Chapter 5

Directional static stability and control

(Lectures 16,17 and 18)

Keywords : Sideslip and yaw ; criteria for equilibrium and static stability about z-axis ; contributions of wing, fuselage, power and vertical tail to $C_{n\beta}$; desirable level of $C_{n\beta}$; critical case for directional control ; rudder lock ; dorsal fin.

Topics

Chapter 5

Directional static stability and control - 1

Lecture 16

Topics

5.1 Introduction

Chapters 2,3 and 4 dealt with longitudinal static stability. In this case, the motion of the airplane takes place in the plane of symmetry i.e. along x- and z-axes and about y- axis. This chapter and the next one, deal with the motions along y-axis and about x- and z-axes. These motions lie outside the plane of symmetry. The translatory motion along y-axis is sideslip and rotations about x- and z-axes are the rolling and yawing respectively. The directional stability and control, deal with the equilibrium and its maintainability about the z-axis. The lateral stability and control, deal with the equilibrium and its maintainability about the x-axis. However, the lateral and directional motions cannot be separated completely because a change in one of them leads to change in the other. For example, when an airplane has a rate of roll, the unequal changes in the drag of the two wing halves create a yawing moment (see subsection 5.8.1). Besides the rolling and yawing motions, the sideslip also creates forces and moments affecting lateral and directional motions. The six effects caused by rolling, yawing and sideslip are listed below.

i.Rolling moment due to rate of roll. It is called damping in roll.

ii.Yawing moment due to rate of yaw. It is called damping in yaw.

iii.Rolling moment due to rate of yaw. It is called cross effect.

iv. Yawing moment due to rate of roll. It is called adverse yaw.

v. Rolling moment due to sideslip. It is called dihedral effect.

vi.Yawing moment due to sideslip. It is called weathercock effect.

The directional static stability and control are considered in this chapter.

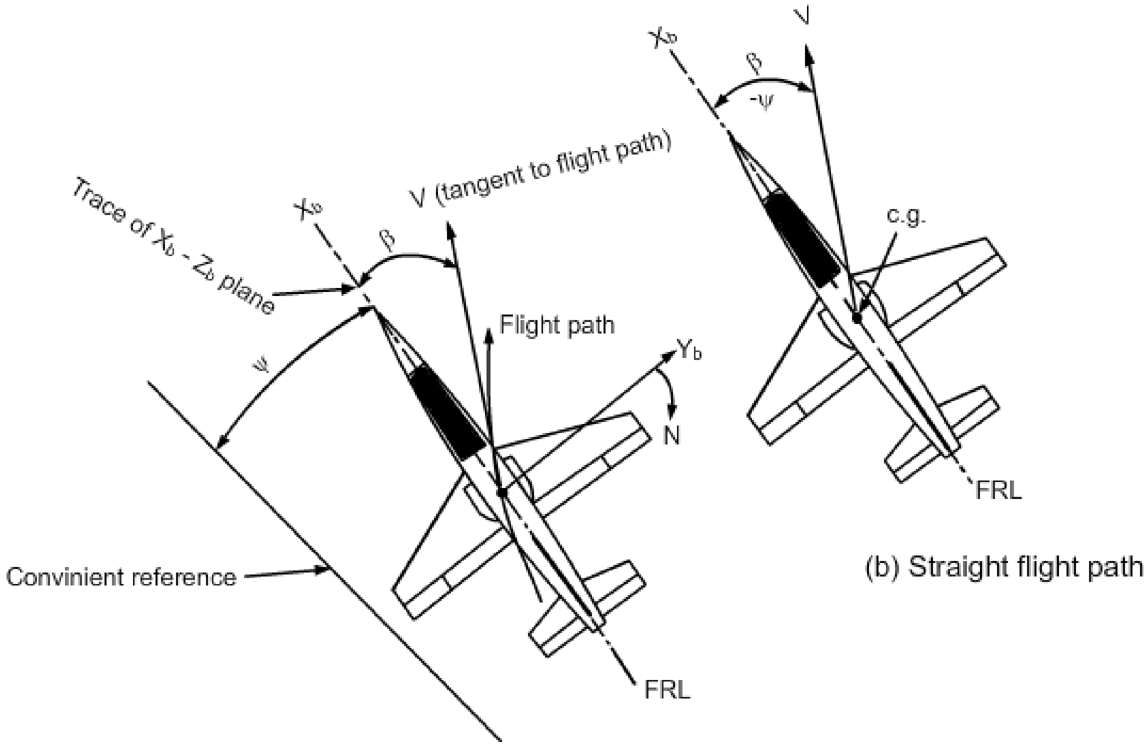
5.2 Criteria for equilibrium and static stability about z-axis

In an equilibrium flight, the airplane flies in the plane of symmetry with sideslip and yawing moment both being zero. Before discussing the criteria for equilibrium and static stability about z- axis, it is useful to recapitulate a few relevant concepts.

5.2.1 Sideslip and yaw

Sideslip is the angle between the plane of symmetry of the airplane and the direction of motion. It is taken as positive in the clockwise sense (Fig.5.1a, see also Fig.1.15). It is denoted by 'β'. It may be recalled that the tangent to the flight path is the direction of motion. It may be further pointed out that a positive β is due to a positive sideslip velocity which is the component of airplane velocity along the y-axis.

Angle of yaw is the angular displacement of the airplane center line, about a vertical axis, from a convenient horizontal reference line. It is measured from the arbitrarily chosen reference direction and taken as positive in the clockwise direction. It is denoted by 'ψ'(Fig.5.1a).



(a) Curved flight path

Fig.5.1 Slide slip and yaw

Remarks:

i) The sideslip angle and the yaw angle are not equal. For example in a 360º turn, the airplane yaws through 360º, but there may not be any sideslip if the airplane axis is aligned with the tangent to the flight path at all points during the turn.

ii) If the flight path is a straight line (Fig.5.1b) and the arbitrary axis chosen to measure the yaw is taken as the direction of flight, then yaw and sideslip angles are equal in magnitude, but opposite in sign (Fig.5.1b).

iii) In wind tunnel tests, the models of airplane are tested by rotating the airplane center line with respect to the air stream and the angle between the plane of symmetry of the airplane and the air stream is called the angle of yaw. The results are reported as variations of yawing moment with ψ.

iv) In flight test work however, sideslip angle β is generally used.

In the subsequent analysis, the case of straight flight path is considered. Even if the flight path is curved, the analysis can be carried out by taking into consideration a small segment of the path; however, the damping due to angular velocity will need to be taken into account. Following Ref.1.1, the subsequent analysis is carried out in terms of β.

5.2.2 Yawing movement and its convention

The moment about z-axis i.e. yawing moment is denoted by N. Considering contributions from major components of the airplane, N can be written as:

$$N = (N)_w + (N)_f + (N)_n + (N)_p + (N)_{vt} \tag{5.1}$$

Where, the suffixes w, f, n, p and v_t indicate contributions from wing, fuselage, nacelle, power and vertical tail.

Equation 5.1 is expressed in non-dimensional form as:

$$C_n = \frac{N}{\frac{1}{2}\rho V^2 Sb} = (C_n)_w + (C_n)_{f,n,p} + (C_n)_{vt} \tag{5.2}$$

Remarks:

i) Convention

The yawing moment (N) is taken positive when the right wing goes back (see Figs.5.1a and 1.8). Recalling the convention for pitching moment, it is observed that in stability analysis a moment is taken positive in a clockwise direction when looking along the axis under consideration.

ii) In Eq.(5.2), the wing span (b) is used as the reference length for non-dimensionalization of yawing moment, whereas the mean aerodynamic chord ($\bar{c}$) is used as the reference length for non-dimensionalization of pitching moment. To explain this difference in choice of reference length, it may be recalled that a moment is the product of a force and the distance perpendicular to the axis. Hence, reference length for non-dimensionalization of a moment should be a representative length perpendicular to the axis under consideration.

5.2.3 Criterion for equilibrium about z-axis

The criteria for equilibrium about z-axis is that the yawing moment should be zero i.e. for equilibrium, $C_n = 0$. (5.3)

5.2.4 Criterion for directional static stability

Figures 5.1a and b show the conventions for positive yawing moment and sideslip (β). Consider that in equilibrium flight, the airplane is flying with β = 0. Now, let a disturbance cause the airplane to develop positive sideslip of Δβ. It is observed that to bring the airplane back to equilibrium position i.e. β = 0, a positive yawing moment (ΔN) should be produced by the airplane. Similarly, a disturbance causing a negative Δβ should result in – ΔN i.e. for static directional stability, $dC_n / d\beta$ or $C_{n\beta}$ should be positive. Hence,

$C_{n\beta} > 0$ for static directional stability
$= 0$ for neutral directional stability and (5.4)
< 0 for directional instability.

Differentiating Eq.(5.2) yields:

$$C_{n\beta} = (C_{n\beta})_w + (C_{n\beta})_{f,n,p} + (C_{n\beta})_{v\,t} \quad (5.5)$$

Remark:

It may be recalled that $C_{m\alpha}$ should be negative for longitudinal static stability whereas $C_{n\beta}$ should be positive for directional static stability. This difference in sign is due to conventions used for α and β. Compare Figs.1.14 and 1.15b. However, it may also be pointed out that when α is positive, the 'w' component of flight velocity is along positive z- direction and when β is positive, the sideslip velocity 'v' is along positive y-axis (see Fig.1.15).

The contributions of major components to c_n and $c_{n\beta}$ are discussed in the next four sections.

5.3 Contribution of wing to $C_{n\beta}$

For straight (or unswept) wings, there is no significant contribution of wing to $C_{n\beta}$. However, for swept wings, there is a small contribution. Reference.3.1 chapter 15, explains this contribution based on certain simplifying assumptions. This approach is explained below.

The explanation is based on the argument that for a swept wing, the component of the free stream velocity normal to the quarter chord line mainly decides the aerodynamic forces. Consider an airplane with wings which have sweep Λ. When this wing is subjected to sideslip β, the components of the free stream velocity normal to the quarter chord line on the two wing halves will be unequal i.e. V cos (Λ-β) on the right wing and V cos (Λ+β) on the left wing. Consequently, even if the two wing halves are at the same angle of attack, they would experience unequal effective dynamic pressures and their drags will be different. This will cause a yawing moment. The contribution of a swept wing to $C_{n\beta}$ can be derived in the following manner.

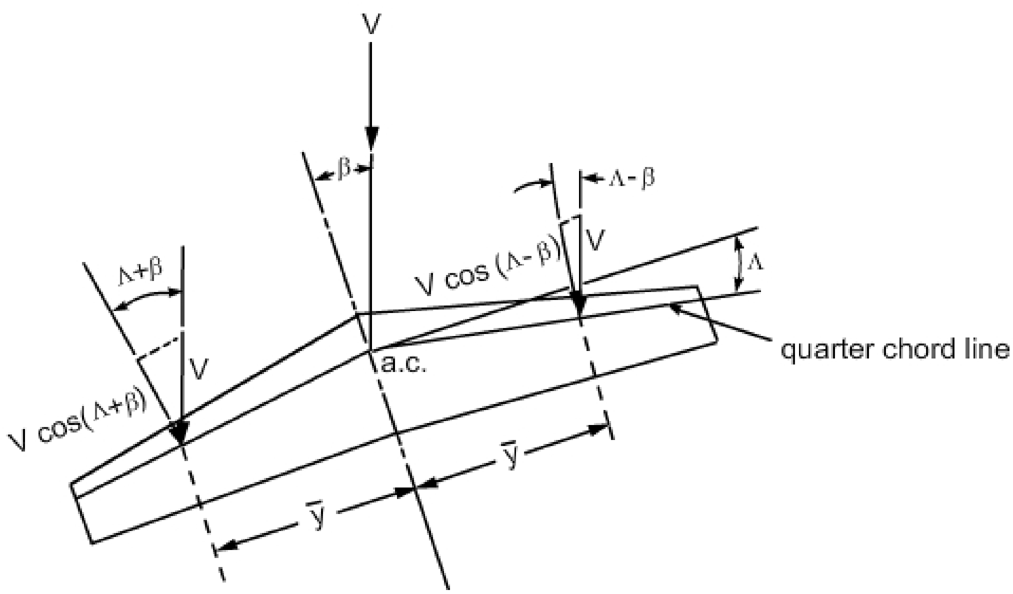


Fig.5.2 Swept wing with sideslip

In general, the chord of the wing and the span wise lift distribution varies with the span wise coordinate (y). However, for the sake of explanation let $\bar{y}$ be the span wise location of the resultant drag on the right wing. Similarly let $-\bar{y}$ be the location of the resultant drag on the left wing. Then, the yawing moments due to the right and the left wing halves $(N_w)_r$ and $(N_w)_l$ are:

$$(N_w)_r = \frac{1}{2}\rho V^2 C_D \frac{S}{2} \bar{y} \cos^2(\Lambda\text{-}\beta) \tag{5.6}$$

$$(N_w)_l = -\frac{1}{2}\rho V^2 C_D \frac{S}{2}\bar{y}\cos^2(\Lambda+\beta) \qquad (5.7)$$

Consequently, total yawing moment due to the wing is:

$$N_w = \frac{1}{2}\rho V^2 C_D \frac{S}{2}\bar{y}\{\cos^2(\Lambda-\beta)-\cos^2(\Lambda+\beta)\}$$

$$=\frac{1}{2}\rho V^2 C_D \frac{S}{2}\bar{y}\{4\cos\Lambda\sin\Lambda\cos\beta\sin\beta\} \qquad (5.8)$$

For small β, sin β = β and cos β = 1. Hence,

$$N_w = C_D S\bar{y}\frac{1}{2}\rho V^2\frac{\beta}{57.3}\sin 2\Lambda;\ \beta \text{ in degrees.} \qquad (5.9)$$

Hence, $(C_n)_w = \frac{N_w}{\frac{1}{2}\rho V^2 Sb} = C_D\frac{\bar{y}}{b}\frac{\beta}{57.3}\sin 2\Lambda$ (5.10)

Differentiating Eq.(5.10) with β yields :

$$(C_{n\beta})_w = C_D\frac{\bar{y}}{b}\frac{1}{57.3}\sin 2\Lambda \qquad (5.11)$$

Remarks:

i) As mentioned earlier Eq.(5.11) is an approximate estimate. Reference 1.12, based on DATCOM (Ref.2.2), gives a more accurate formula for $C_{n\beta w}$ due to sweep. The formula shows that $C_{n\beta w}$ depends also on the wing aspect ratio and the distance between the a.c. and c.g.. Further the contribution is proportional to C_L^2 and would be small during cruise.

ii) A wing with a dihedral also contributes to $C_{n\beta w}$. See section 6.5.1.

iii) It may be noted that the contribution of wing to $C_{n\beta}$ is positive. Since, $C_{n\beta}$ should be positive for directional static stability, a positive contribution to $C_{n\beta}$ is called stabilizing contribution.

5.4 Contribution of fuselage to $C_{n\beta}$

In subsection 2.5.1 it is shown that a fuselage at an angle of attack produces a pitching moment and also contributes to $C_{m\alpha}$. Similarly, a fuselage in sideslip produces a yawing moment and contributes to $C_{n\beta}$ of the airplane. However, in an airplane, the flow past a fuselage is influenced by the flow past the wing. Hence, instead of an isolated fuselage, the contributions of wing and fuselage to $C_{n\beta}$ are estimated togather. It is denoted by $C_{n\beta wf}$. Based on Ref.2.2, section 5.2.3, the following formula is presented for $C_{n\beta wf}$.

$$C_{n\beta wf} = -k_n \ k_{Rl} \frac{S_{fs}}{S_w} \frac{l_f}{b} \ deg^{-1} \tag{5.12}$$

where, k_n is the wing body interference factor which depends on the following fuselage parameters.

(a) Length of fuselage (l_f). (b) Projected side area of fuselage (S_{fs}).

(c) Heights (h_1 and h_2) of fuselage at $l_f/4$ and $3l_f/4$. (d) Distance, from nose, of the station where the height of fuselage is maximum (x_m).

Figure 5.3 illustrates the procedure to obtain k_n when x_m/l_f = 0.586, l_f^2 / S_{fs}= 10, $(h_1/h_2)^{1/2}$ = 1.0 and h / w_f = 1.0.

The quantity k_{Rl} is an empirical factor which depends on the Reynolds number of the fuselage ($R_{lf} = V\rho l_f/\mu$) (see Fig.5.4).

Appendix 'C' illustrates the procedure to obtain $C_{n\beta wf}$.

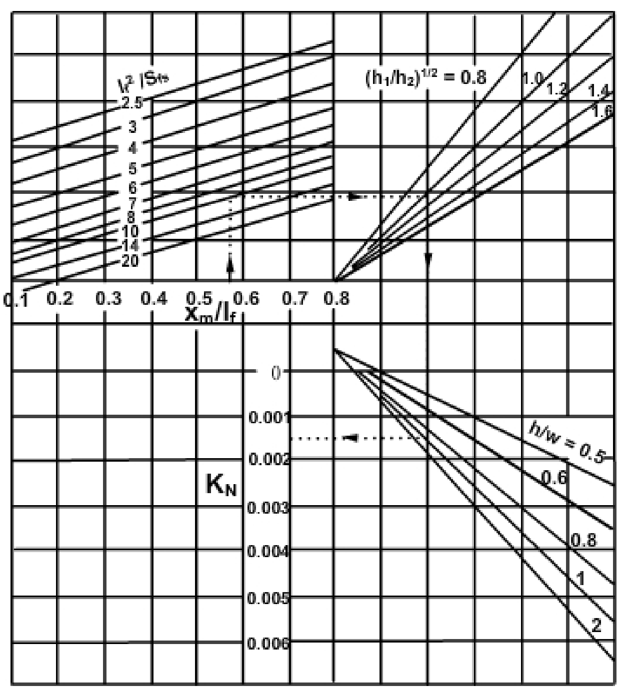


Fig.5.3 Wing body interference factor
(Adapted from Ref.2.2, section 5.2.3.1)

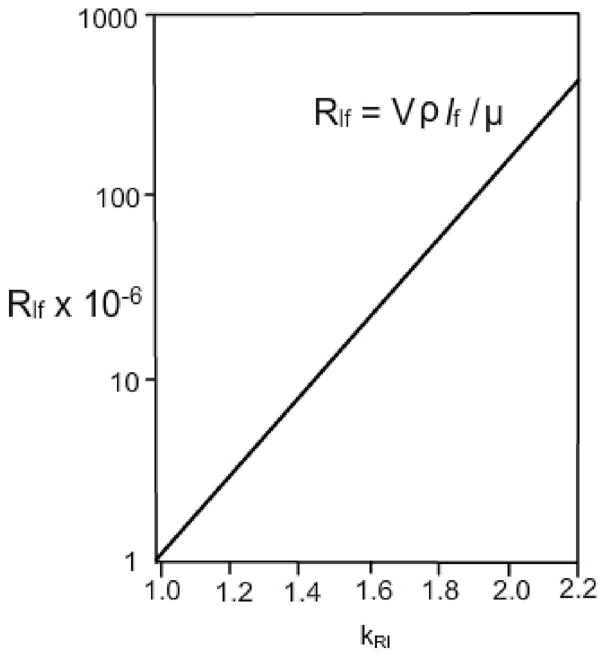


Fig.5.4 Correction factor for Reynolds number

(Adapted from Ref.2.2, section 5.2.1)

Example 5.1

A fuselage has the following dimensions. Obtain its contribution to $C_{n\beta}$ at sea level at a speed of 100 m/s.

l_f = 13.7 m, x_m = 8.0m, w_f = 1.6m, S_{fs} = 15.4 m^2

h =1.6 m, h_1 = 1.6 m, h_2 = 1.07,

Wing: area = 26.81 m^2, span =13.7 m.

Solution:

(I) $(h_1/h_2)^{1/2}$ = 1.223, x_m / l_f = 8/13.7 = 0.584

$l_f^2 / S_{fs} = (13.7)^2/15.4 = 12.19$; $h / w_f = 1.6/1.6 = 1.0$

Using these parameters, Fig.5.3 gives:

$k_n = 0.0017$

(II) Flight speed is 100 m/s at sea level

Hence, $R_{lf} = 100 \times 13.7 / (14.6 \times 10^{-6}) = 93.83 \times 10^6$

From Fig.3.4 $k_{Rl} = 1.96$

$$C_{n\beta wf} = -k_n\, k_{Rl} \frac{S_{fs}}{S_w} \frac{l_f}{b} \ \text{deg}^{-1}$$

$= - 0.0017 \times 1.96(15.4/26.81)(13.7/13.7) = - 0.00191 \text{ deg}^{-1}$.

Remark:

As the flight speed changes R_{lf} also changes. The effect of this change can be assessed as follows.

Let, $V_{min} = 60$ m/s, $V_{max} = 250$ m/s.

Then, the range of R_{lf} at sea level would be 56.3×10^6 to 235×10^6. The value of k_{Rl} (from Fig.5.4) would be between 1.75 to 2.16 and $C_{n\beta wf}$ would be change from 0.00017 to 0.00217.