Chapter 2

Lecture 11

Longitudinal stick–fixed static stability and control – 8

Topics

Example 2.7

2.13 Determination of stick-fixed neutral point from flight tests

Example 2.7

An airplane has elevator power ($C_{m\delta} = -\eta V_H \tau C_{L\alpha t}$) of -0.010 per degree. The c.g. is placed such that the static margin is 10% of m.a.c. Further, the tail setting (i_t) is such that the airplane is in trim, with zero elevator deflection, at $C_L = 0.5$. Plot the curves of C_{mcg} vs. C_L for constant elevator angles of $\delta_e = -20^0$, -10^0, 0^0, $+10^0$ and 20^0. Cross plot these curves to obtain the curve corresponding to δ_{trim} vs. C_L. Note $C_{Lmax} = 1.5$.

Solution:

The given data is:

$$C_{m\delta e} = -0.01 deg^{-1}$$

static margin = 0.1, hence, $dC_{mcg}/dC_L = -0.1$

$$\text{Now}, \delta_e = \delta_{e0CL} - (dC_{mcg}/dC_L)\frac{C_L}{C_{m\delta e}} \quad \text{(E 2.7.1)}$$

Noting that, $\delta_e = 0$ at $C_L = 0.5$, enables calculation of δ_{e0CL} as:

$$0 = \delta_{e0CL} - \frac{(-0.1)}{(-0.01)} \times 0.5$$

$$\text{or } \delta_{e0CL} = 5^0$$

$$C_{mcg} = C_{m0} + C_{m\alpha}\alpha + C_{m\delta e}\delta_e$$

Similarly, $C_{mcg} = (C_{mcg})_{CL=0} + (dC_m/dC_L)C_L + C_{m\delta e}\,\delta_e$

Since, airplane is in equilibrium with zero elevator deflection at $C_L = 0.5$, gives the following result.

$0 = (C_{mcg})_{CL=0} - 0.1\times0.5+0$

Or $(C_{mcg})_{CL=0} = 0.05$

Hence, $C_{mcg} = 0.05 - 0.1C_L - 0.01\times\delta_e$

Then, for

$\delta_e = -20^0 : C_{mcg} = 0.05 - 0.1C_L - 0.01\times(-20^0) = 0.25 - 0.1C_L$

$\delta_e = -10^0 : C_{mcg} = 0.05 - 0.1C_L - 0.01(-10^0) = 0.15 - 0.1C_L$

$\delta_e = 0^0 : C_{mcg} = 0.05 - 0.1C_L - 0.01(0^0) = 0.05 - 0.1C_L$

$\delta_e = +10^0 : C_{mcg} = 0.05 - 0.1C_L - 0.01(+10^0) = -0.05 - 0.1C_L$

$\delta_e = +20^0 : C_{mcg} = 0.05 - 0.1C_L - 0.01(+20^0) = -0.15 - 0.1C_L$

The variations of C_{mcg} with C_L for above values of δ_e are shown in Fig.E2.7a.

Note: All the curves in Fig.E2.7a have same slope as static margin or (dC_m / dC_L) is same for all of them.

The cross plot, δ_{trim} vs. C_L is shown in Fig.E2.7b.

Note: Alternatively from Eq.(E2.7.1) and $\delta_{e0CL} = 5^0$ yield δ_{trim} in degrees as:

$\delta_{trim} = 5 - 10\,C_L$.

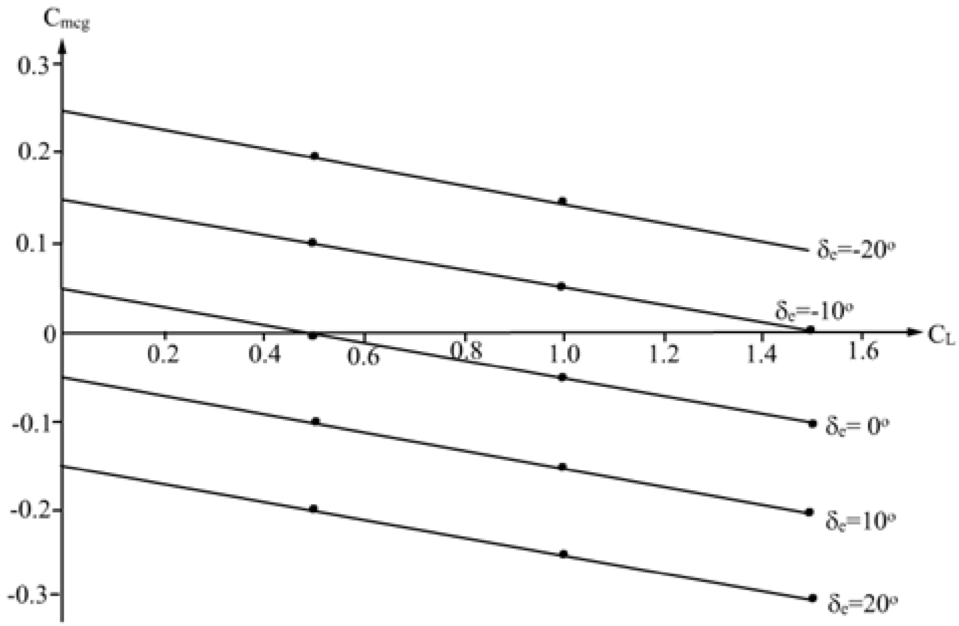

Fig. E2.7a C_{mcg} vs C_L with δ_e as parameter; static margin = 0.1

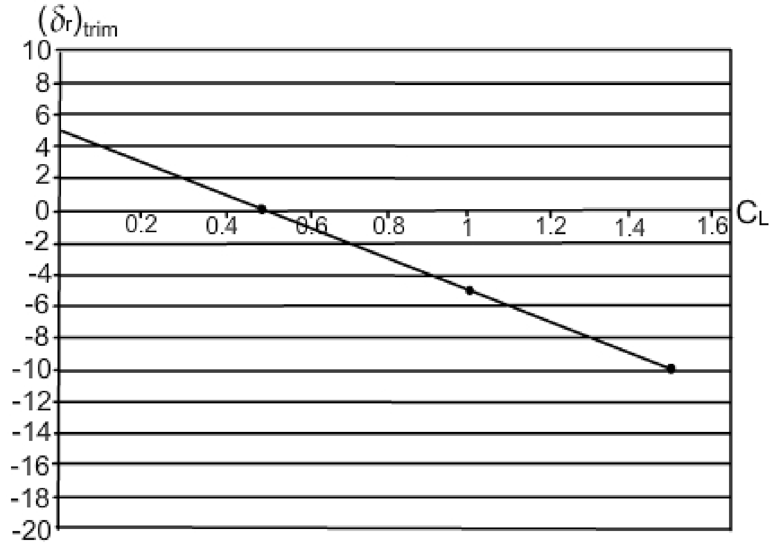

Fig.E2.7b Variation of δ_{trim} with C_L ; static margin = 0.1

2.13 Determination of stick-fixed neutral point from flight tests

It is risky to test an airplane in flight with c.g. at the neutral point. Hence, there should be a way to obtain the neutral point by extrapolation of results from flight tests conducted when the aircraft is stable. A method is suggested by the following equations which have been derived earlier.

$$\delta_{trim} = -\frac{[C_{L\alpha}(C_{m0} + C_{m\alpha}\ \alpha_{0L}) + C_{m\alpha}\ C_{Ltrim}]}{[C_{m\delta e}\ C_{L\alpha} - C_{m\alpha}\ C_{L\delta e}]} \quad (2.82)$$

$$\frac{d\delta_{trim}}{dC_{Ltrim}} = -\frac{C_{m\alpha}}{[C_{m\delta e}\ C_{L\alpha} - C_{m\alpha}\ C_{L\delta e}]} \quad (2.83)$$

Equation (2.83) shows that $C_{m\alpha}$ is proportional to $d\delta_{trim}/dC_L$. Hence, when $d\delta_{trim}/dC_L$ is zero, $C_{m\alpha}$ is also zero. This fact suggests the following way to obtain the neutral point.

(a) Choose a c.g. location for which the airplane is stable. Obtain δ_{trim} at various values of C_L.

(b) Obtain $d\delta_{trim}/ dC_L$ at this c.g. location.

(c) Change the c.g. location and conduct the tests again and obtain δ_{trim} vs C_L and $d\delta_{trim}/dC_L$.

(d) Plot $d\delta_{trim}/ dC_L$ vs. c.g. location. Extrapolate the curve and obtain the c.g. location for which $d\delta_{trim}/ dC_L$ is zero.

The flight test procedure to determine the neutral point could be as follows.

1.Calibrate the instrumentation for measurement of elevator deflection during flight.

2.Obtain the weight and c.g. of the airplane on ground.

3.Take the airplane to a suitable height and attain a steady flight. Note the flight velocity, flight altitude and elevator deflection. Repeat the measurements at various flight speeds. Apply corrections, if required, to the various readings.

4.From the weight and the flight velocities the values of C_L are obtained as :

$$C_L = W / \{\frac{1}{2}\rho_0 V_e^2 S\}; V_e = \text{equivalent speed.}$$

5. Plot δ_{trim} vs C_L.

6. Repeat the tests at different locations of c.g.. The change in c.g. is generally achieved by changing the weight of the ballast in the cargo compartment. Obtain δ_{trim} at various speeds and plot δ_{trim} vs C_L (Fig.2.35).

7. Obtain $d\delta_{trim}/dC_L$ for the various cases and plot the variation of $d\delta_{trim} / dC_L$ with c.g. location. Extrapolate the line. The point where the line cuts the x-axis is the neutral point (Fig.2.36).

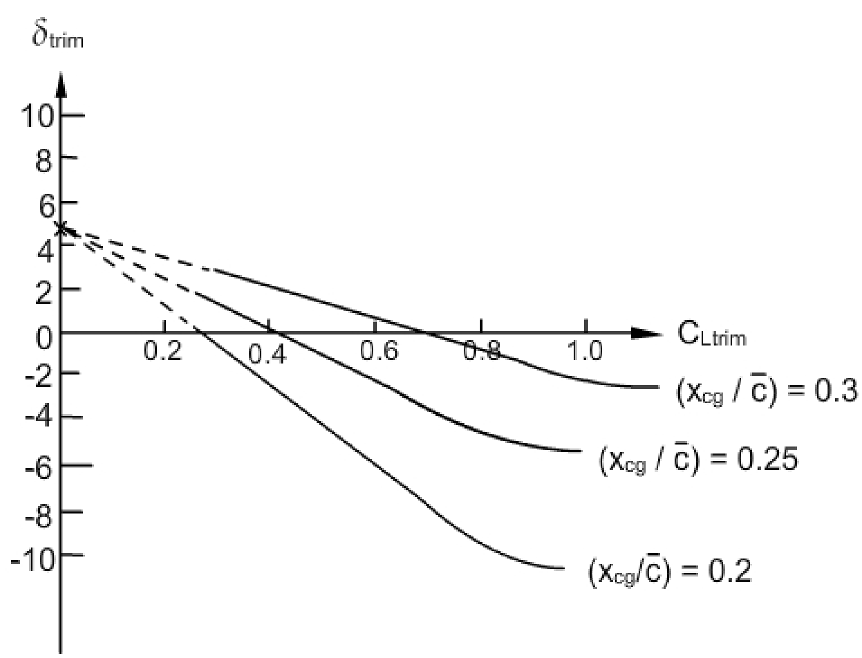

Fig.2.35 Schematic of δ_{trim} vs C_{Ltrim} at different c.g. locations

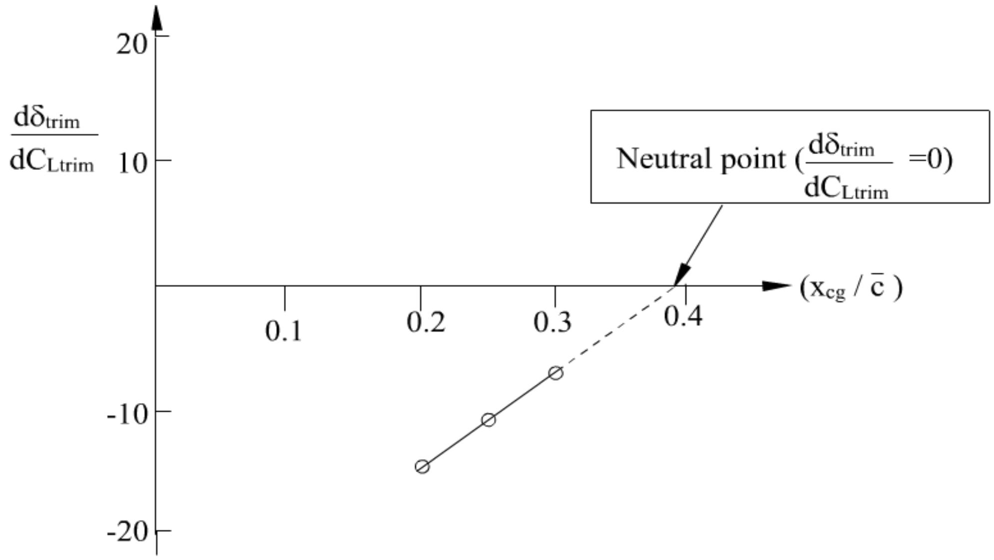

Fig.2.36 Determination of neutral point from flight test data (schematic based on linear portions in Fig.2.35)

Remark:

The above description is based on the simplified treatment of stability analysis wherein the δ_{trim} vs C_{Ltrim} curves for different c.g. locations are straight lines and pass through the same point at $C_L = 0$ (Fig.2.33). However, the data from actual flight test (Ref.2.5) shows that δ_{trim} vs C_{Ltrim} curves for different c.g. locations do not pass through the same point and may not be perfect straight lines. This indicates a weak dependence of the neutral point location on C_L . Figure 2.37, based on data in Ref.2.5 shows that x_{NP} at higher values of C_L may move forward by a few percent of m.a.c.

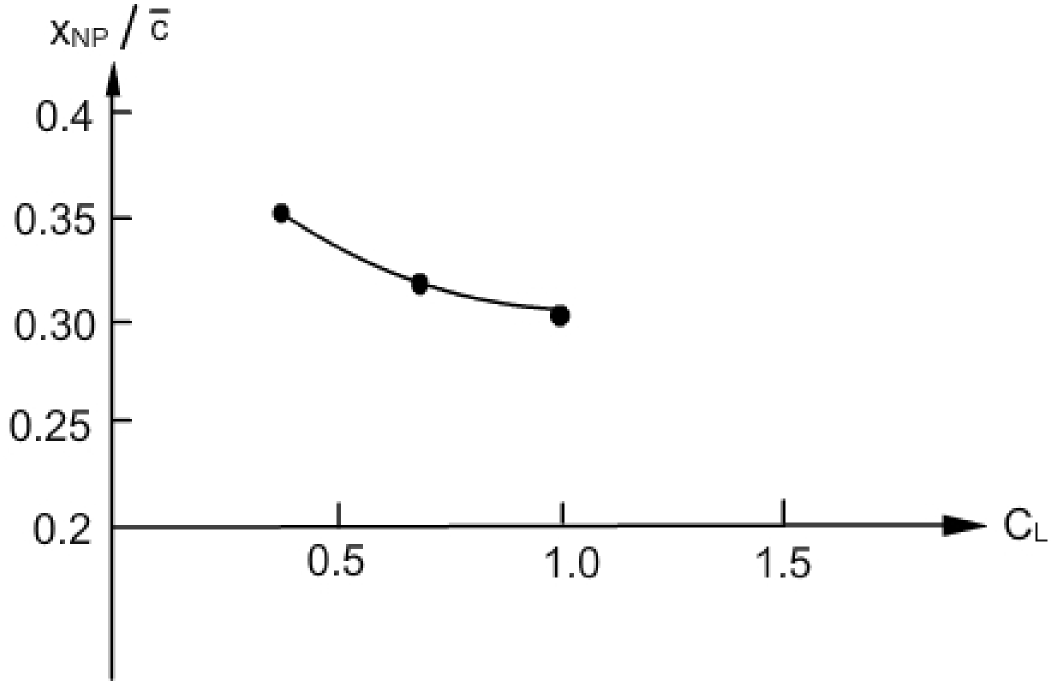

Fig.2.37 Dependence of stick-fixed neutral point on lift coefficient