# Chapter 3

# Lecture 13

# Longitudinal stick–free static stability and control – 2

## Topics



## 3.3.4 Shift in neutral point by freeing the stick

The shift in neutral point by freeing the stick is given by :

$$\frac{x_{NP}}{\bar{c}} - \frac{x'_{NP}}{\bar{c}} = (1 - f)\frac{C_{L\alpha t}}{C_{L\alpha w}} V_H \eta (1 - \frac{d\varepsilon}{d\alpha}) \quad (3.16)$$

## Example 3.1

Obtain the shift in the neutral point for the airplane in example 2.4. The values of some of the parameters are: $V_H$ = 0.738, η = 0.9, $C_{L\alpha w}$ = 4.17 rad$^{-1}$, $C_{L\alpha t}$ = 3.43 rad$^{-1}$, dε/dα = 0.438. Assume $\tau$ = 0.5, $C_{h\delta e}$ = - 0.005 deg$^{-1}$, $C_{h\alpha t}$ = - 0.003 deg$^{-1}$. Substituting various values in Eq.(3.16) yields:

$$f = 1 - 0.5(\frac{-0.003}{-0.005}) = 1 - 0.3 = 0.7 \text{ and}$$

$$\frac{x_{NP}}{\bar{c}} - \frac{x'_{NP}}{\bar{c}} = (1 - 0.7)\times\frac{3.43}{4.17}\times 0.738\times 0.9\,(1 - 0.438) = 0.0921$$

**Remarks:**

i) In this case, by freeing the stick, the neutral point has shifted forward by 0.0921$\bar{c}$ or the static margin has decreased by 0.0921. In other words

$$C_{m\alpha} - C'_{m\alpha} = - 0.0921 C_{L\alpha}$$

ii) If $C_{h\alpha t}$ is positive and $C_{h\delta e}$ is negative then $(1-\frac{C_{h\alpha t}}{C_{h\delta e}}\tau)$ can become more that one and stability will increase on freeing the stick. It is explained in subsection 3.4.4 that for proper variation of stick force gradient, $C_{h\delta e}$ should have a small negative value. Control of $C_{h\alpha t}$ within narrow limits is difficult and generally $C_{h\alpha t}$ and $C_{h\delta e}$ have small negative values.

## Example 3.2

An airplane has the following characteristics.

$C_{L\alpha w} = 0.085\ deg^{-1}$, $C_{L\alpha t} = 0.058\ deg^{-1}$, $dC_L/d\delta_e = 0.032$, $C_{h\alpha t} = -0.003\ deg^{-1}$, $C_{h\delta t} = -0.0055$, $i_w = 0$, $\alpha_{0L} = -2^0$, $i_t = -1^0$, $\varepsilon = 0.5\ \alpha$, $S_t = 0.25\ S$, $l_t = 3\bar{c}$, W/S =1500 N/m$^2$ , a.c. location = $0.25\bar{c}$, $\eta = 1.0$, $(C_{m\alpha})_{f,n,p} = 0.37\ rad^{-1}$.

Obtain

i) Stick-fixed neutral point

ii) Stick-free neutral point

iii) Stick -free neutral point when $C_{h\alpha t}$ is changed to 0.003.

## Solution:

The given data is:

$C_{L\alpha w} = 0.085$, $C_{L\alpha t} = 0.058$, $dC_{Lt}/d\delta_e = 0.032$

$C_{h\delta e} = -0.0055$, $C_{h\alpha t} = -0.003$

$i_w = 0$, $\alpha_{0L} = -2^0$, $i_t = -1^0$, $\varepsilon = 0.5\ \alpha$,

$S_t = 0.25 S$, $l_t = 3\bar{c}$, a.c. at $0.25\bar{c}$,

$\eta = 1.0$, $(C_{m\alpha})_{f,n,p} = 0.37\ rad^{-1}$

$C_{L\alpha w} = 0.085\ deg^{-1} = 4.87\ rad^{-1}$

$C_{L\alpha t} = 0.058\ deg^{-1} = 3.323\ rad^{-1}$

$$\frac{d\varepsilon}{d\alpha} = 0.5,\ V_H = \frac{S_t}{S}\frac{l_t}{\bar{c}} = 0.25\times 3 = 0.75$$

$\tau = C_{L\delta e} / C_{L\alpha t} = 0.032 / 0.058 = 0.552$

(i) Stick-fixed neutral point:

$$\frac{x_{NP}}{\bar{c}} = \frac{x_{ac}}{\bar{c}} - \frac{1}{C_{L\alpha w}}(C_{m\alpha})_{f,n,p} + \eta V_H \frac{C_{L\alpha t}}{C_{L\alpha w}}(1 - \frac{d\varepsilon}{d\alpha})$$

$$= 0.25 - \frac{1}{4.87} \times 0.37 + 1.0 \times 0.75 \times \frac{3.323}{4.87}(1 - 0.5)$$

$$= 0.25 - 0.0759 + 0.256 = 0.4301$$

(ii) Stick-free neutral point:

$$\frac{x_{NP}}{\bar{c}} = \frac{x_{ac}}{\bar{c}} - \frac{1}{C_{L\alpha w}}(C_{m\alpha})_{f,n,p} + \eta V_H \frac{C_{L\alpha t}}{C_{L\alpha w}}(1 - \frac{d\varepsilon}{d\alpha})(1 - \tau \frac{C_{h\alpha t}}{C_{h\delta e}})$$

$$= 0.25 - 0.0759 + 0.256 \{1 - 0.552 (\frac{-0.003}{-0.0055})\}$$

$$= 0.25 - 0.759 + 0.181 = 0.355$$

(iii) Stick-free neutral point when $C_{h\alpha t} = 0.003$

$$\frac{x'_{NP}}{\bar{c}} = 0.25 - 0.0759 + 0.256\{1 - 0.552 (\frac{0.003}{-0.0055})\}$$

$$= 0.25 - 0.0759 + 0.331 = 0.5051$$

## 3.4 Stick force and stick force gradient

Figure 3.1 shows the schematic of the control surface, the control stick, the hinge moment ($H_e$) due to pressure distribution and the stick force (F). As mentioned earlier, a nose up hinge moment is taken as positive. The convention for the stick force is that a pull force at the stick is taken as positive.

The relation between F and $H_e$ is given by :

$$F = GH_e = G \frac{1}{2}\rho V^2 \eta S_e \bar{c}_e C_{he} \qquad (3.17)$$

where, G is the gearing ratio. It may be pointed out that G is not dimensionless; it has the dimension of $m^{-1}$.

Recall that:

$$C_{he} = C_{h\alpha t} \alpha_t + C_{h\delta e} \delta_e + C_{h\delta t} \delta_t \qquad (3.4)$$

$$\alpha_t = \alpha_{0Lw} + i_t - i_w + \frac{C_L}{C_{L\alpha w}}(1 - \frac{d\varepsilon}{d\alpha}); C_L \approx C_{LW} \qquad (2.44)$$

$$(\frac{dC_m}{dC_L})_{stick\text{-}fix} = \frac{1}{C_{L\alpha w}}(C_{m\alpha})_{stick\text{-}fix} \qquad (2.71)$$

$$\delta_{trim} = \delta_{e0CL} - \frac{1}{C_{m\delta e}}\left(\frac{dC_m}{dC_L}\right)_{stick\text{-}fix} C_L \tag{2.84}$$

Substituting from Eqs.(2.44),(2.71)and (2.84) in Eq.(3.4) yields :

$$C_{he} = C_{h\alpha t}\left\{\alpha_{0Lw} + i_t - i_w + \frac{C_L}{C_{L\alpha w}}\left(1-\frac{d\varepsilon}{d\alpha}\right)\right\} + C_{h\delta t}\delta_t + C_{h\delta e}\,\delta_{e0CL} - \frac{C_{h\delta e}}{C_{m\delta e}}\left(\frac{dC_m}{dC_L}\right)_{stick\text{-}fix} C_L \tag{3.19}$$

Rearranging yields:

$$C_{he} = [C_{h\alpha t}(\alpha_{0Lw} + i_t - i_w) + C_{h\delta e}\delta_{e0CL}] + C_{h\delta t}\delta_t - \frac{C_{h\delta e}}{C_{m\delta e}}\, C_L\left[\left(\frac{dC_m}{dC_L}\right)_{stick\text{-}fix} - \frac{C_{m\delta e}}{C_{h\delta e}}\frac{C_{h\alpha t}}{C_{L\alpha w}}\left(1-\frac{d\varepsilon}{d\alpha}\right)\right] \tag{3.20}$$

Substituting $C_{m\delta e} = -V_H\,\eta_t\,C_{L\alpha t}\,\tau$, gives :

$$\left(\frac{dC_m}{dC_L}\right)_{stick\text{-}fix} - \frac{C_{m\delta e}}{C_{h\delta e}}\frac{C_{h\alpha t}}{C_{L\alpha w}}\left(1-\frac{d\varepsilon}{d\alpha}\right)$$

$$= \left(\frac{dC_m}{dC_L}\right)_{stick\text{-}fix} + V_H\,\eta_t\,C_{L\alpha t}\,\tau\,\frac{C_{h\alpha t}}{C_{h\delta e}}\frac{1}{C_{L\alpha w}}\left(1-\frac{d\varepsilon}{d\alpha}\right) \tag{3.21}$$

From Eq.(3.14a) the r.h.s of Eq.(3.21) is $\left(\frac{dC_m}{dC_L}\right)_{stick\text{-}free}$

Substituting from Eq.(3.21) in Eq.(3.20) gives:

$$C_{he} = A + C_{h\delta t}\,\delta_t - \frac{C_{h\delta e}}{C_{m\delta e}}\,C_L\left(\frac{dC_m}{dC_L}\right)_{stick\text{-}free} \tag{3.22}$$

$$\text{where, } A = C_{h\alpha t}\,(\alpha_{0Lw} + i_t - i_w) + C_{h\delta e}\,\delta_{e0CL} \tag{3.23}$$

Substituting, from Eq.(3.22) in Eq.(3.17) and noting $C_L = \frac{W}{\frac{1}{2}\rho V^2 S}$, yields:

$$F = G\,\frac{1}{2}\rho V^2\,\eta\,S_e\bar{c}_e\left\{A + C_{h\delta t}\,\delta_t - \frac{C_{h\delta e}}{C_{m\delta e}}\frac{W}{\frac{1}{2}\rho V^2 S}\left(\frac{dC_m}{dC_L}\right)_{stick\text{-}free}\right\}$$

$$\text{Or } F = K\frac{1}{2}\rho V^2\{A + C_{h\delta t}\,\delta_t\} - K\frac{W}{S}\frac{C_{h\delta e}}{C_{m\delta e}}\left(\frac{dC_m}{dC_L}\right)_{stick\text{-}free} \tag{3.24}$$

$$\text{where, } K = G\,\eta\,S_e\bar{c}_e \tag{3.25}$$

$$\text{Or } \frac{F}{q} = G\,\eta\,S_e\bar{c}_e\left\{A + C_{h\delta t}\,\delta_t - \frac{C_{h\delta e}}{C_{m\delta e}}\,C_L\left(\frac{dC_m}{dC_L}\right)_{stick\text{-}free}\right\};\; q = \frac{1}{2}\rho V^2 \tag{3.26}$$

Hence, $\frac{d(F/q)}{dC_L} = - G\eta S_e \bar{c}_e \frac{C_{h\delta e}}{C_{m\delta e}}(\frac{dC_m}{dC_L})_{stick-free}$ (3.27)

### 3.4.1 Dependence of stick force on flight velocity and airplane size

The first term in Eq.(3.24) depends on $V^2$ and hence, the stick force increases rapidly with flight speed. The constant K in Eq. (3.24) involves the product $S_e \bar{c}_e$ in it. The quantities $\bar{c}_e$ and $S_e$ are roughly proportional to the linear dimension of the airplane and its square respectively. Thus, the product $S_e \bar{c}_e$ is proportional to the cube of the linear dimension of the airplane. Hence, the control force which depends on $S_e \bar{c}_e$ could be very large for large airplanes. Manual control is not possible in such cases (see section 6.12).

### 3.4.2 Tab deflection for zero stick force

In section 3.1 it was noted that the stick force can be made zero by proper tab deflection. An expression for this deflection is obtained below.

Consider the second term in Eq.(3.24). Noting that (a) $C_{h\delta e}$ is generally negative (see section 3.4.4), (b) $C_{m\delta e}$ is negative and (c) $(dC_m / dC_L)_{stick-ree}$ is negative for a stable airplane, the second term in Eq.(3.24) is positive for a stable airplane. Further, the first term in Eq.(3.24) depends on V and $\delta_t$. Hence, at a given V, the stick force can be reduced to zero by proper choice of $\delta_t$ (Fig.3.5). The operation of making stick force zero by proper tab deflection, is called trimming the stick. Equating r.h.s. of Eq.(3.24) to zero yields $(\delta_t)_{trim}$ for chosen $V_{trim}$ i.e.

$$0 = K\frac{1}{2}\rho V_{trim}^2 \{A + C_{h\delta t}(\delta_t)_{trim}\} - K\frac{W}{S}\frac{C_{h\delta e}}{C_{m\delta e}}(\frac{dC_m}{dC_L})_{stick-free}$$

Or $(\delta_t)_{trim} = -\frac{1}{C_{h\delta t}}\{A - \frac{C_{h\delta e}}{C_{m\delta e}}\frac{W/S}{\frac{1}{2}\rho V_{trim}^2}(\frac{dC_m}{dC_L})_{stick-free}\}$ (3.28)

Differentiating Eq.(3.28) with $C_L$ yields :

$$\frac{d(\delta_t)_{trim}}{dC_L} = \frac{C_{h\delta e}}{C_{h\delta t}} \frac{1}{C_{m\delta e}} \left(\frac{dC_m}{dC_L}\right)_{stick\text{-}free} \quad (3.29)$$

Substituting $\delta_{trim}$ from Eq.(3.28) in Eq.(3.24) , the stick force becomes:

$$F = K \frac{W}{S} \frac{C_{h\delta e}}{C_{m\delta e}} \left(\frac{dC_m}{dC_L}\right)_{stick\text{-}free} \left(\frac{V^2}{V_{trim}^2} - 1\right) \quad (3.30)$$

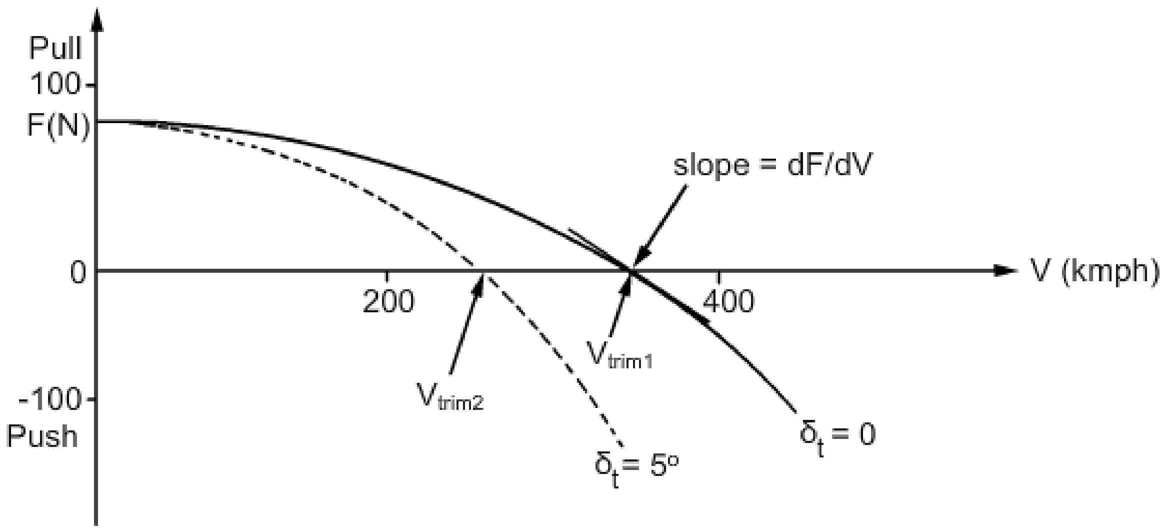


Fig.3.5 Variation of stick force with velocity for different tab deflections-schematic

### 3.4.3 Stick force gradient

The stick force gradient is defined as dF/dV. Differentiating Eq.(3.30) with V gives:

$$\frac{dF}{dV} = 2K \frac{W}{S} \frac{C_{h\delta e}}{C_{m\delta e}} \left(\frac{dC_m}{dC_L}\right)_{stick\text{-}free} \frac{V}{V_{trim}^2} \quad (3.31)$$

When V = $V_{trim}$, the stick force gradient, $\left(\frac{dF}{dV}\right)_{trim}$, is :

$$\left(\frac{dF}{dV}\right)_{trim} = 2K \frac{W}{S} \frac{C_{h\delta e}}{C_{m\delta e}} \left(\frac{dC_m}{dC_L}\right)_{stick\text{-}free} \frac{1}{V_{trim}} \quad (3.32)$$

Figure 3.5 shows the variation of the stick force with V and the gradient (dF/dV) at V = $V_{trim}$. See example 3.3.