# Appendix B

## Lecture 39

## Performance analysis of a subsonic jet transport –2

## Topics



### 3 Engine characteristics

To calculate the performance, the variations of thrust and SFC with speed and altitudes are needed. Chapter 9 of Ref.3 contains these variations for turbofan engines with various bypass ratios. The thrust variations versus Mach number with altitude as parameter are given, in non-dimensional form, for take-off, cruise and climb ratings. The values were read from those curves, interpolated and later smoothed. The values multiplied by 97.9 kN, the sea level static thrust rating for the chosen engine, are shown in Figs.3 and 4. Figure 3 also contains (a) the variation of thrust with Mach number at sea level with take-off rating and (b) variations of climb thrust with Mach number at various altitudes. The values at h = 38000 ft and 39000 ft are obtained by interpolating the values at 36000 ft and 40000 ft and are used for computation of performance.

The SFC variation is also given in Ref.3, but is taken as 0.6 $hr^{-1}$ under cruise conditions based on the trend shown in Fig.3.3 of Ref.4.

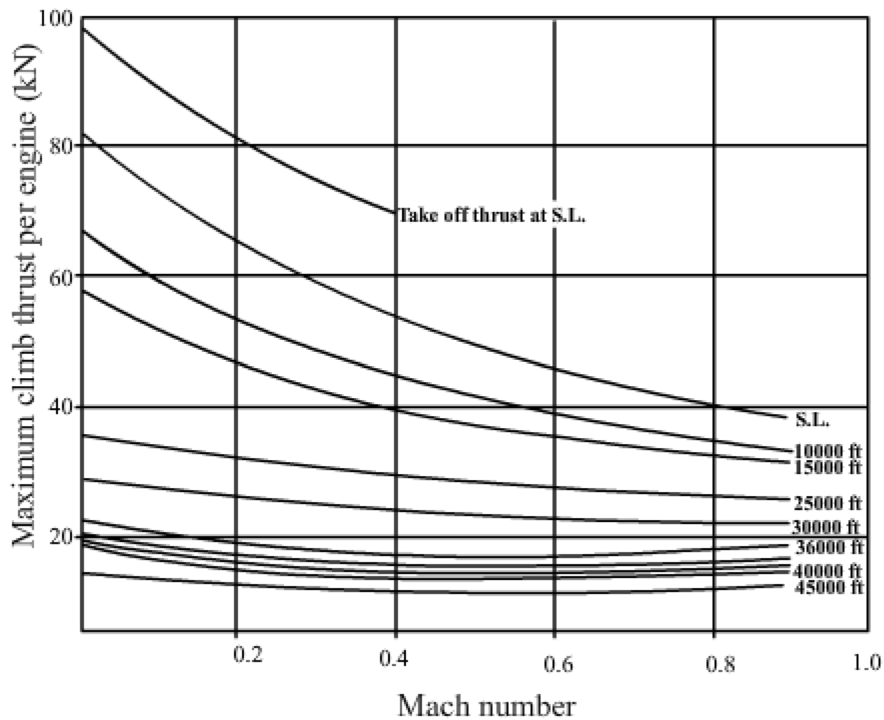



Fig.3 Output for single engine – take-off thrust at sea level and climb thrust at various altitudes.

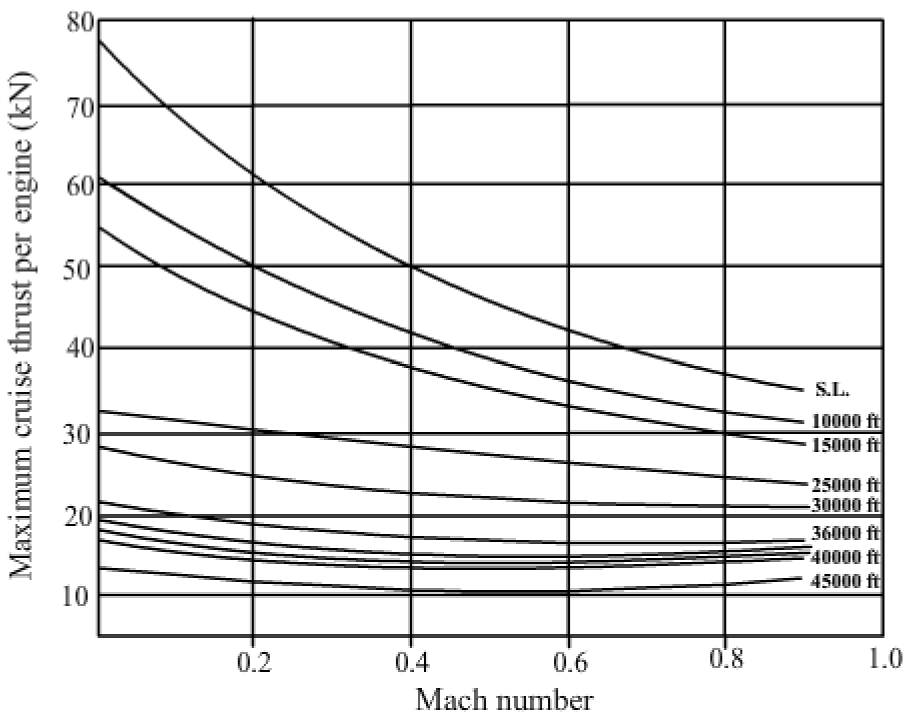
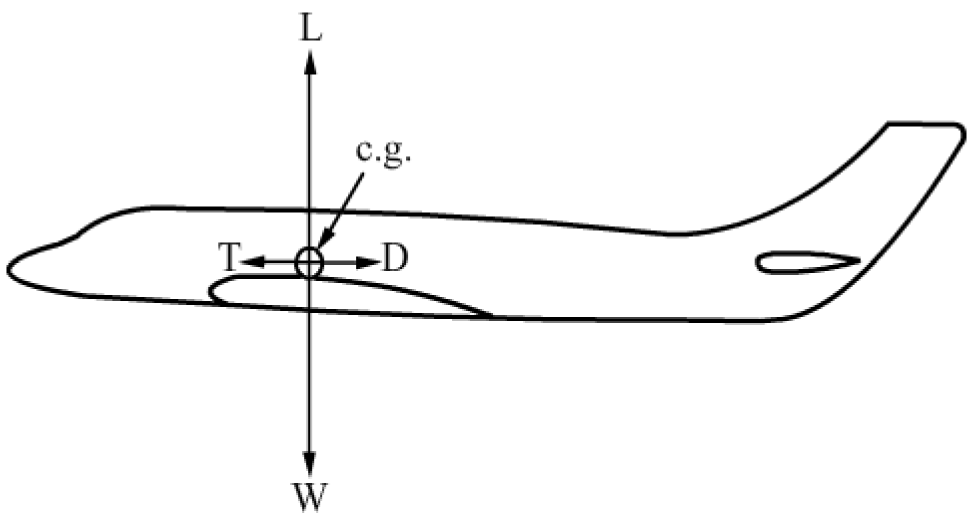



Fig.4 Output of single engine – cruise thrust at various altitudes

## 4 Level flight performance




Forces on an airplane in steady level flight

In steady level flight, the equations of motion, in standard notation, are:

$$T - D = 0 \tag{5}$$

$$L - W = 0 \tag{6}$$

$$L = W = \frac{1}{2}\rho V^2 S C_L \tag{7}$$

$$D = \frac{1}{2}\rho V^2 S C_D = T \tag{8}$$

### 4.1 Stalling speed

In level flight,

$$V = \sqrt{\frac{2W}{\rho S C_L}} \tag{9}$$

Since, $C_L$ cannot exceed $C_{Lmax}$, there is a flight speed below which level flight is not possible. The flight speed at $C_L = C_{Lmax}$ is called the stalling speed and is denoted by $V_s$

$$V_s = \sqrt{\frac{2W}{\rho S C_{max}}} \tag{10}$$

Since, ρ decreases with altitude, $V_s$ increases with height. It may be noted that

W/S = 5195 $N/m^2$, $C_{Lmax}$ = 2.7 with landing flaps and $C_{Lmax}$ = 1.4 without flaps. The values of stalling speed at different altitudes and flap settings are tabulated in Table 1 and shown in Fig.5.

| h (m) | ρ ($kg/m^3$) | $V_s$ ($C_{Lmax}$ = 1.4) (m/s) | $V_s$ ($C_{Lmax}$ = 2.7) (m/s) |
|---|---|---|---|
| 0 | 1.225 | 77.83 | 56.04 |
| 2000 | 1.006 | 85.86 | 61.83 |
| 4000 | 0.819 | 95.18 | 68.54 |
| 6000 | 0.659 | 106.06 | 76.37 |
| 8000 | 0.525 | 118.87 | 85.59 |
| 10000 | 0.412 | 134.09 | 96.56 |
| 11000 | 0.363 | 142.80 | 102.83 |
| 12000 | 0.310 | 154.52 | 111.27 |

Table 1 Variation of stalling speed with altitude

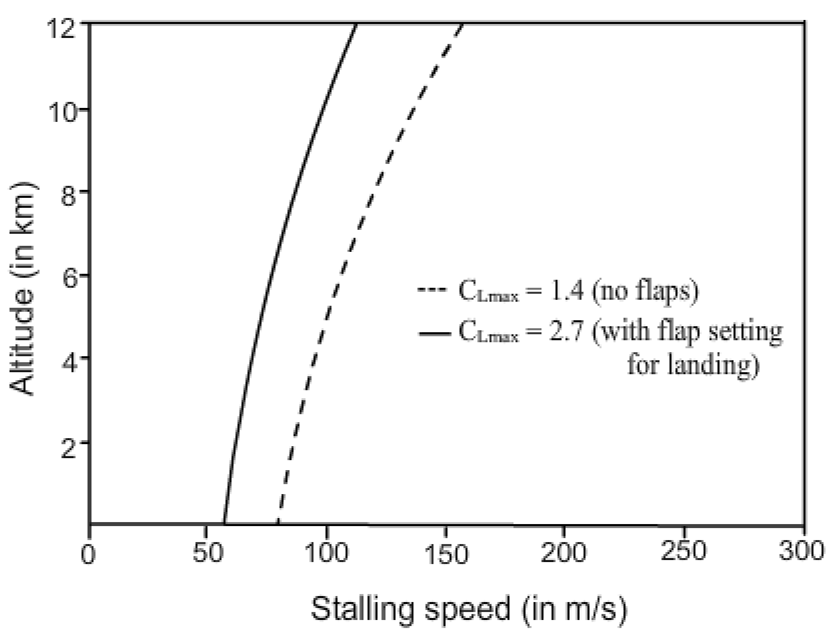



Fig.5 Stalling speed vs altitude

### 4.2 Variations of $V_{min}$ and $V_{max}$ with altitude

To determine the $V_{min}$ and $V_{max}$ at each altitude, the following procedure is adopted. The engine thrust as a function of velocity at each altitude is obtained from the smoothed data. The drag at each altitude is obtained as a function of velocity using the drag polar and the level flight formulae given below.

$$C_L = \frac{2 \times (W/S)}{\rho V^2} \tag{11}$$

$$C_D = C_{Do} + K\,C_L^2 \tag{12}$$

$$\text{Thrust required} = \text{Drag} = \frac{1}{2}\rho V^2 S C_D \tag{13}$$

$$\text{Thrust available} = T_a = f(M) \tag{14}$$

where, $C_{Do}$ = 0.0159 and K = 0.04244.

However, the cruise Mach number ($M_{cruise}$) for this airplane is 0.8. Hence, $C_{Do}$ and K are expected to become functions of Mach number above $M_{cruise}$. To get some guidelines about variations of $C_{Do}$ and K, the drag polars of B-727 given in Volume VI, Chapter 5 of Ref.6 are considered. These drag polars are shown in the Fig.6 as discrete points.

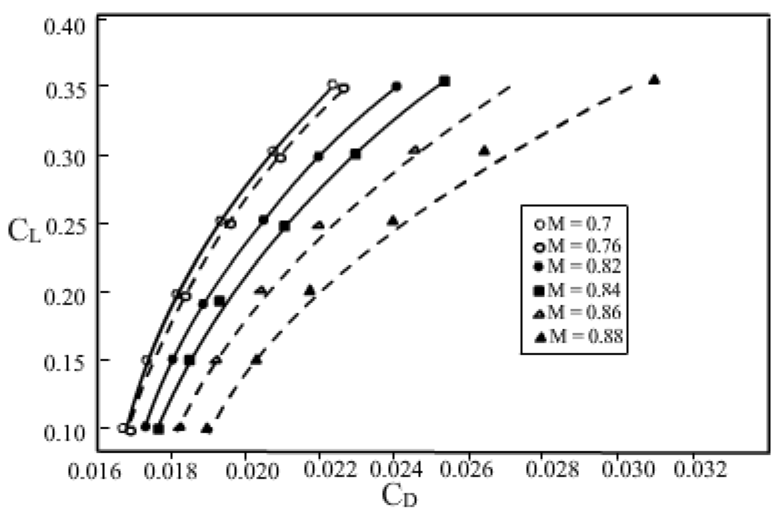



Fig.6 Drag polars at different Mach numbers for B727-100; Symbols are data from Ref.6 and various lines are the parabolic fits.

These polars were approximated by the parabolic polar expression namely $C_D = C_{Do} + KC_L^2$. The values of $C_{Do}$ and K at various Mach numbers, obtained by least square method, are given in the Table 2. The parabolic fits are also shown in Fig.6.

| M | $C_{Do}$ | K |
|---|---|---|
| 0.7 | 0.01631 | 0.04969 |
| 0.76 | 0.01634 | 0.05257 |
| 0.82 | 0.01668 | 0.06101 |
| 0.84 | 0.01695 | 0.06807 |
| 0.86 | 0.01733 | 0.08183 |
| 0.88 | 0.01792 | 0.10300 |

Table 2 Variations of $C_{Do}$ and K with Mach number (Parabolic fit)

The variations of $C_{Do}$ and K with Mach number are plotted in Figs.7 and 8. It is seen that there is no significant increase in $C_{Do}$ and K upto M = 0.76. This is expected to be the cruise Mach number for the airplane (B727-100). Following analytical expressions have been found to closely represent the changes in $C_{Do}$ and K from M = 0.76 to M = 0.86.

$$C_{Do} = 0.01634 - 0.001 \times (M - 0.76) + 0.11 \times (M - 0.76)^2 \quad (15)$$

$$K = 0.05257 + (M - 0.76)^2 + 20.0 \times (M - 0.76)^3 \quad (16)$$

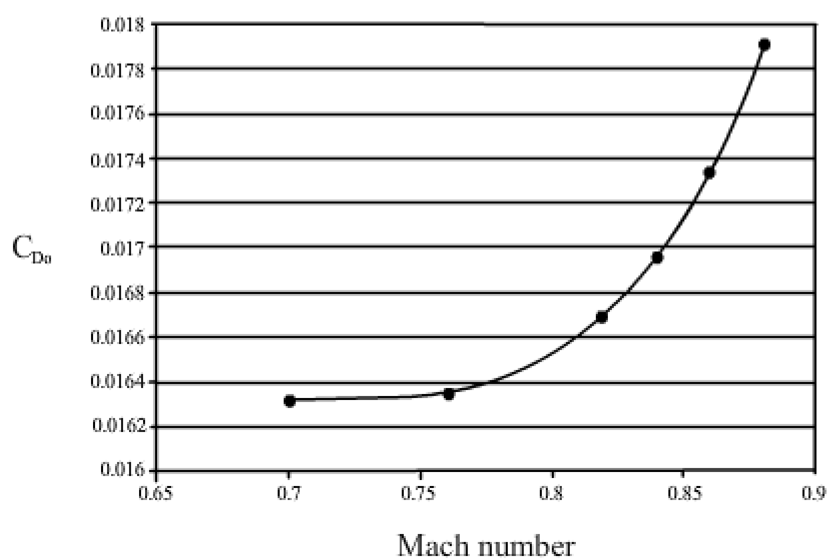



Fig.7 Variation of $C_{Do}$ with Mach number

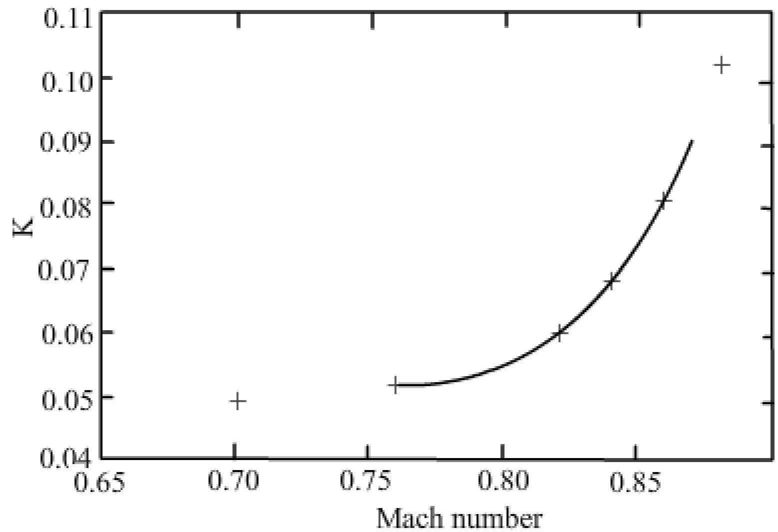



Fig.8 Variation of K with Mach number

In the case of the present airplane, the cruise Mach number is 0.8. The variations of $C_{Do}$ and K above $M_{cruise}$ and upto M = 0.9, based on the B727-100 data are taken as follows.

$$C_{Do} = 0.0159 - 0.001 \times (M - 0.80) + 0.11 \times (M - 0.80)^2 \quad (17)$$

$$K = 0.04244 + (M - 0.80)^2 + 20.0 \times (M - 0.80)^3 \quad (18)$$

The thrust available and thrust required curves are plotted at each altitude as a function of velocity. The points of intersection give the $(V_{min})_e$ and $V_{max}$ at each altitude from thrust available consideration (Figs.9 – 14).

However, to arrive at the minimum speed ($V_{min}$), the stalling speed ($V_s$) also needs to be taken in to account. Since, the drag polar is not valid below $V_s$, in the Figs.9 to 14, the thrust required curves are plotted only for $V \geq V_s$. Stalling speed is taken for $C_{Lmax}$ without flaps.

The calculations are carried out for h = 0, 10000, 15000, 25000, 30000 and 36000 ft, i.e S.L, 3048, 4572, 7620, 9144 and 10972.8 m using $T_a$ as both climb thrust ($T_{climb}$) and as cruise thrust ($T_{cr}$). Results in Figs.9 – 14 are presented only for climb thrust case. The variations of $V_s$, $(V_{min})_e$ and $V_{max}$ are tabulated in Table 3 and presented in Fig.15.

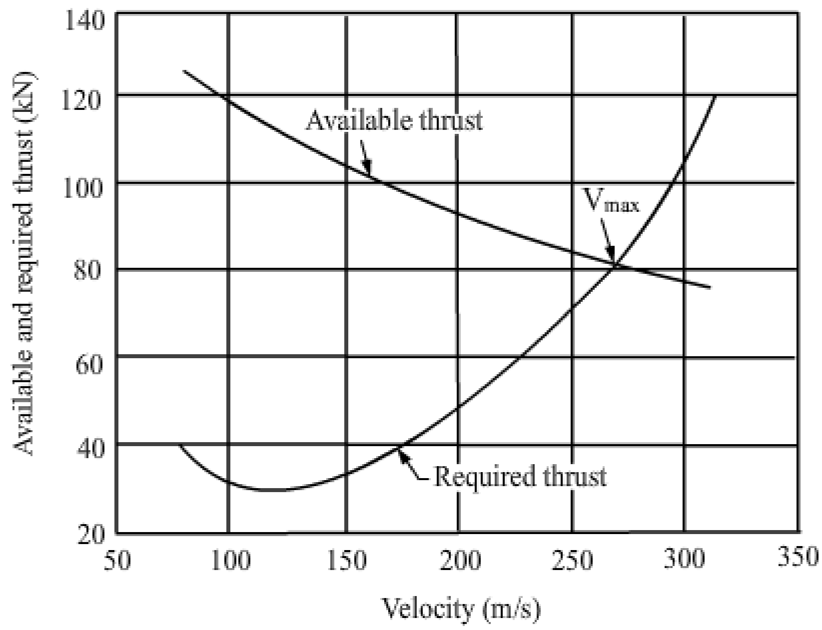



Fig.9 Available and required thrust at S.L

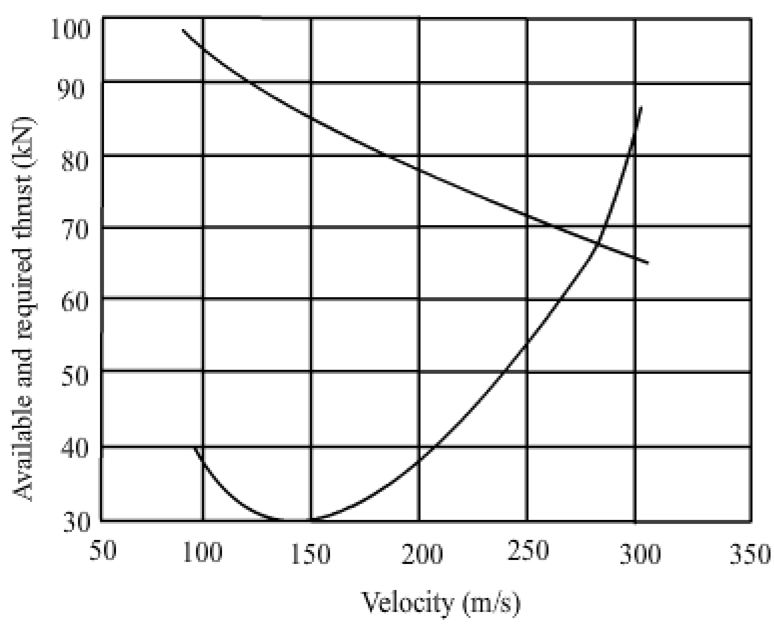



Fig.10 Available and required thrust at h = 3048 m

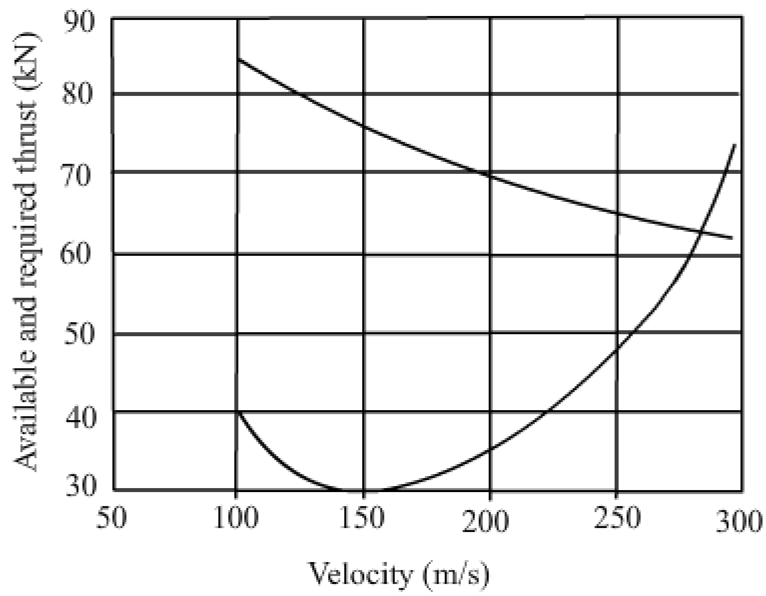



Fig.11 Available and required thrust at h = 4572 m

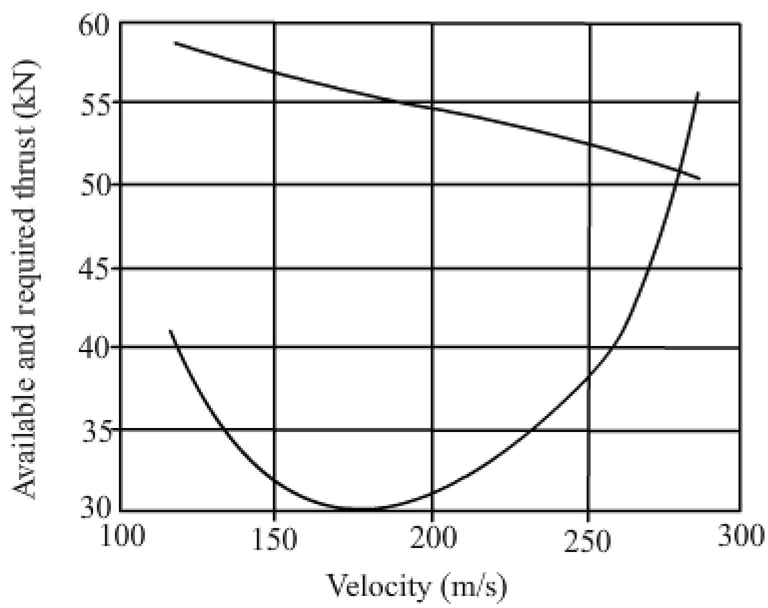



Fig.12 Available and required thrust at h = 7620 m

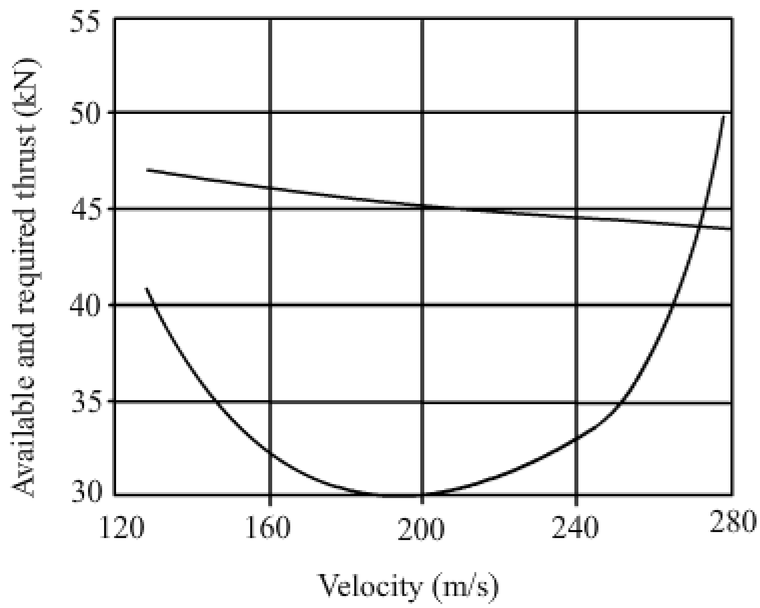



Fig.13 Available and required thrust at h = 9144 m

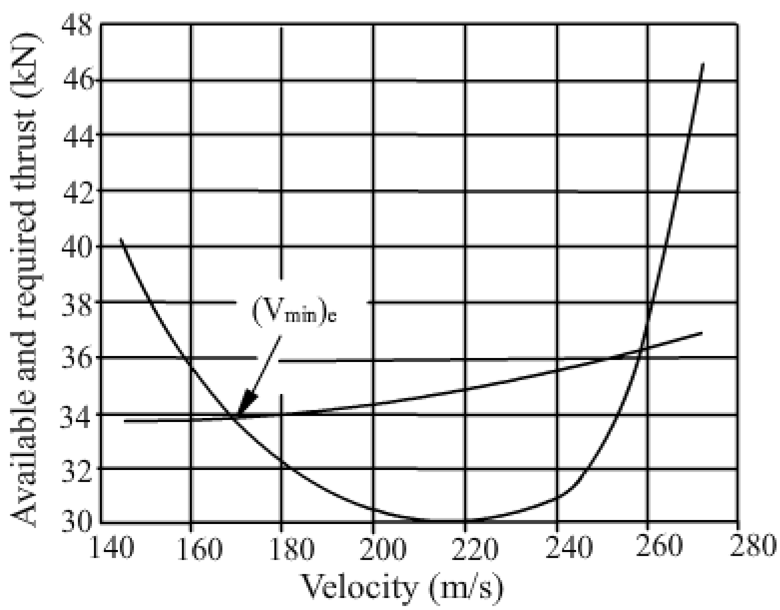



Fig.14 Available and required thrust at h = 10973 m

| h (in ft) | h (in m) | $V_s$ (m/s) | $(V_{min})_e$ (m/s) $T = T_{cr}$ | $(V_{min})_e$ (m/s) $T=T_{climb}$ | $V_{max}$(m/s) $T = T_{cr}$ | $V_{max}$(m/s) $T=T_{climb}$ | $V_{max}$(kmph) $T=T_{climb}$ |
|---|---|---|---|---|---|---|---|
| S.L | 0 | 77.833 | < $V_s$ | < $V_s$ | 258.711 | 269.370 | 969.7 |
| 10000 | 3048 | 90.579 | < $V_s$ | < $V_s$ | 272.060 | 280.595 | 1010.1 |
| 15000 | 4572 | 98.131 | < $V_s$ | < $V_s$ | 275.613 | 283.300 | 1019.9 |
| 25000 | 7620 | 116.292 | < $V_s$ | < $V_s$ | 272.929 | 279.291 | 1005.4 |
| 30000 | 9144 | 127.278 | < $V_s$ | < $V_s$ | 267.854 | 271.755 | 978.3 |
| 36000 | 10973 | 142.594 | 176.054 | 169.071 | 253.671 | 258.154 | 929.4 |
| 38000 | 11582 | 149.557 | 217.386 | 200.896 | 243.676 | 248.630 | 895.1 |
| 38995 | 11884 | 153.159 | 235.48 | 229.865 | 235.48 | 238.649 | 859.1 |
| 39220 | 11954 | 153.950 | ---- | 236.40 | ------ | 236.40 | 851.04 |

Table 3 Variations of $V_s$, $(V_{min})_e$ , $V_{min}$ and $V_{max}$

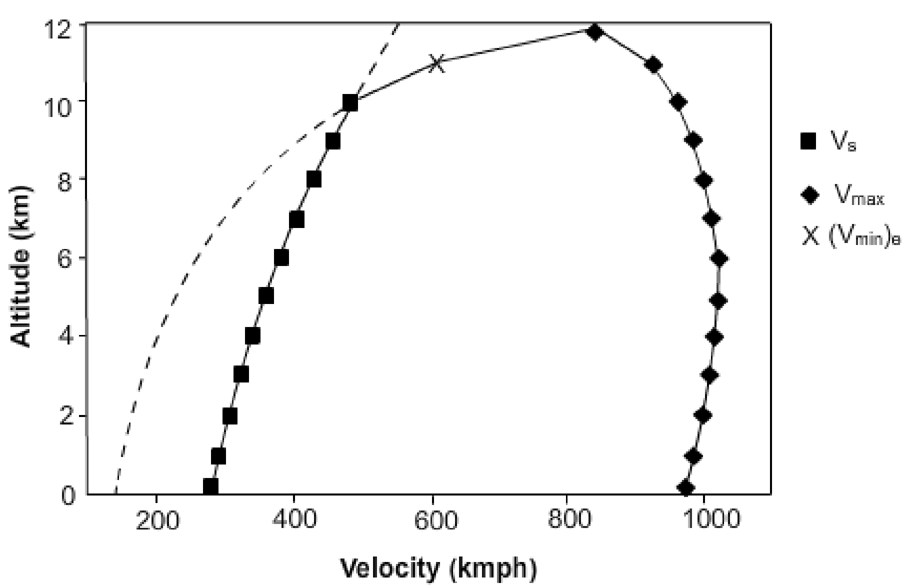
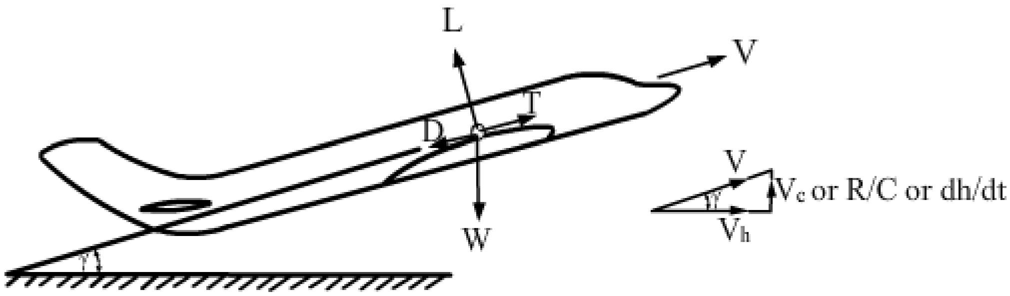



Fig.15 Variations of $V_{min}$ and $V_{max}$ with altitude

## 5 Steady climb




Forces on an airplane in a steady climb

In this flight, the C.G of the airplane moves along a straight line inclined to the horizontal at an angle $\gamma$ . The velocity of flight is assumed to be constant during the climb. Since, the flight is steady, the acceleration is zero and the equations of motion can be written as:

$$T - D - W \sin\gamma = 0 \qquad (19)$$

$$L - W \cos\gamma = 0 \qquad (20)$$

To calculate the variation of rate of climb with flight velocity at different altitudes, the following procedure is adopted.

Choose an altitude.

Choose a flight speed.

Noting that $C_L = 2W \cos\gamma / \rho SV^2$, gives :

$$C_D = C_{Do} + K\left(\frac{2W\cos\gamma}{\rho SV^2}\right)^2$$

Also,

$$V_c = V \sin\gamma$$

Hence, $$\cos\gamma = \sqrt{1-\frac{V_c^2}{V^2}}$$

Substituting various quantities in Eq.(19) yields :

$$T_a = \frac{1}{2}\rho V^2 S\left\{C_{DO} + \frac{KW^2}{\frac{1}{2}\rho V^2 S}\left[1-\left(\frac{V_c}{V}\right)^2\right]\right\} + W\frac{V_c}{V}$$

$$\text{Or} \quad A\left(\frac{V_c}{V}\right)^2 + B\left(\frac{V_c}{V}\right) + C = 0 \tag{21}$$

$$A = \frac{KW^2}{\frac{1}{2}\rho V^2 S};\ B = -W;\ C = T_a - \frac{1}{2}\rho V^2 S C_{Do} - \frac{2KW^2}{\rho V^2 S}, T_a = \text{Thrust available} \tag{22}$$

Since, altitude and flight velocity have been chosen, the thrust available is read from the climb thrust curves in Fig.3. Further, the variation of $C_{Do}$ and K with Mach number is taken as in Eqs.17 and 18. Equation 21 gives 2 values of $V_c/V$ . The value which is less than 1.0 is chosen, as sin $\gamma$ cannot be greater than unity. Hence ,

$$\gamma = \sin^{-1}(V_c/V) \tag{23}$$

$$\text{and} \quad V_c = V \sin\gamma \tag{24}$$

This procedure is repeated for various speeds between $V_{min}$ and $V_{max}$. The entire procedure is then repeated for various altitudes. The variations of (R/C) and $\gamma$ with velocity and with altitude as parameter are shown in Figs.16 and 18. The variations of $(R/C)_{max}$ and $\gamma_{max}$ with altitude are shown in Figs.17 and 19. The variations of $V_{(R/C)max}$ and $V_{\gamma max}$ with altitude are shown in Figs.20 and 21. A summary of results is presented in Table 4.

| h (ft) | h (m) | $(R/C)_{max}$ (m/min) | $V_{(R/C)max}$ (m/s) | $\gamma_{max}$ (degrees) | $V_{\gamma max}$ (m/s) |
|---|---|---|---|---|---|
| 0 | 0.0 | 1086.63 | 149.7 | 8.7 | 88.5 |
| 10000 | 3048.0 | 867.34 | 167.5 | 6.0 | 111.6 |
| 15000 | 4572.0 | 738.16 | 174.0 | 4.7 | 125.7 |
| 25000 | 7620.0 | 487.41 | 198.2 | 2.6 | 164.1 |
| 30000 | 9144.0 | 313.43 | 212.2 | 1.5 | 188.0 |
| 36000 | 10972.8 | 115.57 | 236.1 | 0.5 | 230.2 |
| 38000 | 11582.4 | 41.58 | 236.9 | 0.2 | 234.0 |
| 38995 | 11885.7 | 1.88 | 236.5 | 0.0076 | 236.0 |
| 39220 | 11954.0 | 0 | 236.40 | 0 | 236.40 |

Table 4 Climb performance

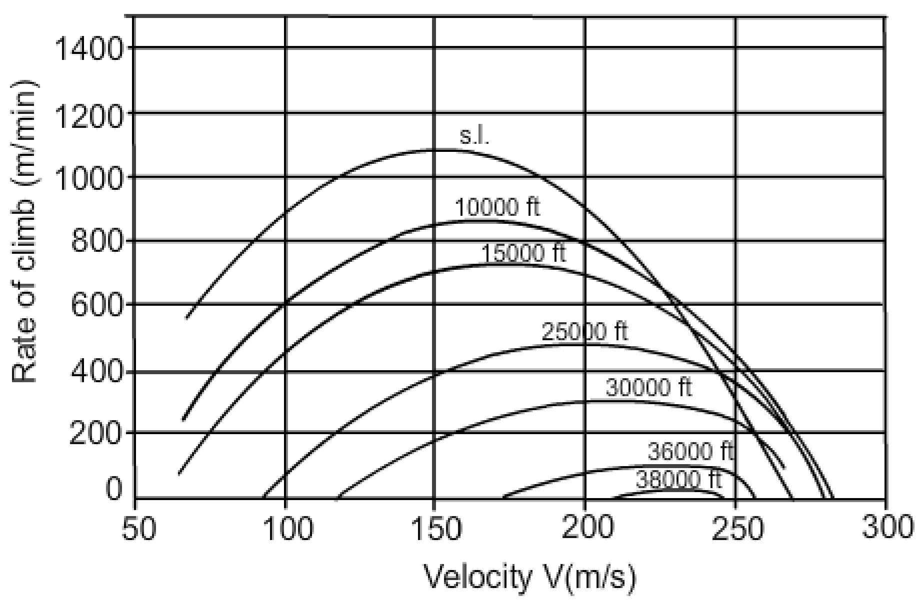



Fig.16 Rate of climb vs velocity for various altitudes

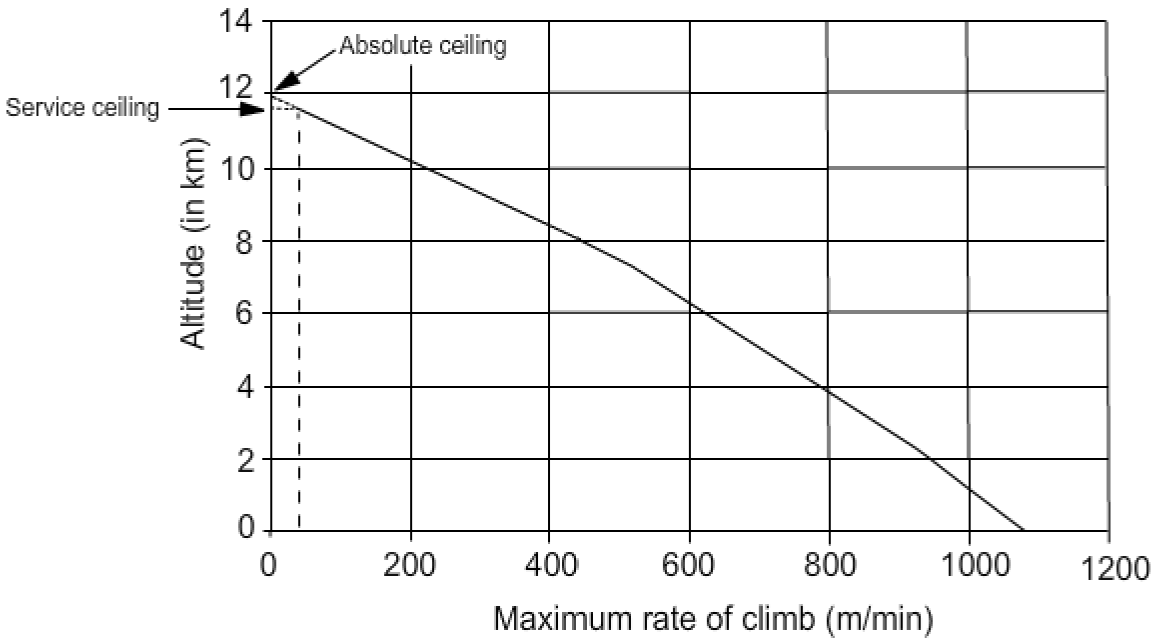



Fig.17 Maximum rate of climb vs altitude

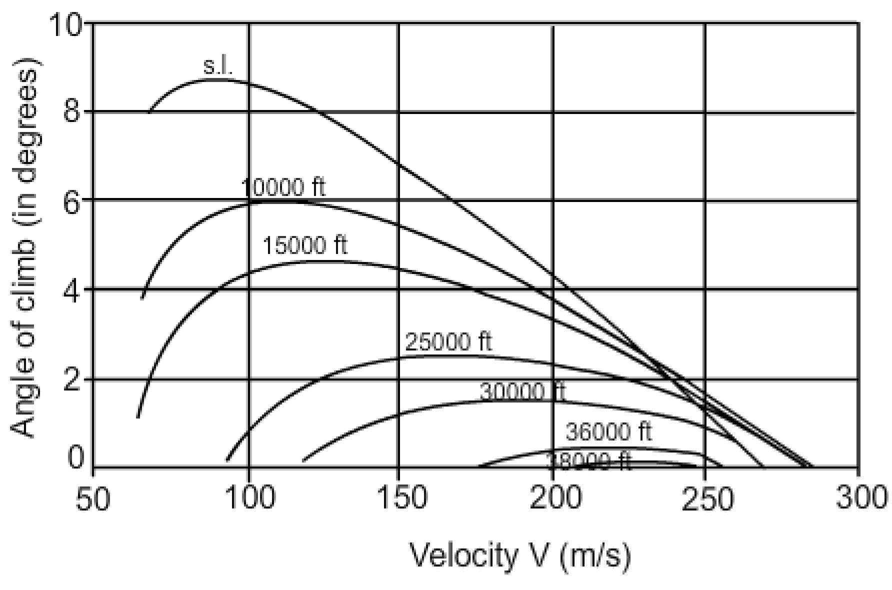



Fig.18 Angle of climb vs velocity for various altitudes

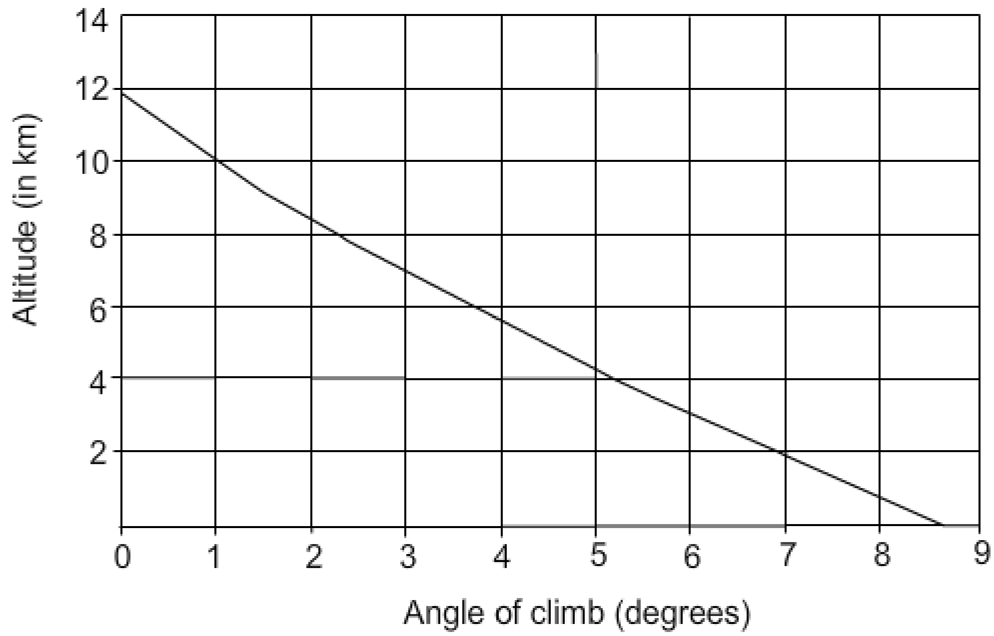



Fig.19 Maximum angle of climb vs altitude

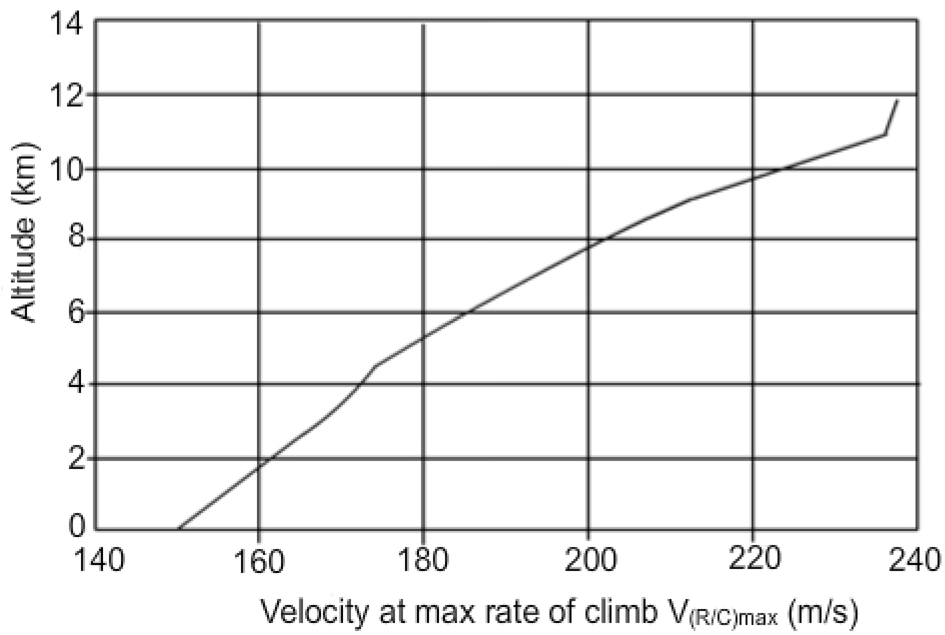



Fig.20 Velocity at maximum rate of climb vs altitude

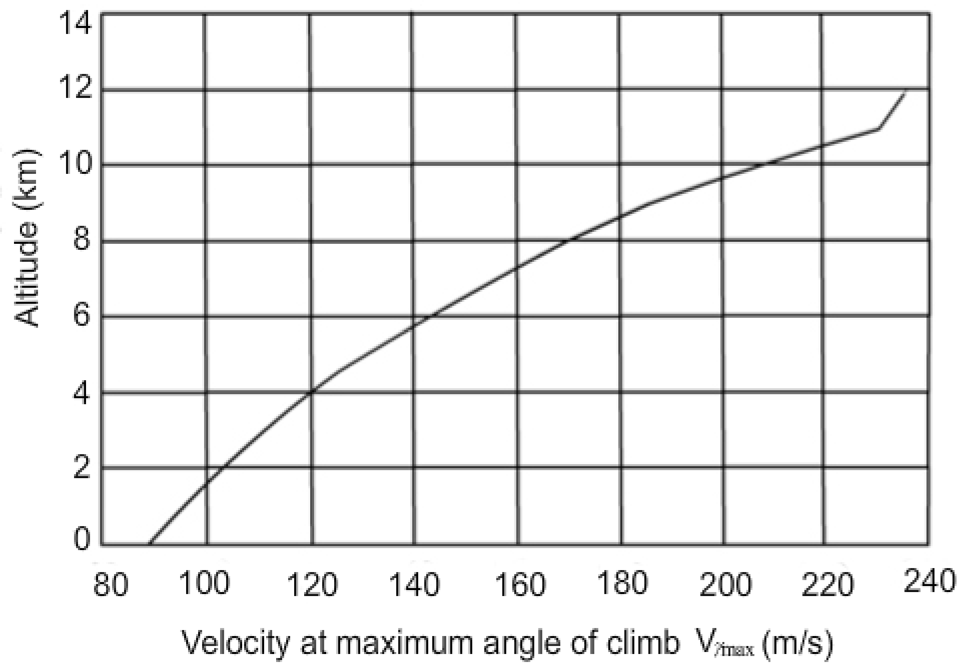


Fig.21 Velocity at maximum angle of climb vs altitude

**Remarks:**

i) The discontinuity in slope in Figs.20 and 21 at high velocities are due to the change in drag polar as the Mach number exceeds 0.8.

ii) From Fig.17, the absolute ceiling (at which $(R/C)_{max}$ is zero) is 11.95 km. The service ceiling at which $(R/C)_{max}$ equals 100 ft /min (30.5 m/min) is 11.71 km.