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Appendix B

Lecture 38

Performance analysis of a subsonic jet transport –1

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1 Airplane Details

1.1 Overall Dimensions

Length	: 34.32 m
Wing span	: 32.22 m
Height above ground	: 11.17 m
Wheel base	: 13.2 m
Wheel track	: 5.8 m

1.2 Engine details

Similar to CFM 56 - 2B

Seal level static thrust	: 97.9 kN per engine
By pass ratio	: 6.5 (For which the engine characteristics are given in Ref.3*)
SFC	: 0.6 hr^{-1} at M = 0.8 and h = 10973 m (36000 ft)

1.3 Weights

Gross weight	: 59175 kgf (580506.8 N)
Empty weight	: 29706 kgf (291415.9 N)
Fuel weight	: 12131 kgf (119005.1 N)
Payload	: 17338 kgf (170085.8 N)
Maximum landing weight	: 50296 kgf (493403.8 N)

1.4 Wing Geometry

Planform shape	: Cranked wing
Span	: 32.22 m
Area (S_{ref})	: 111.63 m^2
Airfoil	: NASA - SC(2) series, t/c = 14%, C_{lopt} = 0.5
Root chord	: 5.59 m (Equivalent trapezoidal wing)
Tip chord	: 1.34 m (Equivalent trapezoidal wing)
Root chord of cranked wing	: 7.44 m
Portion of wing with straight trailing edge	: 11.28 m

* Reference numbers in this Appendix relate to those given on page 40.

Mean aerodynamic chord	: 3.9 m
Quarter chord sweep	: 27.69^{o}
Dihedral	: 5^{o}
Twist	: 3^{o}
Incidence	: 1.4^{o}
Taper ratio	: 0.24 (Equivalent trapezoidal wing)
Aspect ratio	: 9.3

1.5 Fuselage geometry

Length	: 33 m
Maximum diameter	: 3.59 m

1.6 Nacelle geometry

No. of nacelles	: 2
Nacelle diameter	: 1.62 m
Cross sectional area	: 2.06 m^2
Length of nacelle	: 3.3 m (based on B737 Nacelle)

1.7 Horizontal tail geometry

Span	: 11.98 m
Area	: 28.71 m^2
Mean aerodynamic chord	: 2.67 m
Quarter chord sweep	: 32^{o}
Root chord	: 3.80 m
Tip chord	: 0.99 m
Taper ratio	: 0.26
Aspect ratio	: 5

1.8 Vertical tail geometry

Span	: 6.58 m
Area	: 25.43 m^2
Root chord	: 5.90 m
Tip chord	: 1.83 m
Mean aerodynamic chord	: 4.22 m
Quarter chord sweep	: 37^{o}

Taper ratio : 0.31

Aspect ratio : 1.70

1.9 Other details

C_{Lmax} without flap : 1.4

C_{Lmax} with landing flaps : 2.7

C_{Lmax} with T.O flaps : 2.16

Maximum load factor (n_{max}) : 3.5

1.10 Flight condition

Altitude : 10973 m (36000 ft)

Mach number : 0.8

Kinematic viscosity : 3.90536 x10^{-5} m^2/s

Density : 0.3639 kg/m^3

Speed of sound : 295.07 m/s

Flight speed : 236.056 m/s

Weight of the airplane : 59175 kgf (580506.8 N)

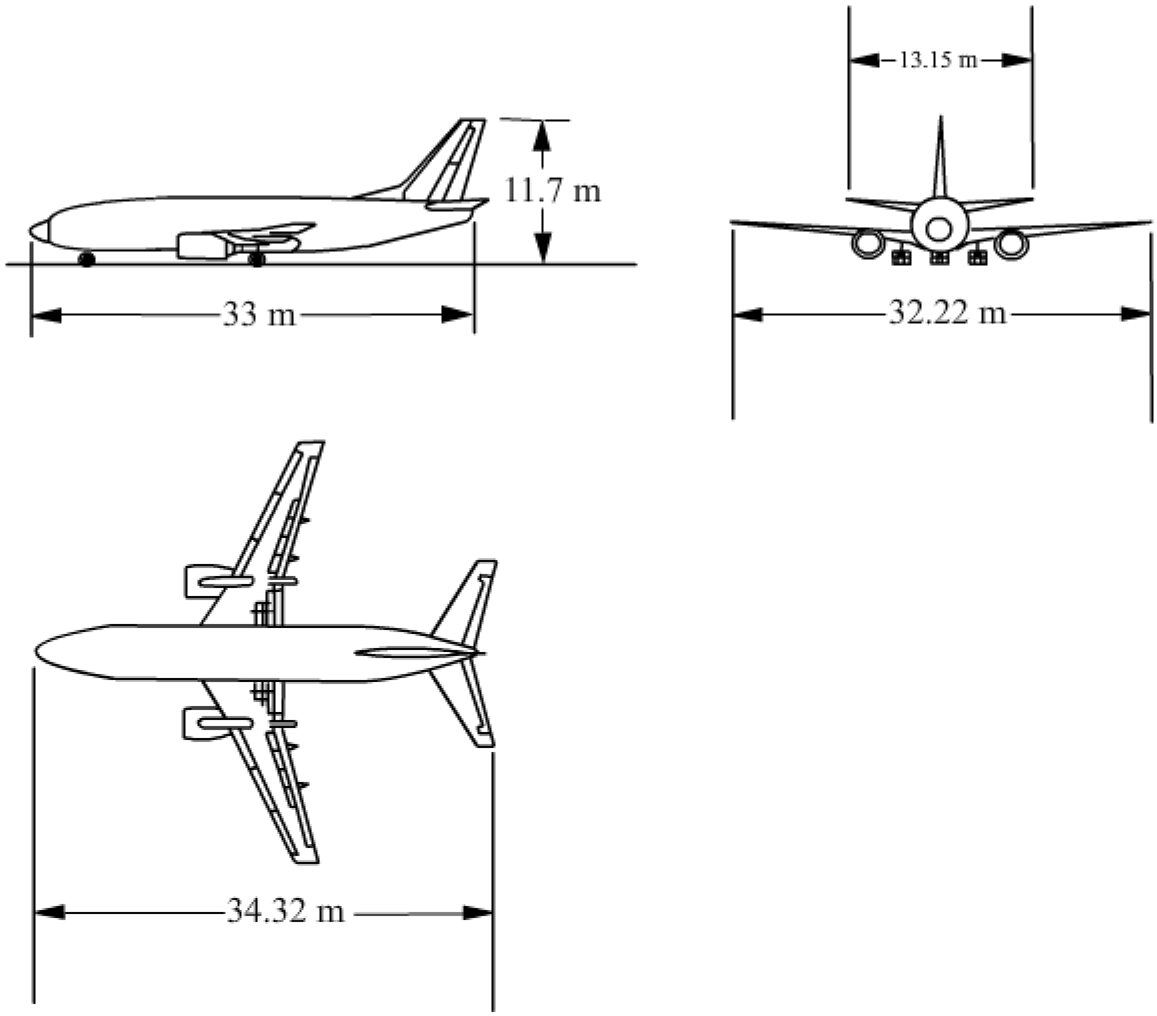

Fig.1 Three-view drawing of the airplane

2 Estimation of drag polar

The drag polar is assumed to be of the form:

$$C_D = C_{Do} + \frac{C_L^2}{\pi Ae}$$

The quantity C_{DO} is assumed to be given by:

$$C_{Do} = (C_{Do})_{WB} + (C_{Do})_V + (C_{Do})_H + (C_{Do})_{Misc} \quad (1)$$

where suffices WB, V, H, Misc denote wing-body combination, vertical tail, horizontal tail, and miscellaneous contributions respectively.

2.1 Estimation of $(C_{Do})_{WB}$

Initially, the drag polar is obtained at a Mach number of 0.6 as suggested in Ref.5, section 3.1.2. $(C_{Do})_{WB}$ is given as :

$$(C_{Do})_{WB} = (C_{Do})_w + (C_{Do})_B \frac{S_B}{S_{ref}}$$

The suffix B denotes fuselage and S_B is the maximum frontal area of fuselage. $(C_{DO})_W$ is given as :

$$(C_{Do})_W = C_{fW}\left[1+L(\frac{t}{c})\right](\frac{S_{wet}}{S_{ref}})_{wing}$$

where, C_{fw} is the turbulent flat plate skin friction coefficient. The Reynolds number used to determine it (C_{fw}) is lower of the two Reynolds numbers viz. Reynolds number based on the mean aerodynamic chord of the exposed wing (R_e) and $R_{ecuttoff}$ based on surface roughness. Further, $(S_{wet})_e$ is the wetted area of the exposed wing.

In the present case, c_r = 5.59m, c_t= 1.34m, b/2 = 16.11m and d_{fus} = 3.59m. Hence,

Root chord of exposed wing = c_{re} = $5.59 - \frac{5.59-1.34}{16.11} \times \frac{3.59}{2} = 5.116$ m

$$\lambda_e = \frac{1.34}{5.116} = 0.262$$

Hence, mean aerodynamic chord of exposed wing ($\overline{c}_e$) is :

$$\overline{c}_e = \frac{2}{3}[5.116(\frac{1+0.262+0.262^2}{1+0.262})] = 3.596 \text{ m}$$

Span of exposed wing = $(b/2)_e$ = 16.11 – 1.795 = 14.315m

Further, M = 0.6, a = 295.07m/s. Hence, V = 177.12m/s.

Also $\nu = 3.90536 \text{ X } 10^{-5}\ m^2/s$.

Hence,

$$R_e = \frac{177.12 \times 3.596}{3.90536 \times 10^{-5}} = 16.31 \text{ x } 10^6$$

The height of roughness corresponding to the standard camouflage paint, average application, is $k = 1.015 \text{ x } 10^{-5}$m (Ref.5, table 3.1). Hence, l/k in this case is:

$$\frac{l}{k} = \frac{3.596}{1.015 \times 10^{-5}} = 3.543 \times 10^5$$

The $R_{ecutoff}$ corresponding to the above l/k is 30 x 10^6. Consequently, C_{fw} corresponding to $R_e = 16.31 \text{ x } 10^6$ is obtained from Fig.3.1 of Ref.5, as :

$C_{fw} = 0.00265$.

$(t/c)_{avg}$ = 14% and $(t/c)_{max}$ occurs at x/c > 0.3 Hence, L = 1.2 and

$$S_{exposedplanform} = 14.314(\frac{5.116 + 1.341}{2}) \times 2 = 92.41m^2$$

$$S_{wet\,W} = 2 \text{ x } 92.41(1+1.2 \text{ x } 0.14) = 215.8m^2$$

Hence,

$$(C_{Df})_w = 0.00265\ (1+1.2 \text{ x } 0.14)\frac{215.8}{111.63} = 0.00598$$

$(C_{Do})_B$ is given as:

$$(C_{Do})_B = (C_{Df})_B + (C_{Dp})_B + C_{Db}$$

$$(C_{Do})_B = C_{fB}[1 + \frac{60}{(l_b/d)^3} + 0.0025(\frac{l_b}{d})](\frac{S_{wet}}{S_B})_{fus} + C_{Db}\frac{S_{base}}{S_{ref}}$$

In the present case , l_f = 33.0m , d_{max} = 3.59m ,

$$R_{eb} = \frac{177.12 \times 33}{3.905 \times 10^{-5}} = 149.6 \text{ x } 10^6$$

$$\frac{l}{k} = \frac{33}{1.015 \times 10^{-5}} = 32.51 \text{x} 10^5$$

The $R_{ecutoff}$ corresponding to the above l/k is 2.6 x 10^8. The C_{fw} corresponds to $R_{eb} = 149.6 \text{ x } 10^6$ measured from the graph in Ref.5, Fig.3.1 is:

$C_{fw} = 0.0019$

$(S_{wet})_{fus}$ = 0.75 x π x 3.59 x 33 = 279m^2

S_B = $\frac{\pi}{4}$ x 3.59^2 = 10.12m^2

Hence,

$$(C_{Df})_B = 0.0019 \times \frac{279}{10.12} = 0.0524$$

$$(C_{Dp})_B = 0.0019[\frac{60}{(33/3.59)^3} + 0.0025 \times (33/3.59)]\frac{279}{10.12} = 0.00524$$

Since, base area is almost zero, C_{Db} is assumed to be zero. Hence,

$$(C_{Do})_B = 0.0524 + 0.00524 + 0 = 0.0576$$

$(\Delta C_D)_{canopy}$ is taken as 0.002. Hence, $(C_{Do})_B$ = 0.0596

Finally,

$$(C_{Do})_{WB} = 0.00598 + 0.0596\frac{10.12}{111.63} = 0.01138$$

2.2 Estimation of $(C_{Do})_V$ and $(C_{Do})_H$

The estimation of $(C_{Do})_H$ and $(C_{Do})_V$ can be done in a manner similar to that for the wing. However, the details regarding the exposed tail area etc. would be needed. In the absence of the detailed data on the shape of fuselage at rear, a simplified approach given in Ref.5, section 2.2 is adopted, wherein C_{Df} = 0.0025 for both horizontal and vertical tails.

$$S_W = 2(S_h + S_v)$$

Hence,

$$(C_{Do})_{HV} = 0.0025(28.71 + 25.43)\frac{2}{111.63} = 0.0024 \qquad (2)$$

2.3 Estmation of misc drag - nacelle

For calculating drag due to the nacelles the short cut method is used i.e.:

$$(C_{Do})_{nacelle} = 0.006 \times \frac{S_{wet}}{S_{ref}}$$

where, S_{wet} is the wetted area of nacelle. Here, S_{wet} = 16.79m^2. Since, there are two nacelles, the total drag will be twice of this. Finally,

$$(C_{Do})_{nacelle} = 0.006 \times \frac{16.79}{111.63} \times 2 = 0.0018$$

2.4 C_{Do} of the airplane

Taking 2% for miscellaneous roughness and protuberances(Ref.5, section 3.4.6), the C_{Do} of the airplane is:

$$C_{Do} = 1.02\ [0.01138 + 0.0024 + 0.0018] = 0.0159 \qquad (3)$$

2.5 Induced drag

The induced drag component has the Oswald's efficiency factor e which is estimated by adding the effect of all the airplane components on induced drag (Ref.5, section 2.3).The rough estimate of e can be obtained as follows :

Figure 2.4 of Ref.5 is useful only for estimating e_{wing} of unswept wings of low speed airplanes. For the present case of swept wing, the following expression given in Ref.2 , chapter 7 is used.

$$e_{wing} = (e_{wing})_{\Lambda=0}\ \cos(\Lambda - 5)$$

where Λ is the quarter chord sweep. Ref.1, chapter 1 is used to estimate $(e_{wing})_{\Lambda=0.}$ In the present case, with A = 9.3 and λ = 0.24, the value of $(e_{wing})_{\Lambda=0}$ is 0.97.

Hence, e_{wing} = 0.97 x cos (27.69 - 5) = 0.8948.

From Ref.5, section 2.3, $\dfrac{1/e_{fus}}{(S_f/S)} = 0.8$ for a round fuselage. Hence,

$$\frac{1}{e_{fus}} = 0.8 \times \frac{10.122}{111.63} = 0.0725$$

Further, from Ref.5, section 2.3, $\dfrac{1}{e_{other}} = 0.05$

Finally,

$$e = \frac{1}{0.8948^{-1} + 0.0725 + 0.05} = 0.8064$$

Hence,

$$K = \frac{1}{\pi A e} = \frac{1}{\pi \times 9.3 \times 0.8064} = 0.04244$$

Remark:

Based on Ref.7, a detailed estimates of e_{wing} and $e_{fuselage}$ are given in Ref.5, section 3.3.

For an untwisted wing the value of e_{wing} is given as:

$$e_{wing} = \frac{1.1(C_{L\alpha w}/A)}{R(\frac{C_{L\alpha w}}{A}) + (1-R)\pi}$$

where,

$$C_{L\alpha W} = \frac{2\pi A}{2 + \sqrt{\frac{A^2\beta^2}{\kappa^2}\left(1 + \frac{\tan^2\Lambda_{\frac{1}{2}}}{\beta^2}\right) + 4}}$$

$C_{L\alpha W}$ = slope of lift curve of wing per radian

A = aspect ratio of wing

R = a factor which depends on (a) Reynolds number based on leading edge radius, (b) leading edge sweep (Λ_{LE}), (c) Mach number (M), (d) wing aspect ratio (A) and (e) taper ratio (λ).

$\beta = \sqrt{1-M^2}$

$\Lambda_{1/2}$ = sweep of semi-chord line

κ = ratio of the slope of lift curve of the airfoil used on wing divided by 2π. It is generally taken as unity.

In the present case,

M= 0.6, h= 10973 m (36000 ft), V= 177.12 m, $\nu = 3.90536\times10^{-5} m^2/s$, S = 111.63 m^2

b = 32.22 m, c_{re} = 5.59 m, c_t = 1.34 m, $\Lambda_{1/4}$ = 27.69 deg,

Hence, A = 9.3, λ =0.24, β =0.8 , $\tan \Lambda_{1/2}$ = 0.4589, $\cos \Lambda_{LE}$ = 0.8609 ,

Average chord = 3.615 m

The airfoil is NASA – SC(2) with 14 % thickness. From Ref.8 the leading edge radius is 3 % of the chord.

From these data:

$C_{L\alpha W}$ = 5.404

R_{Ler} = Reynolds number based on leading edge radius = 4.974 x 10^4

$R_{Ler} \times \cot \Lambda_{LE} \times \sqrt{1-M^2\cos^2 \Lambda_{LE}}$ = 7.198 x 10^5

$$\frac{A\lambda}{\cos\Lambda_{LE}} = 2.592$$

Corresponding to these data, R = 0.943 is obtained from Ref.5, Fig.3.14. Consequently,

$$e_{wing} = \frac{1.1 \times 5.404/9.3}{0.943 \times \left(\frac{5.404}{9.3}\right) + (1 - 0.943) \times \pi} = 0.8793$$

This value of e_{wing} is close to the value of 0.8948 obtained by the simpler approach. However, detailed approach is recommended for wings with sweep of above 35°. Reference 7, section 4.5.3 contains guidelines for estimating drag of wing-body-tail combination with allowance for trim drag.

2.6 Final drag polar

$$C_D = 0.0159 + 0.04244\ C_L^2 \quad (4)$$

The drag polar is presented in Fig.2.

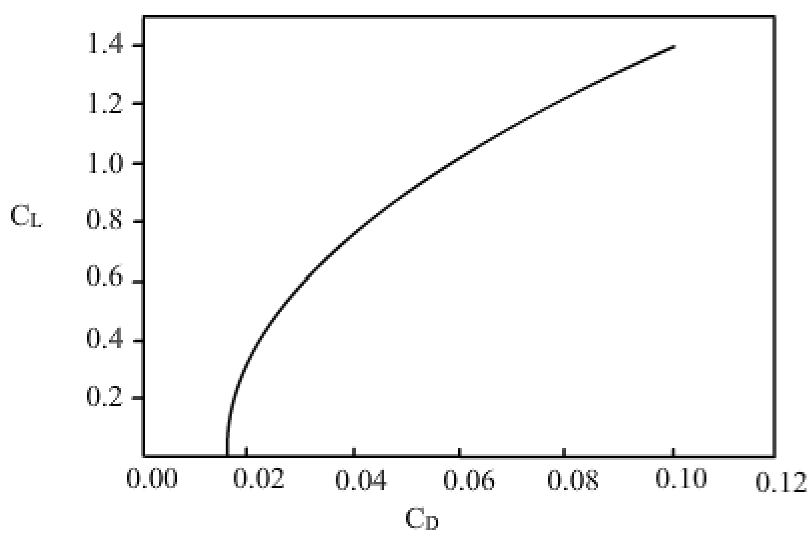

Fig.2 Drag polar at sub-critical Mach numbers

Remarks:

i) The polar given by Eq.(4) is valid at subcritical Mach numbers. The increase in C_{DO} and K at higher Mach numbers is discussed in section 4.2.

ii) The maximum lift to drag ratio ($(L/D)_{max}$) is given by:

$$(L/D)_{max} = \frac{1}{2\sqrt{C_{Do}K}}$$

Using C_{DO} and K from Eq.(4), (L/D)max is 19.25, which is typical of modern jet transport airplanes.

iii) It may be noted that the parabolic polar is an approximation and is not valid beyond C_{Lmax}. It is also not accurate close to $C_L = 0$ and $C_L = C_{Lmax}$.