Appendix B

Lecture 40

Performance analysis of a subsonic jet transport – 3

Topics

6 Range and endurance
7 Turning performance
8 Take-off distance
9 Landing distance
10 Concluding remarks

6 Range and endurance

In this section, the range of the airplane in a constant altitude and constant velocity cruise is studied. The range is given by the following formula.

$$R = \frac{7.2\ E_{max}\ V}{TSFC} \tan^{-1}\left[\frac{E_1\ \zeta}{2E_{max}\ (1-KC_{L_1}\ E_1\ \zeta)}\right] \tag{25}$$

where, $E_{max} = \frac{1}{2\sqrt{K\ C_{D_O}}}$; K and C_{Do} are at Mach number corresponding to V.

$$\zeta = \frac{W_f}{W_1} = 1 - \frac{W_2}{W_1}$$

$$E_1 = \frac{C_{L1}}{C_{D1}} \quad , \quad C_{L1} = \frac{W_1}{\frac{1}{2}\rho V^2 S} \quad ,$$

C_{D1} = Drag coefficient at C_{L1} and Mach number corresponding to V.

W_1 is the weight of the airplane at the start of the cruise and W_2 is the weight of the airplane at the end of the cruise.

The cruising altitude is taken as h = 10973 m (36000 ft). TSFC is taken to be constant as $0.6 hr^{-1}$. The variation of drag polar above M = 0.8 is given by Eqs.17 and 18.

$W_1 = W_o = 59175 \times 9.81 = 580506.8$ N , $W_f = 0.205 \times W_1$

Allowing 6% fuel as trapped fuel, W_2 becomes

$W_2 = W_1 - 0.94 \times W_f$ or $\zeta = 0.94 \times 0.205 = 0.1927$

The values of endurance (in hours) are obtained by dividing the expression for range by 3.6V where V is in m/s. The values of range (R) and endurance (E) in flights at different velocities are presented in Table 5 and are plotted in Figs.22 and 23.

M	V (m/s)	C_{Do}	K	E_{max}	C_{L1}	C_{D1}	E_1	R (km)	E (hr)
0.50	147.53	0.0159	0.04244	19.25	1.312	0.089	14.75	2979.0	5.61
0.55	162.29	0.0159	0.04244	19.25	1.085	0.066	16.48	3608.0	6.18
0.60	177.04	0.0159	0.04244	19.25	0.911	0.051	17.82	4189.6	6.57
0.65	191.79	0.0159	0.04244	19.25	0.777	0.041	18.72	4691.7	6.80
0.70	206.54	0.0159	0.04244	19.25	0.670	0.035	19.17	5095.6	6.85
0.75	221.30	0.0159	0.04244	19.25	0.583	0.030	19.23	5396.5	6.77
0.80	236.05	0.0159	0.04244	19.25	0.513	0.027	18.95	5599.8	6.59
0.81	239.00	0.0159	0.04256	19.22	0.500	0.02654	18.84	5619.7	6.53
0.82	241.95	0.01592	0.04300	19.11	0.488	0.02616	18.65	5621.6	6.45
0.83	244.90	0.01597	0.04388	18.89	0.476	0.02591	18.37	5597.7	6.35
0.84	247.85	0.01604	0.04532	18.54	0.465	0.02584	18.00	5544.1	6.21
0.85	250.80	0.01613	0.04744	18.08	0.454	0.02591	17.52	5460.4	6.05
0.86	253.75	0.01624	0.05036	17.48	0.444	0.02617	16.97	5349.3	5.86
0.87	256.71	0.01637	0.05420	16.79	0.433	0.02653	16.32	5210.1	5.64
0.88	259.66	0.01652	0.05908	16.00	0.424	0.02714	15.62	5051.1	5.40

Table 5 Range and endurance in constant velocity flights at h = 10973 m (36000 ft)

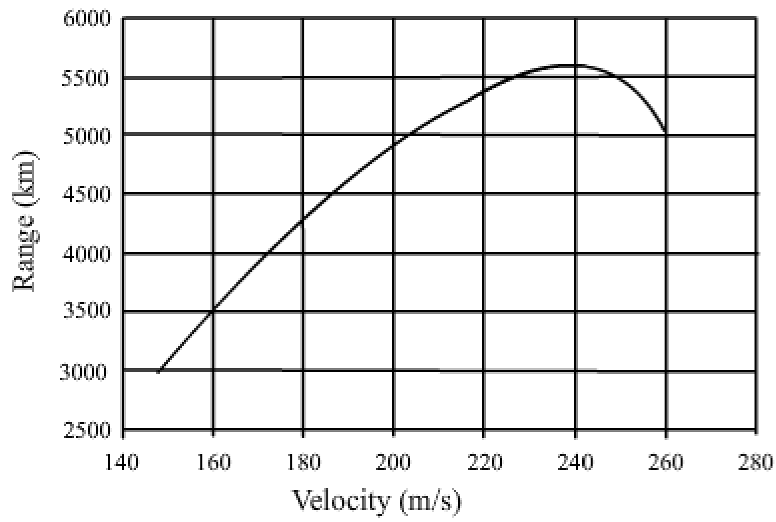


Fig.22 Range in constant velocity flights at h = 10973 m

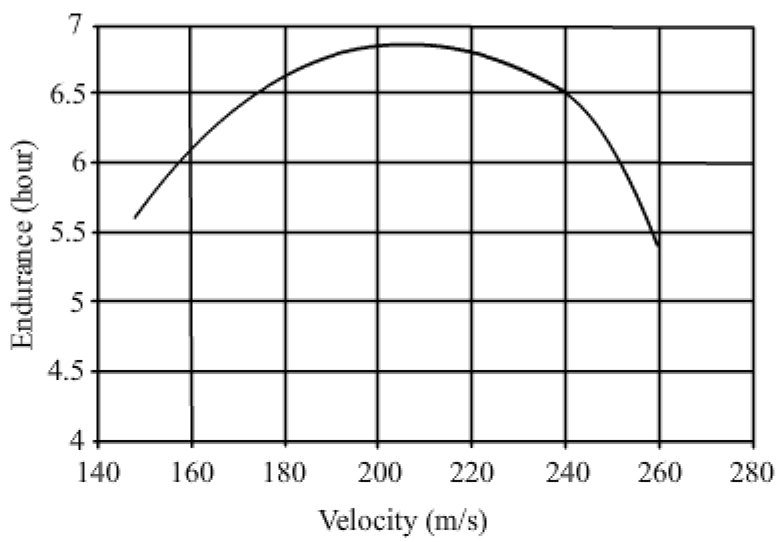


Fig.23 Endurance in constant velocity flights at h = 10973 m

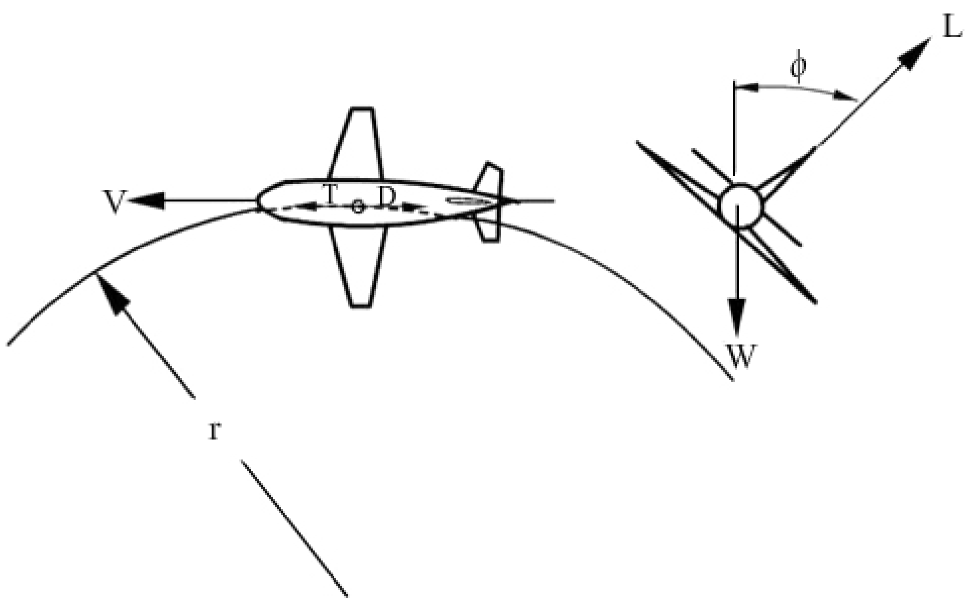
Remarks:

i) It is observed that the maximum range of 5620 km is obtained around a velocity of 240 m/s (864 kmph). Corresponding Mach number is 0.82 which is slightly higher than the Mach number beyond which C_{Do} and K increase. This can be explained based on two factors namely (a) the range increases as the flight speed increases and (b) after M_{cruise} is exceeded, C_{Do} and K increase thus, reducing $(L/D)_{max}$.

ii) The range calculated above is the gross still air range. The safe range would be about two-thirds of this. In the present case, the safe range would be around 3750 km.

iii) The maximum endurance of 6.85 hours occurs in a flight at V = 206 m/s. (742 kmph). It is observed that the endurance is roughly constant over a speed range of 190 m/s to 230 m/s (684 to 828 kmph).

7 Turning performance



Forces acting on an airplane in turning flight

In this section, the performance of the airplane in a steady level, co-ordinated-turn is studied. The equations of motion in this case are:

$$T - D = 0$$

$$W - L\cos\phi = 0$$

$$L \sin \phi = \frac{W}{g}\frac{V^2}{r}$$

where ϕ is the angle of bank.

These equations give:

$$r = \frac{V^2}{g\tan\phi}$$

$$\dot{\psi} = \frac{V}{r} = \frac{g\tan\phi}{V}$$

$$\text{Load factor} = n = \frac{L}{W} = \frac{1}{\cos\phi}$$

where, $\dot{\psi}$ is the rate of turn and r is the radius of turn.

The following procedure is used to obtain r_{min} and $\dot{\psi}_{max}$.

1) A flight speed and altitude are chosen and the level flight lift coefficient C_{LL} is obtained as :

$$C_{LL} = \frac{2(W/S)}{\rho V^2}$$

2) If $C_{Lmax}/C_{LL} < n_{max}$, where n_{max} is the maximum load factor for which the airplane is designed, then the turn is limited by C_{Lmax} and $C_{LT1} = C_{Lmax}$. However, if $C_{Lmax}/C_{LL} > n_{max}$, then the turn is limited by n_{max}, and $C_{LT1} = n_{max}C_{LL}$.

3) From the drag polar, C_{DT1} is obtained corresponding to C_{LT1} . Then,

$$D_{T1} = \frac{1}{2}\rho V^2 S C_{DT1}\,.$$

If $D_{T1} > T_a$, where T_a is the available thrust at that speed and altitude, then the turn is limited by the engine output. In this case, the maximum permissible value of C_D in turning flight is found from

$$C_{DT} = \frac{T_a}{\frac{1}{2}\rho V^2 S}$$

From drag polar, the value of C_{LT} is calculated as

$$C_{LT} = \sqrt{\frac{C_{DT} - C_{Do}}{K}}$$

However, if $D_{T1} < T_a$, then the turn is not limited by the engine output and the value of C_{LT} calculated in step (2) is retained.

4. Once C_{LT} is known, the load factor during the turn is determined as

$$n = \frac{C_{LT}}{C_{LL}}$$

Once n is known, the values of ϕ, r and $\dot{\psi}$ can be calculated using the equations given above.

The above steps are repeated for various speeds and altitudes. A typical turning flight performance estimation is presented in Table 6. In these calculations, C_{Lmax} = 1.4 and n_{max} = 3.5 are assumed. The variation of turning performance with altitude is shown in Table 7. Figures 24, 25, 26 and 27 respectively present (a) radius of turn vs velocity with altitude as parameter, (b) V_{rmin} vs altitude, (c) rate of turn vs velocity with altitude as parameter and (d) $V_{\dot{\psi}max}$ vs altitude.

V (m/s)	C_{LL}	$\frac{C_{Lmax}}{C_{LL}}$	C_{LT1}	C_{DT1}	T_1 (N)	T_a (N)	C_{DT}	C_{LT}	n	ϕ	r (m)	$\dot{\psi}$ (rad/s)
78.8	1.365	1.026	1.4	0.0991	42106	126250	0.0991	1.4	1.026	12.9	2768	0.0285
98.8	0.868	1.612	1.4	0.0991	66182	118125	0.0991	1.4	1.612	51.7	787	0.1255
118.8	0.602	2.331	1.4	0.0991	95678	113750	0.0991	1.4	2.331	64.6	684	0.1738
138.8	0.440	3.181	1.4	0.0991	130595	106611	0.0809	1.238	2.813	69.2	747	0.1858
158.8	0.336	4.164	1.177	0.0747	128778	101539	0.0589	1.006	2.993	70.5	912	0.1742
178.8	0.265	5.279	0.928	0.0525	114709	97041	0.0444	0.819	3.089	71.1	1115	0.1603
198.8	0.215	6.527	0.751	0.0398	107635	92606	0.0343	0.661	3.080	71.1	1384	0.1437
218.8	0.177	7.905	0.620	0.0322	105461	89483	0.0273	0.519	2.930	70.0	1772	0.1235
238.8	0.149	9.415	0.521	0.0274	106860	86229	0.0221	0.383	2.573	67.1	2452	0.0974
241.8	0.145	9.655	0.508	0.0268	107282	85779	0.0215	0.362	2.494	66.4	2609	0.0927

Table 6 A typical turning flight performance at sea level

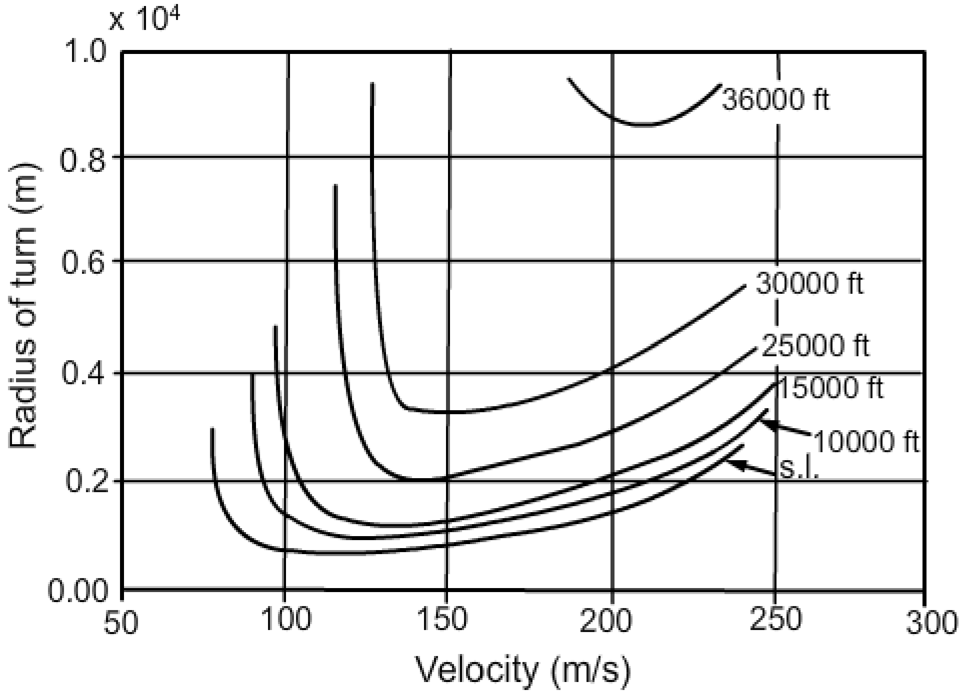


Fig.24 Radius of turn vs velocity at various altitudes

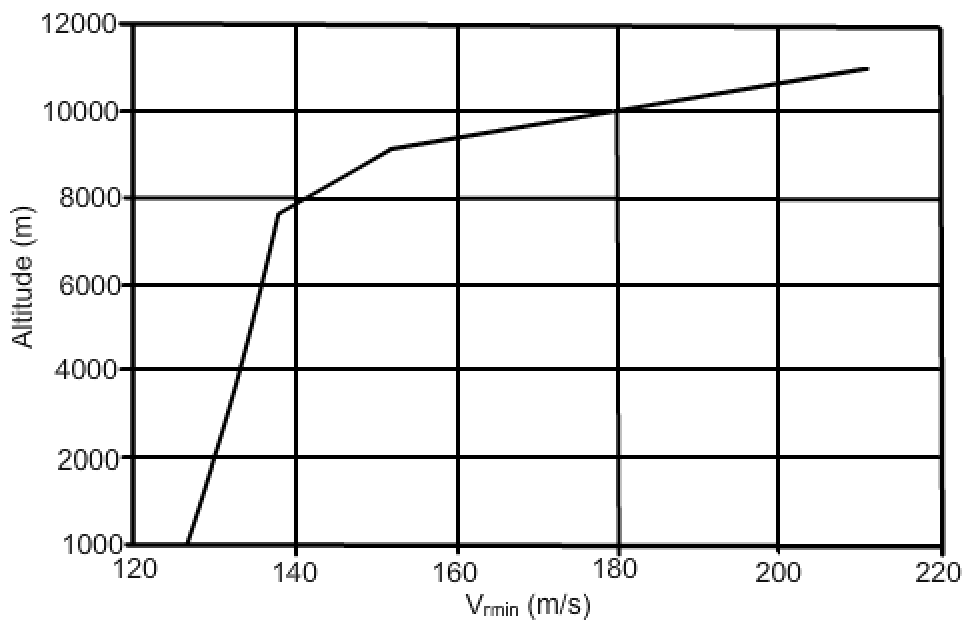


Fig.25 Velocity at r_{min} vs altitude

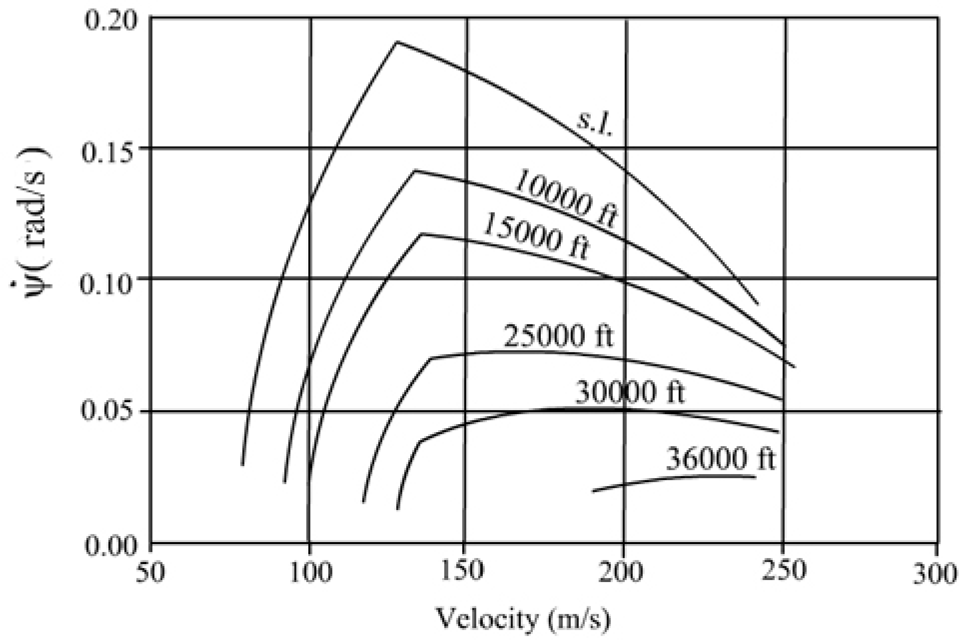


Fig.26 Rate of turn ($\dot{\psi}$) vs speed at various altitudes

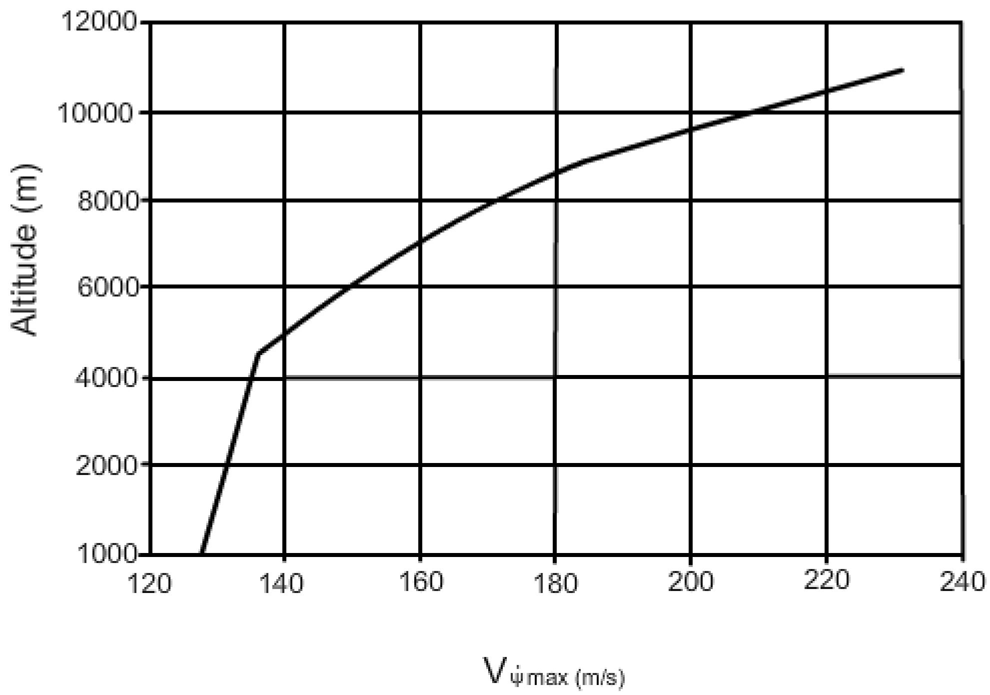


Fig.27 Velocity at $\dot{\psi}_{max}$ vs altitude

h (m)	r_{min} (m)	V_{rmin} (m/s)	$\dot{\psi}_{max}$ (rad/s)	$V_{\dot{\psi} max}$ (m/s)
0.0	666	126.8	0.1910	127.8
3048.0	945	132.6	0.1410	133.6
4572.0	1155	135.1	0.1170	136.1
7620.0	1971	138.3	0.0731	165.3
9144.0	3247	151.3	0.0513	187.3
10972.8	8582	211.0	0.0256	231.0

Table 7 Turning flight performance

Remarks:

i) The maximum value of $\dot{\psi}$ is 0.191 and occurs at a speed of 127.8m/s at sea level.

ii) The minimum radius of turn is 666 m and occurs at a speed of 126.8m/s at sea level.

iii) The various graphs show a discontinuity in slope when the criterion which limits the turn, changes from n_{max} to thrust available.

8 Take-off distance

In this section, the take-off performance of the airplane is evaluated. The take-off distance consists of take-off run, transition and climb to screen height. Rough estimates of the distance covered in these phases can be obtained by writing down the appropriate equations of motion. However, the estimates are approximate and Ref.4 chapter 5 recommends the following formulae for take-off distance and balance field length based on the take-off parameter.

This parameter is defined as:

$$\text{Take-off parameter} = \frac{W/S}{\sigma C_{LTo}(T/W)} \qquad (26)$$

where W/S is wing loading in lb/ft^2, C_{LTO} is 0.8 x C_{Land} and σ is the density ratio at take-off altitude. In the present case:

$$\frac{W}{S} = 5195 N/m^2 = 108.2 lb/ft^2;\ C_{LTO} = 0.8 \times 2.7 = 2.16;\ \sigma = 1.0 \text{(sea level)}$$

and $$\frac{T}{W} = \frac{2 \times 97900}{59175 \times 9.81} = 0.3373$$

Hence, take-off parameter $= \dfrac{108.2}{1.0 \times 2.16 \times 0.3373} = 148.86$ (27)

From Ref.4, chapter 5, the take-off distance, over 50', is 2823' or 861m. The balance field length for the present case of two engined airplane is 6000' or 1829m.

Remark:

It may be noted that the balance field length in this case, is more than twice the take-off distance.

9 Landing distance

In this section the landing distance of the airplane is calculated. From Ref.4, chapter 5, the landing distance for commercial airliners is given by the formula:

$$s_{land} = 80\,(\frac{W}{S})\frac{1}{\sigma C_{Lmax}} + 1000 \text{ ft} \quad (28)$$

where W/S is in lbs/ft^2. In the present case:

$(W/S)_{land} = 0.85 \times (W/S)_{takeoff} = 0.85 \times 108.5 = 92.225\ lb/ft^2$

$C_{Lmax} = 2.7$, $\sigma = 1.0$

Hence,

$$s_{land} = 80 \times 92.225\ \frac{1}{1.0 \times 2.7} + 1000 = 3732 \text{ ft} = 1138 \text{ m} \quad (29)$$

10 Concluding remarks

1. Performance of a typical commercial airliner has been estimated for stalling speed, maximum speed, minimum speed, steady climb, range, endurance, turning, take-off and landing.
2. The performance approximately corresponds to that of B737-200.
3. Figure 28 presents the variations with altitude of the characteristic velocities corresponding to :

   stalling speed, Vs

   maximum speed, V_{max}

   minimum speed as dictated by thrust, $(V_{min})_e$

   maximum rate of climb, $V_{(R/C)max}$

   maximum angle of climb, $V_{\gamma\, max}$

   maximum rate of turn, $V_{\dot{\psi}\, max}$

   minimum radius of turn, V_{rmin}

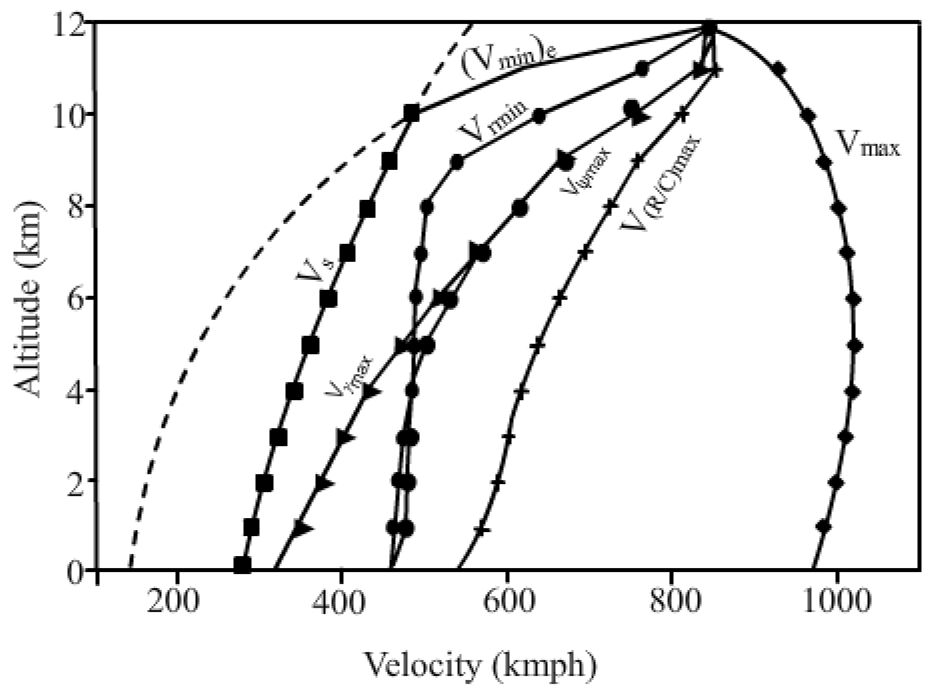


Fig.28 Variations of characteristic velocities with altitude