Chapter 2

Lecture 5

Earth's atmosphere – 2

Topics

2.4 Variations of properties with altitude in ISA

For calculation of the variations of pressure, temperature and density with altitude, the following equations are used.

The equation of state $p = \rho R T$ (2.1)

The hydrostatic equation $dp/dh = - \rho g$ (2.2)

Remark:

The hydrostatic equation can be easily derived by considering the balance of forces on a small fluid element.

Consider a cylindrical fluid element of area A and height Δh as shown in Fig.2.2.

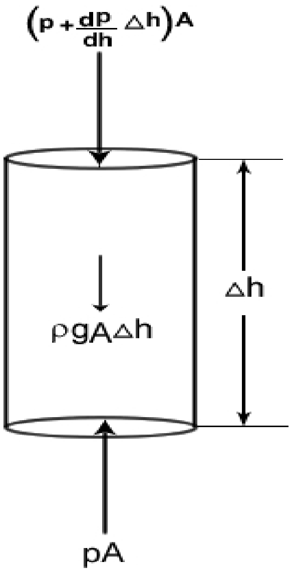

Fig.2.2 Equilibrium of a fluid element.

The forces acting in the vertical direction on the element are the pressure forces and the weight of the element.

For vertical equilibrium of the element,

$$pA - \{p + (dp/dh)\,\Delta h\}\,A - \rho\, g\, A\, \Delta h = 0$$

Simplifying, $dp/dh = -\rho g$

2.4.1 Variations of pressure and density with altitude

Substituting for ρ from the Eq.(2.1) in Eq.(2.2) gives:

$$dp / dh = -(p/RT)\, g$$

$$\text{Or } (dp/p) = -g\, dh/RT \qquad (2.3)$$

Equation (2.3) is solved separately in troposphere and stratosphere, taking into account the temperature variations in each region. For example, in the troposphere, the variation of temperature with altitude is given by the equation

$$T = T_0 - \lambda h \qquad (2.4)$$

where T_0 is the sea level temperature, T is the temperature at the altitude h and λ is the temperature lapse rate in the troposphere.

Substituting from Eq.(2.4) in Eq.(2.3) gives:

$$(dp/p) = -g\,dh / R\,(T_0 - \lambda h) \qquad (2.5)$$

Taking 'g' as constant, Eq.(2.5) can be integrated between two altitudes h_1 and h_2. Taking h_1 as sea level and h_2 as the desired altitude (h), the integration gives the following equation, the intermediate steps are left as an exercise.

$$(p/p_0) = (T/T_0)^{(g/\lambda R)} \qquad (2.6)$$

where T is the temperature at the desired altitude (h) given by Eq.(2.4).
Equation (2.6) gives the variation of pressure with altitude.

The variation of density with altitude can be obtained using Eq.(2.6) and the equation of state. The resulting variation of density with temperature in the troposphere is given by:

$$(\rho/\rho_0) = (T/T_0)^{(g/\lambda R)-1} \qquad (2.7)$$

Thus, both the pressure and density variations are obtained once the temperature variation is known.
As per the ISA, R = 287.05287 $m^2 sec^{-2}$ K and g = 9.80665 m/s^2.
Using these and λ = 0.0065 K/m in the troposphere yields $(g/R\lambda)$ as 5.25588.
Thus, in the troposphere, the pressure and density variations are :

$$(p/p_0) = (T/T_0)^{5.25588} \qquad (2.8)$$

$$(\rho/\rho_0) = (T/T_0)^{4.25588} \qquad (2.9)$$

Note: T= 288.15 - 0.0065 h; h in m and T in K.

In order to obtain the variations of properties in the lower stratosphere (11 to 20 km altitude), the previous analysis needs to be carried-out afresh with λ = 0 i.e., 'T' having a constant value equal to the temperature at 11 km (T = 216.65 K). From this analysis the pressure and density variations in the lower stratosphere are obtained as :

$$(p / p_{11}) = (\rho / \rho_{11}) = \exp \{ -g (h - 11000) / RT_{11} \} \qquad (2.10)$$

where p_{11}, ρ_{11} and T_{11} are the pressure, density and temperature respectively at 11 km altitude.
In the middle stratosphere (20 to 32 km altitude), it can be shown that (note in this case λ = -0.001 K / m):

$$(p / p_{20}) = (T / T_{20})^{-34.1632} \qquad (2.11)$$

$$(\rho / \rho_{20}) = (T/ T_{20})^{-35.1632} \qquad (2.12)$$

where p_{20}, ρ_{20} and T_{20} are pressure, density and temperature respectively at 20 km altitude.

Thus, the pressure and density variations have been worked out in the troposphere and the stratosphere of ISA. Table 2.1 presents these values.

Remark:

Using Eqs.(2.1) and (2.2) the variations of pressure and density can be worked out for other variations of temperature with height (see exercise 2.1).

2.4.2. Variations with altitude of pressure ratio, density ratio, speed of sound, coefficient of viscosity and kinematic viscosity

The ratio (p/p_0) is called pressure ratio and is denoted by δ. Its value in ISA can be obtained by using Eqs.(2.8),(2.10) and (2.11). Table 2.1 includes these values.

The ratio (ρ / ρ_0) is called density ratio and is denoted by σ. Its values in ISA can be obtained using Eqs.(2.9),(2.10) and (2.12). Table 2.1 includes these values.

The speed of sound in air, denoted by 'a', depends only on the temperature and is given by:

$$a = (\gamma RT)^{0.5} \tag{2.13}$$

where γ is the ratio of specific heats; for air $\gamma = 1.4$. The values of 'a' in ISA can be obtained by using appropriate values of temperature. Table 2.1 includes these values.

The kinematic viscosity (ν) is given by:

$\nu = \mu / \rho$ where μ is the coefficient of viscosity.

The coefficient of viscosity of air (μ) depends only on temperature. Its variation with temperature is given by the following Sutherland formula.

$$\mu = 1.458\text{X}10^{-6}\left[\frac{T^{3/2}}{T+110.4}\right], \text{ where T is in Kelvin and } \mu \text{ is in kg m}^{-1}\text{ s}^{-1} \tag{2.14}$$

Table 2.1 includes the variation of kinematic viscosity with altitude.

Example 2.1

Calculate the temperature (T), pressure (p), density (ρ), pressure ratio (δ), density ratio (σ), speed of sound (a), coefficient of viscosity (μ) and kinematic viscosity (ν) in ISA at altitudes of 8 km, 16 km and 24 km.

Solution:

It may be noted that the three altitudes specified in this example, viz. 8 km, 16 km and 24 km, lie in troposphere, lower stratosphere and middle stratosphere regions of ISA respectively.

(a) h = 8 km

Let the quantities at 8 km altitude be denoted by the suffix '8'.

In troposphere: $T = T_0 - \lambda h$

where, $T_0 = 288.15$ K, $\lambda = 0.0065$ K/m

Hence, $T_8 = 288.15 - 0.0065 \times 8000 = 236.15$K

From Eq.(2.8)

$$\left(\frac{p_8}{p_0}\right) = \delta_8 = (T/T_0)^{5.25588} = (236.15/288.15)^{5.25588} = 0.35134$$

Or $\quad p_8 = 0.35134 \times 101325 = 35599.5 \text{ N/m}^2$

$$\rho_8 = p_8/(RT_8) = \frac{35599.5}{287.05287 \times 236.15} = 0.52516 \text{ kg/m}^3$$

$$\sigma_8 = \rho_8/\rho_0 = 0.52516/1.225 = 0.42870$$

$$a_8 = (\gamma RT_8)^{0.5} = (1.4 \times 287.05287 \times 236.15)^{0.5} = 308.06 \text{ m/s}$$

From Eq.(2.14):

$$\mu_8 = 1.458 \times 10^{-6}\left[\frac{T_8^{1.5}}{T_8 + 110.4}\right] = 1.458 \times 10^{-6}\left[\frac{236.15^{1.5}}{236.15 + 110.4}\right] = 1.5268 \times 10^{-5} \text{ kg m}^{-1}\text{s}^{-1}$$

$$\nu_8 = \mu_8/\rho_8 = 1.5268 \times 10^{-5}/0.52516 = 2.9072 \times 10^{-5} \text{ m}^2\text{/s}$$

Remarks:

(i) The values calculated above and those in Table 2.1 may differ from each other in the last significant digit. This is due to the round-off errors in the calculations.

(ii) Consider an airplane flying at 8 km altitude at a flight speed of 220 m/s. The Mach number of this flight would be: 220/308.06 = 0.714

(iii) Further if the reference chord of the wing (c_{ref}) of this airplane be 3.9 m, the Reynolds number in this flight, based on c_{ref}, would be:

$$R_e = \frac{V c_{ref}}{\nu} = \frac{220\times3.9}{2.9072\times10^{-5}} = 29.51\times10^{6}$$

(iv) For calculation of values at 16 km altitude, the values of temperature, pressure and density are needed at the tropopause viz. at h=11 km.

Now $T_{11} = 288.15 - 0.0065\times11000 = 216.65\text{ K}$

$$p_{11} = 101325\left(216.65/288.15\right)^{5.25588} = 22632\text{ N/m}^2$$

$$\rho_{11} = 22632/\left(287.05287\times216.65\right) = 0.36392\text{ kg/m}^3$$

(b) h = 16 km

In lower stratosphere Eq.(2.10) gives :

$$\frac{p}{p_{11}} = \frac{\rho}{\rho_{11}} = \exp\left\{-g\left(h-11000\right)/RT_{11}\right\}$$

Consequently,

$$\frac{p_{16}}{p_{11}} = \frac{\rho_{16}}{\rho_{11}} = \exp\left\{-9.80665\left(16000-11000\right)/\left(287.05287\times216.65\right)\right\} = 0.45455$$

Or $p_{16} = 22632\times0.45455 = 10287\text{ N/m}^2$

$$\rho_{16} = 0.36392\times0.45455 = 0.16541\text{ kg/m}^3$$

$$\delta_{16} = 10287/101325 = 0.10153$$

$$\sigma_{16} = 0.16541/1.225 = 0.13503$$

$$a_{16} = \left(1.4\times287.05287\times216.65\right)^{0.5} = 295.07\text{ m/s}$$

$$\mu_{16} = 1.458\times10^{-6}\left[\frac{216.65^{1.5}}{216.65+110.4}\right] = 1.4216\times10^{-5}\text{ kg m}^{-1}\text{ s}^{-1}$$

$$\nu_{16} = 1.4216\times10^{-5}/0.16541 = 8.594\times10^{-5}\text{ m}^2\text{/s}$$

Remark :

To calculate the required values at 24 km altitude, the values of T and p are needed at h = 20 km. These values are :

$T_{20} = 216.65$

$$\frac{p_{20}}{p_{11}} = \exp\{-9.80665(20000-11000)/(287.05287\times216.65)\} = 0.24191$$

Or $p_{20} = 22632\times0.24191 = 5474.9\ \text{N/m}^2$

(c) h = 24 km

$$T_{24} = 216.65+0.001(24000-20000) = 220.65\text{K}$$

From Eq.(2.11):

$$\frac{p_{24}}{p_{20}} = (T_{24}/T_{20})^{-34.1632}$$

Or $p_{24} = 5474.9(220.65/216.65)^{-34.1632} = 2930.5\text{N/m}^2$

$$\rho_{24} = 2930.5/(287.05287\times220.65) = 0.04627$$

Hence, $\delta_{24} = 2930.5/101325 = 0.02892$

and $\sigma_{24} = 0.04627/1.225 = 0.03777$

$$a_{24} = (1.4\times287.05287\times220.65)^{0.5} = 297.78\ \text{m/s}$$

$$\mu_{24} = 1.458\times10^{-6}\left[\frac{220.65^{1.5}}{220.65+110.4}\right] = 1.4435\times10^{-5}\ \text{kg m}^{-1}\text{s}^{-1}$$

$$\nu_{24} = 1.4435\times10^{-5}/0.04627 = 3.12\times10^{-4}\ \text{m}^2/\text{s}$$

Answers:

h (km)	8	16	24
T (K)	236.15	216.65	220.65
p (N/m^2)	35599.5	10287.0	2930.5
$\delta = p/p_0$	0.35134	0.10153	0.02892
$\rho(\text{kg/m}^3)$	0.52516	0.16541	0.04627
$\sigma = \rho/\rho_0$	0.42870	0.13503	0.03777
a (m/s)	308.06	295.07	297.78
$\mu(\text{kg m}^{-1}\text{ s}^{-1})$	1.5268×10^{-5}	1.4216×10^{-5}	1.4435×10^{-5}
$\nu(\text{m}^2/\text{s})$	2.9072×10^{-5}	8.594×10^{-5}	3.12×10^{-4}

2.5 Geopotential altitude

The variations of pressure, temperature and density in the atmosphere were obtained by using the hydrostatic equation (Eq.2.2). In this equation 'g' is assumed to be constant. However, it is known that 'g' decreases with altitude. Equation (1.1) gives the variation as:

$$g = g_0 \left(\frac{R}{R+h_G}\right)$$

where 'R' is the radius of earth and 'h_G' is the geometric altitude above earth's surface.

Thus, the values of p and ρ obtained by assuming $g = g_0$ are at an altitude slightly different from the geometrical altitude (h_G). This altitude is called geopotential altitude, which for convenience is denoted by 'h'. Following Ref.1, the geopotential altitude can be defined as the height above earth's surface in units, proportional to the potential energy of unit mass (geopotential), relative to sea level. It can be shown that the geopotential altitude (h) is given, in terms of geometric altitude (h_G), by the following relation. Reference 1.13, chapter 3 may be referred to for derivation.

$$h_G = \frac{R}{R-h}\, h$$

It may be remarked that the actual difference between h and h_G is small for altitudes involved in flight dynamics; for h of 20 km, h_G would be 20.0627 km. Hence, the difference is ignored in performance analysis.

2.6 General remarks:

2.6.1 Atmospheric properties in cases other than ISA

It will be evident from chapters 4 to 10 that the engine characteristics and the airplane performance depend on atmospheric characteristics. Noting that ISA only represents average atmospheric conditions, other atmospheric models have been proposed as guidelines for extreme conditions in arctic and tropical regions. Figure 2.3 shows the temperature variations with altitude in arctic and tropical atmospheres along with ISA. It is seen that the arctic minimum atmosphere has the following features. (a) The sea level temperature is -50^0C (b) The temperature increases at the rate of 10 K per km up to 1500 m altitude. (c) The temperature remains constant at -35^{0}C up to 3000 m altitude. (d) Then the temperature decreases at the rate of 4.72 K per km up to 15.5 km altitude (e) The tropopause in this case is at 15.5 km and the temperature there is -94^{0}c.

The features of the tropical maximum atmosphere are as follows.

(a) Sea level temperature is 45^0 C.

(b) The temperature decreases at the rate of 6.5 K per km up to 11.54 km and then remains constant at -30^0 C.

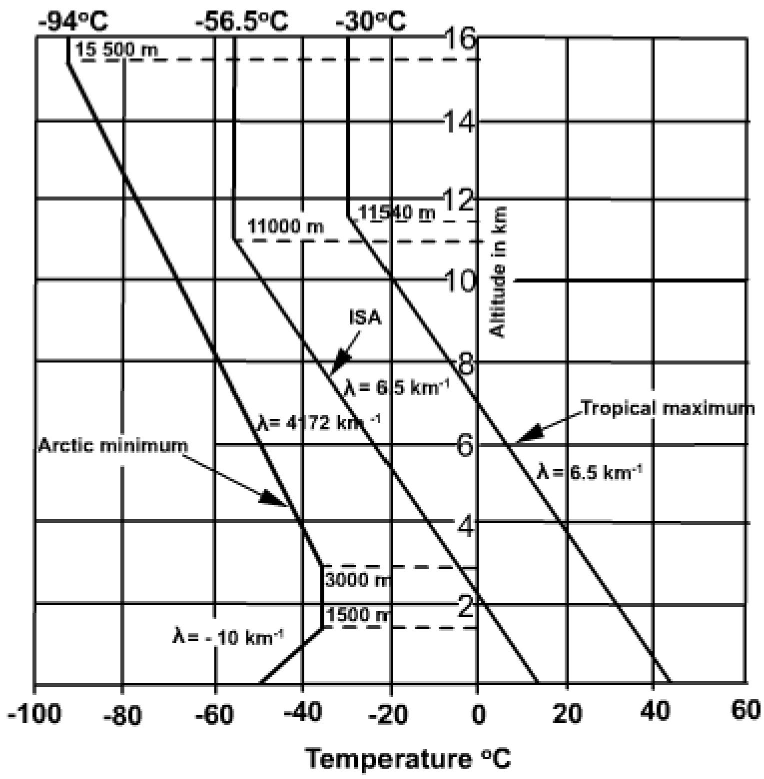

Fig.2.3 Temperature variations in arctic minimum, ISA and tropical maximum atmospheres (Reproduced from Ref.1.7, Chapter 3 with permission of author)

Note:

(a) The local temperature varies with latitude but the sea level pressure (p_0) depends on the weight of air above and is taken same at all the places i.e. 101325 N/m^2. Knowing p_0 and T_0, and the temperature lapse rates, the pressure, temperature and density in tropospheres of arctic minimum and tropical maximum can be obtained using Eqs. (2.4), (2.6) and (2.7). (see also exercise 2.1).

(b) Some airlines/ air forces may prescribe intermediate values of sea level temperature e.g. ISA +15^{0}C or ISA +20^{0}C. The variations of pressure, temperature and density with altitude in these cases can also be worked out from the aforesaid equations.

2.6.2 Stability of atmosphere

It is generally assumed that the air mass is stationary. However, some packets of air mass may acquire motion due to local changes. For example, due to absorption of solar radiation by the earth's surface, an air mass adjacent to the surface may become lighter and buoyancy may cause it to rise. If the atmosphere is stable, a rising packet of air must come back to its original position. On the other hand, if the air packet remains in the disturbed position, then the atmosphere is neutrally stable. If the rising packet continues to move up then the atmosphere is unstable.

Reference 1.7, chapter 3 analyses the problem of atmospheric stability and concludes that if the temperature lapse rate is less than 9.75 K per km, then the atmosphere is stable. It is seen that the three atmospheres, representing different conditions, shown in Fig.2.3 are stable.

Altitude (m)	Temperature (K)	Pressure (N/m^2)	δ (p/p_o)	Density (kg/m^3)	σ (ρ/ρ_o)	speed of sound (m/s)	Kinematic viscosity (m^2/s)
0	288.15	101325.0	1.00000	1.22500	1.00000	340.29	1.4607E-005
200	286.85	98945.3	0.97651	1.20165	0.98094	339.53	1.4839E-005
400	285.55	96611.0	0.95348	1.17864	0.96216	338.76	1.5075E-005
600	284.25	94321.6	0.93088	1.15598	0.94365	337.98	1.5316E-005
800	282.95	92076.3	0.90872	1.13364	0.92542	337.21	1.5562E-005
1000	281.65	89874.4	0.88699	1.11164	0.90746	336.43	1.5813E-005
1200	280.35	87715.4	0.86568	1.08997	0.88977	335.66	1.6069E-005
1400	279.05	85598.6	0.84479	1.06862	0.87234	334.88	1.6331E-005
1600	277.75	83523.3	0.82431	1.04759	0.85518	334.10	1.6598E-005
1800	276.45	81489.0	0.80423	1.02688	0.83827	333.31	1.6870E-005
2000	275.15	79494.9	0.78455	1.00649	0.82162	332.53	1.7148E-005
2200	273.85	77540.6	0.76527	0.98640	0.80523	331.74	1.7432E-005
2400	272.55	75625.4	0.74636	0.96663	0.78908	330.95	1.7723E-005
2600	271.25	73748.6	0.72784	0.94716	0.77319	330.16	1.8019E-005
2800	269.95	71909.7	0.70969	0.92799	0.75754	329.37	1.8321E-005
3000	268.65	70108.2	0.69191	0.90912	0.74214	328.58	1.8630E-005
3200	267.35	68343.3	0.67450	0.89054	0.72697	327.78	1.8946E-005
3400	266.05	66614.6	0.65744	0.87226	0.71205	326.98	1.9269E-005
3600	264.75	64921.5	0.64073	0.85426	0.69736	326.18	1.9598E-005
3800	263.45	63263.4	0.62436	0.83655	0.68290	325.38	1.9935E-005
4000	262.15	61639.8	0.60834	0.81912	0.66867	324.58	2.0279E-005
4200	260.85	60050.0	0.59265	0.80197	0.65467	323.77	2.0631E-005
4400	259.55	58493.7	0.57729	0.78510	0.64090	322.97	2.0990E-005
4600	258.25	56970.1	0.56225	0.76850	0.62735	322.16	2.1358E-005
4800	256.95	55478.9	0.54753	0.75217	0.61402	321.34	2.1734E-005

Table 2.1 Atmospheric properties in ISA (Cont..)

5000	255.65	54019.4	0.53313	0.73611	0.60091	320.53	2.2118E-005
5200	254.35	52591.2	0.51903	0.72031	0.58801	319.71	2.2511E-005
5400	253.05	51193.7	0.50524	0.70477	0.57532	318.90	2.2913E-005
5600	251.75	49826.4	0.49175	0.68949	0.56285	318.08	2.3324E-005
5800	250.45	48488.8	0.47855	0.67446	0.55058	317.25	2.3744E-005
6000	249.15	47180.5	0.46564	0.65969	0.53852	316.43	2.4174E-005
6200	247.85	45900.9	0.45301	0.64516	0.52666	315.60	2.4614E-005
6400	246.55	44649.5	0.44066	0.63088	0.51501	314.77	2.5064E-005
6600	245.25	43425.9	0.42858	0.61685	0.50355	313.94	2.5525E-005
6800	243.95	42229.6	0.41677	0.60305	0.49229	313.11	2.5997E-005
7000	242.65	41060.2	0.40523	0.58949	0.48122	312.27	2.6480E-005
7200	241.35	39917.1	0.39395	0.57617	0.47034	311.44	2.6974E-005
7400	240.05	38799.9	0.38292	0.56308	0.45965	310.60	2.7480E-005
7600	238.75	37708.1	0.37215	0.55021	0.44915	309.75	2.7998E-005
7800	237.45	36641.4	0.36162	0.53757	0.43884	308.91	2.8529E-005
8000	236.15	35599.2	0.35134	0.52516	0.42870	308.06	2.9073E-005
8200	234.85	34581.2	0.34129	0.51296	0.41875	307.21	2.9629E-005
8400	233.55	33586.9	0.33148	0.50099	0.40897	306.36	3.0200E-005
8600	232.25	32615.8	0.32189	0.48923	0.39937	305.51	3.0784E-005
8800	230.95	31667.6	0.31254	0.47768	0.38994	304.65	3.1383E-005
9000	229.65	30741.9	0.30340	0.46634	0.38069	303.79	3.1997E-005
9200	228.35	29838.2	0.29448	0.45521	0.37160	302.93	3.2627E-005
9400	227.05	28956.1	0.28577	0.44428	0.36268	302.07	3.3272E-005
9600	225.75	28095.2	0.27728	0.43355	0.35392	301.20	3.3933E-005
9800	224.45	27255.2	0.26899	0.42303	0.34533	300.33	3.4611E-005
10000	223.15	26435.7	0.26090	0.41270	0.33690	299.46	3.5307E-005
10200	221.85	25636.2	0.25301	0.40256	0.32862	298.59	3.6020E-005
10400	220.55	24856.4	0.24531	0.39262	0.32050	297.71	3.6752E-005
10600	219.25	24096.0	0.23781	0.38286	0.31254	296.83	3.7503E-005

Table 2.1 Atmospheric properties in ISA (Cont..)

10800	217.95	23354.4	0.23049	0.37329	0.30473	295.95	3.8274E-005
11000	216.65	22631.5	0.22336	0.36391	0.29707	295.07	3.9065E-005
11200	216.65	21929.4	0.21643	0.35262	0.28785	295.07	4.0316E-005
11400	216.65	21248.6	0.20971	0.34167	0.27892	295.07	4.1608E-005
11600	216.65	20588.9	0.20320	0.33106	0.27026	295.07	4.2941E-005
11800	216.65	19949.7	0.19689	0.32079	0.26187	295.07	4.4317E-005
12000	216.65	19330.4	0.19078	0.31083	0.25374	295.07	4.5736E-005
12200	216.65	18730.2	0.18485	0.30118	0.24586	295.07	4.7202E-005
12400	216.65	18148.7	0.17911	0.29183	0.23823	295.07	4.8714E-005
12600	216.65	17585.3	0.17355	0.28277	0.23083	295.07	5.0275E-005
12800	216.65	17039.4	0.16817	0.27399	0.22366	295.07	5.1886E-005
13000	216.65	16510.4	0.16294	0.26548	0.21672	295.07	5.3548E-005
13200	216.65	15997.8	0.15789	0.25724	0.20999	295.07	5.5264E-005
13400	216.65	15501.1	0.15298	0.24925	0.20347	295.07	5.7035E-005
13600	216.65	15019.9	0.14823	0.24152	0.19716	295.07	5.8862E-005
13800	216.65	14553.6	0.14363	0.23402	0.19104	295.07	6.0748E-005
14000	216.65	14101.8	0.13917	0.22675	0.18510	295.07	6.2694E-005
14200	216.65	13664.0	0.13485	0.21971	0.17936	295.07	6.4703E-005
14400	216.65	13239.8	0.13067	0.21289	0.17379	295.07	6.6776E-005
14600	216.65	12828.7	0.12661	0.20628	0.16839	295.07	6.8916E-005
14800	216.65	12430.5	0.12268	0.19988	0.16317	295.07	7.1124E-005
15000	216.65	12044.6	0.11887	0.19367	0.15810	295.07	7.3403E-005
15200	216.65	11670.6	0.11518	0.18766	0.15319	295.07	7.5754E-005
15400	216.65	11308.3	0.11160	0.18183	0.14844	295.07	7.8182E-005
15600	216.65	10957.2	0.10814	0.17619	0.14383	295.07	8.0687E-005
15800	216.65	10617.1	0.10478	0.17072	0.13936	295.07	8.3272E-005
16000	216.65	10287.5	0.10153	0.16542	0.13504	295.07	8.5940E-005
16200	216.65	9968.1	0.09838	0.16028	0.13084	295.07	8.8693E-005
16400	216.65	9658.6	0.09532	0.15531	0.12678	295.07	9.1535E-005

Table 2.1 Atmospheric properties in ISA (Cont..)

16600	216.65	9358.8	0.09236	0.15049	0.12285	295.07	9.4468E-005
16800	216.65	9068.2	0.08950	0.14581	0.11903	295.07	9.7495E-005
17000	216.65	8786.7	0.08672	0.14129	0.11534	295.07	1.0062E-004
17200	216.65	8513.9	0.08403	0.13690	0.11176	295.07	1.0384E-004
17400	216.65	8249.6	0.08142	0.13265	0.10829	295.07	1.0717E-004
17600	216.65	7993.5	0.07889	0.12853	0.10492	295.07	1.1060E-004
17800	216.65	7745.3	0.07644	0.12454	0.10167	295.07	1.1415E-004
18000	216.65	7504.8	0.07407	0.12068	0.09851	295.07	1.1780E-004
18200	216.65	7271.9	0.07177	0.11693	0.09545	295.07	1.2158E-004
18400	216.65	7046.1	0.06954	0.11330	0.09249	295.07	1.2547E-004
18600	216.65	6827.3	0.06738	0.10978	0.08962	295.07	1.2949E-004
18800	216.65	6615.4	0.06529	0.10637	0.08684	295.07	1.3364E-004
19000	216.65	6410.0	0.06326	0.10307	0.08414	295.07	1.3793E-004
19200	216.65	6211.0	0.06130	0.09987	0.08153	295.07	1.4234E-004
19400	216.65	6018.2	0.05939	0.09677	0.07900	295.07	1.4690E-004
19600	216.65	5831.3	0.05755	0.09377	0.07654	295.07	1.5161E-004
19800	216.65	5650.3	0.05576	0.09086	0.07417	295.07	1.5647E-004
20000	216.65	5474.9	0.05403	0.08803	0.07187	295.07	1.6148E-004
20200	216.85	5305.0	0.05236	0.08522	0.06957	295.21	1.6694E-004
20400	217.05	5140.5	0.05073	0.08251	0.06735	295.34	1.7257E-004
20600	217.25	4981.3	0.04916	0.07988	0.06521	295.48	1.7839E-004
20800	217.45	4827.1	0.04764	0.07733	0.06313	295.61	1.8440E-004
21000	217.65	4677.9	0.04617	0.07487	0.06112	295.75	1.9060E-004
21200	217.85	4533.3	0.04474	0.07249	0.05918	295.89	1.9701E-004
21400	218.05	4393.4	0.04336	0.07019	0.05730	296.02	2.0363E-004
21600	218.25	4257.9	0.04202	0.06796	0.05548	296.16	2.1046E-004
21800	218.45	4126.8	0.04073	0.06581	0.05372	296.29	2.1752E-004
22000	218.65	3999.7	0.03947	0.06373	0.05202	296.43	2.2480E-004
22200	218.85	3876.7	0.03826	0.06171	0.05038	296.56	2.3232E-004

Table 2.1 Atmospheric properties in ISA (Cont..)

22400	219.05	3757.6	0.03708	0.05976	0.04878	296.70	2.4009E-004
22600	219.25	3642.3	0.03595	0.05787	0.04724	296.83	2.4811E-004
22800	219.45	3530.5	0.03484	0.05605	0.04575	296.97	2.5639E-004
23000	219.65	3422.4	0.03378	0.05428	0.04431	297.11	2.6494E-004
23200	219.85	3317.6	0.03274	0.05257	0.04291	297.24	2.7376E-004
23400	220.05	3216.1	0.03174	0.05091	0.04156	297.38	2.8287E-004
23600	220.25	3117.8	0.03077	0.04931	0.04026	297.51	2.9228E-004
23800	220.45	3022.6	0.02983	0.04776	0.03899	297.65	3.0198E-004
24000	220.65	2930.4	0.02892	0.04627	0.03777	297.78	3.1200E-004
24200	220.85	2841.1	0.02804	0.04482	0.03658	297.92	3.2235E-004
24400	221.05	2754.6	0.02719	0.04341	0.03544	298.05	3.3302E-004
24600	221.25	2670.8	0.02636	0.04205	0.03433	298.19	3.4404E-004
24800	221.45	2589.6	0.02556	0.04074	0.03325	298.32	3.5542E-004
25000	221.65	2510.9	0.02478	0.03946	0.03222	298.45	3.6716E-004
25200	221.85	2434.7	0.02403	0.03823	0.03121	298.59	3.7927E-004
25400	222.05	2360.9	0.02330	0.03704	0.03024	298.72	3.9178E-004
25600	222.25	2289.4	0.02259	0.03589	0.02929	298.86	4.0468E-004
25800	222.45	2220.1	0.02191	0.03477	0.02838	298.99	4.1800E-004
26000	222.65	2153.0	0.02125	0.03369	0.02750	299.13	4.3174E-004
26200	222.85	2087.9	0.02061	0.03264	0.02664	299.26	4.4593E-004
26400	223.05	2024.9	0.01998	0.03163	0.02582	299.40	4.6056E-004
26600	223.25	1963.9	0.01938	0.03064	0.02502	299.53	4.7566E-004
26800	223.45	1904.7	0.01880	0.02969	0.02424	299.66	4.9124E-004
27000	223.65	1847.3	0.01823	0.02878	0.02349	299.80	5.0732E-004
27200	223.85	1791.8	0.01768	0.02788	0.02276	299.93	5.2391E-004
27400	224.05	1737.9	0.01715	0.02702	0.02206	300.07	5.4102E-004
27600	224.25	1685.8	0.01664	0.02619	0.02138	300.20	5.5868E-004
27800	224.45	1635.2	0.01614	0.02538	0.02072	300.33	5.7690E-004
28000	224.65	1586.2	0.01565	0.02460	0.02008	300.47	5.9569E-004

Table 2.1 Atmospheric properties in ISA (Cont…)

28200	224.85	1538.7	0.01519	0.02384	0.01946	300.60	6.1508E-004
28400	225.05	1492.6	0.01473	0.02311	0.01886	300.74	6.3508E-004
28600	225.25	1448.0	0.01429	0.02239	0.01828	300.87	6.5572E-004
28800	225.45	1404.8	0.01386	0.02171	0.01772	301.00	6.7700E-004
29000	225.65	1362.9	0.01345	0.02104	0.01718	301.14	6.9896E-004
29200	225.85	1322.2	0.01305	0.02040	0.01665	301.27	7.2161E-004
29400	226.05	1282.8	0.01266	0.01977	0.01614	301.40	7.4497E-004
29600	226.25	1244.7	0.01228	0.01916	0.01564	301.54	7.6906E-004
29800	226.45	1207.6	0.01192	0.01858	0.01517	301.67	7.9391E-004
30000	226.65	1171.8	0.01156	0.01801	0.01470	301.80	8.1954E-004
30200	226.85	1137.0	0.01122	0.01746	0.01425	301.94	8.4598E-004
30400	227.05	1103.3	0.01089	0.01693	0.01382	302.07	8.7324E-004
30600	227.25	1070.6	0.01057	0.01641	0.01340	302.20	9.0136E-004
30800	227.45	1038.9	0.01025	0.01591	0.01299	302.33	9.3035E-004
31000	227.65	1008.1	0.00995	0.01543	0.01259	302.47	9.6026E-004
31200	227.85	978.3	0.00966	0.01496	0.01221	302.60	9.9109E-004
31400	228.05	949.5	0.00937	0.01450	0.01184	302.73	1.0229E-003
31600	228.25	921.4	0.00909	0.01406	0.01148	302.87	1.0557E-003
31800	228.45	894.3	0.00883	0.01364	0.01113	303.00	1.0895E-003
32000	228.65	867.9	0.00857	0.01322	0.01079	303.13	1.1243E-003

Table 2.1 Atmospheric properties in ISA

Note: Following values / expressions have been used while preparing ISA table.

$R = 287.05287\, m^2 sec^{-2} K$

$g = 9.80665\, m/s^2$

Sutherland formula for viscosity:

$$\mu = 1.458 X 10^{-6} \left[\frac{T^{3/2}}{T+110.4}\right]$$

In troposphere (h = 0 to 11000 m): T= 288.15 - 0.0065 h.

$$p = 101325 [1-0.000022588h]^{5.25588}$$

$$\rho = 1.225 [1-0.000022588h]^{4.25588} .$$

In lower stratosphere (h = 11000 to 20000 km): T=216.65 K.

$$p = 22632 \exp \{-0.000157688 (h-11000)\}$$

$$\rho = 0.36391 \exp \{-0.000157688 (h-11000)\}$$

In middle stratosphere (h = 20000 to 32000 km):

$$T = 216.65 + 0.001h$$

$$p = 5474.9 [1+0.000004616(h-20000)]^{-34.1632}$$

$$\rho = 0.08803 [1+0.000004616(h-20000)]^{-35.1632}$$