}{C_L} \text{ and } P_r = \frac{1}{1000}\sqrt{\frac{2W^3}{\rho S}}\frac{C_D}{C_L^{3/2}}.$$

The drag coefficient (C_D)depends on the lift coefficient (C_L) and the Mach number. The relationship between C_D and C_L, the drag polar, is already known from the estimation of the aerodynamic characteristics of the airplane.

Thus, when the drag polar, the weight of the airplane and the wing area are prescribed, the thrust required and the power required in steady level flight at various speeds and altitudes can be calculated for any airplane using the above equations. The steps are as follows.

i) Choose an altitude (h).

ii) Choose a flight velocity (V).

iii) For the chosen values of V and h, and given values of the weight of airplane (W) and the wing area(S) calculate C_L as :

$$C_L = \frac{2W}{\rho S V^2}$$

where ρ corresponds to the density at the chosen 'h'.

iv) Calculate Mach number from M = V/a; 'a' is the speed of sound at the chosen altitude.

v) For the values of C_L and M, calculated in steps (iii) and (iv), obtain C_D from the drag polar. It (drag polar) may be given in the form of Eqs.(3.45) or (3.49). The drag polar can also be given in the form of a graph or a table.

vi) Knowing C_D, The thrust required (T_r) and power required (P_r) can now be calculated using Eqs.(5.3) and (5.4).