Appendix A

Lecture 37

Performance analysis of a piston engined airplane – 3

Topics

7 Turning performance

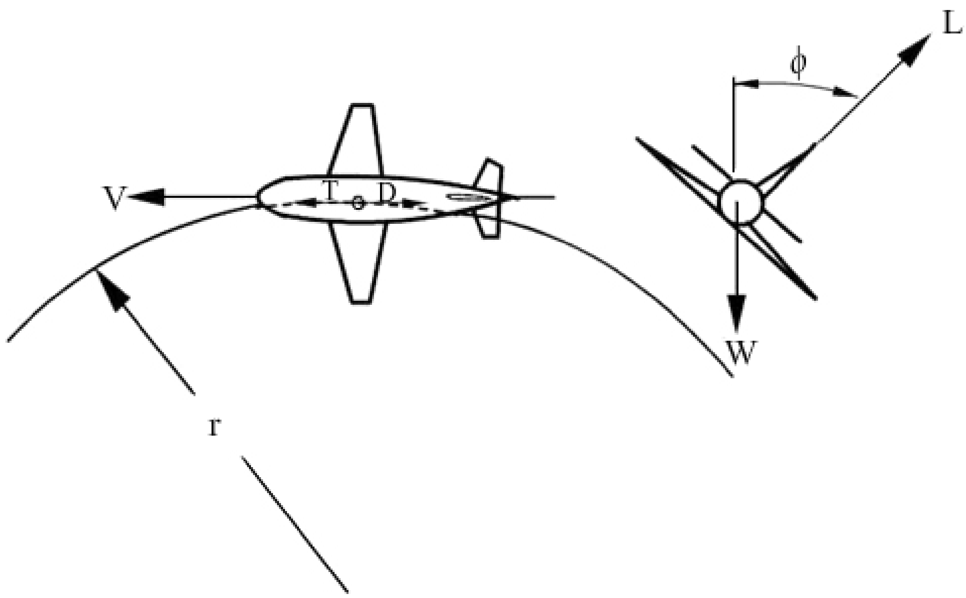

Fig.21 Forces on an airplane in turning flight

In this section, the performance of the airplane in a steady level co-ordinated-turn is studied.

The forces acting on the airplane are shown in Fig.21.

The equations of motion in this flight are:

$$T - D = 0, \text{ as it is a steady flight} \tag{28}$$

$$W - L\cos\phi = 0, \text{ as it is a level flight} \tag{29}$$

$$L\sin\phi = \frac{W}{g}\frac{V^2}{r}, \text{ as it is a co-ordinated-turn} \tag{30}$$

These equations give:

$$\text{Radius of turn} = r = \frac{W}{g}\frac{V^2}{L\sin\phi} = \frac{V^2}{g\tan\phi} \tag{31}$$

$$\text{Rate of turn} = \dot{\psi} = \frac{V}{r} = V \Big/ \left(\frac{V^2}{g\tan\phi}\right) = \frac{g\tan\phi}{V} \tag{32}$$

$$\text{Load Factor} = n = \frac{L}{W} = \frac{1}{\cos\phi} \quad (33)$$

In the following calculations, C_{Lmax} =1.33 and n_{max} = 3.5 are assumed ; where n_{max} is the maximum load factor for which the airplane is designed. The following procedure is used to obtain r_{min} and $\dot{\psi}_{max}$.

1. The flight speed and altitude are chosen. The lift coefficient in level flight (C_{LL}) is obtained as :

$$C_{LL} = \frac{2(W/S)}{\rho V^2}$$

2. Obtain $\frac{C_{Lmax}}{C_{LL}}$. If $\frac{C_{Lmax}}{C_{LL}} < n_{max}$, then the turn is limited by C_{Lmax} and $C_{LT1} = C_{Lmax}$.

However, if $C_{Lmax}/C_{LL} > n_{max}$, then the turn is limited by n_{max}, and $C_{LT1} = n_{max}\ C_{LL}$.

3. From the drag polar, C_{DT1} is obtained corresponding to C_{LT1}. Then,

$$D_{T1} = \frac{1}{2}\rho V^2 S C_{DT1}$$

If $D_{T1} > T_a$, where T_a is the available thrust at chosen speed and altitude, then the turn is limited by the engine output. The maximum permissible value of C_D in this case is found from:

$$C_{DT} = \frac{2T_a}{\rho V^2 S}$$

From the drag polar, the value of C_{LT} is calculated as:

$$C_{LT} = \sqrt{\frac{C_{DT} - C_{DO}}{K}}$$

However, if $D_{T1} < T_a$, then the turn is not limited by the engine output and the value of C_{LT1} calculated in step (2) is taken as C_{LT}.

4. Once C_{LT} is known, the load factor n, which satisfies the three constraints namely of C_{Lmax}, n_{max} and T_a, is given by:

$$n = \frac{C_{LT}}{C_{LL}}$$

5. Knowing n, the values of the radius of turn (r) and the rate of turn ($\dot{\psi}$) can be calculated from Eqs.(31), (32) and (33).

6. The above steps are repeated for various speeds at the same altitude and subsequently the procedure is repeated at various altitudes.

Sample calculations of turning performance at sea level are represented in Table 9. Figures 22 and 23 present turning performance at various altitudes.

V (m/s)	C_{LL}	$\frac{C_{Lmax}}{C_{LL}}$	C_{LT1}	C_{DT1}	D_{T1} (N)	THP_1 (kW)	η_p	THP_a (kW)	C_{LT}	n	ϕ (deg)	r (m)	$\dot{\psi}$ (rad/s)
30	1.30	1.021	1.33	0.168	1380	41.4	0.578	78.0	1.33	1.02	11.6	445	0.067
35	0.96	1.390	1.33	0.168	1879	65.8	0.635	85.7	1.33	1.39	44.0	129	0.270
38	0.81	1.638	1.33	0.168	2215	84.2	0.666	89.9	1.33	1.64	52.4	113	0.335
40	0.73	1.815	1.33	0.168	2454	98.2	0.685	92.4	1.28	1.75	55.1	114	0.351
45	0.58	2.297	1.33	0.168	3106	139.8	0.727	98.2	1.05	1.82	56.6	136	0.330
50	0.47	2.836	1.33	0.168	3834	191.7	0.762	102.9	0.86	1.83	56.9	166	0.300
55	0.39	3.432	1.33	0.168	4639	255.2	0.789	106.5	0.69	1.77	55.5	212	0.260
60	0.33	4.084	1.14	0.133	4359	261.5	0.805	108.7	0.52	1.60	51.2	295	0.203
65	0.28	4.793	0.97	0.106	4082	265.3	0.809	109.3	0.34	1.23	35.7	600	0.108

Table 9 Turning performance calculations at sea level

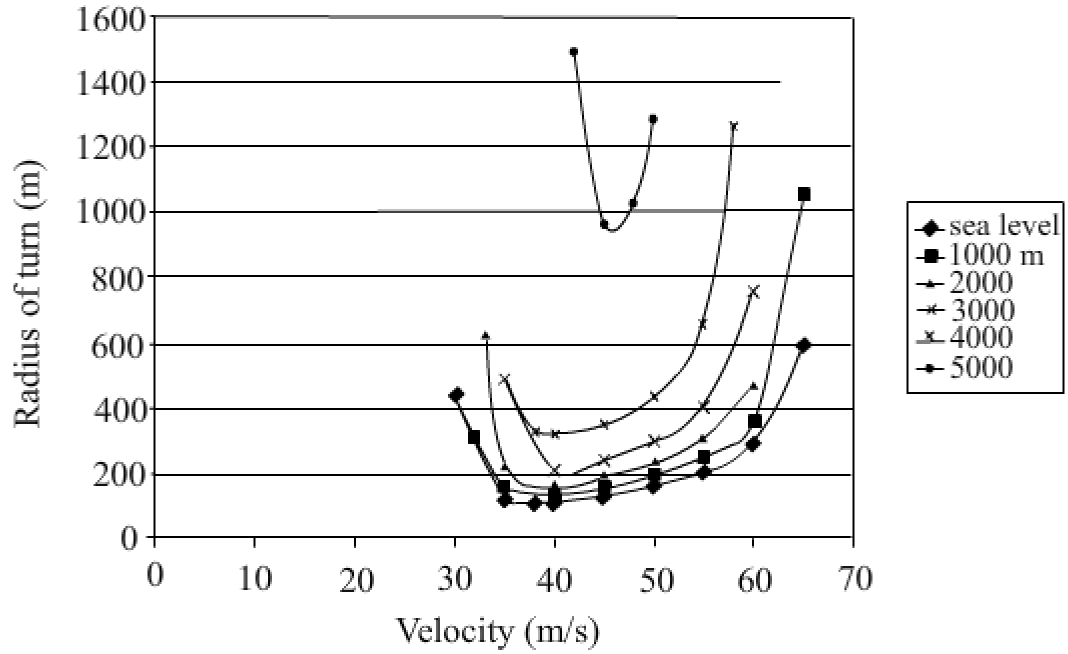

Fig.22 Variations of radius of turn with velocity at various altitudes

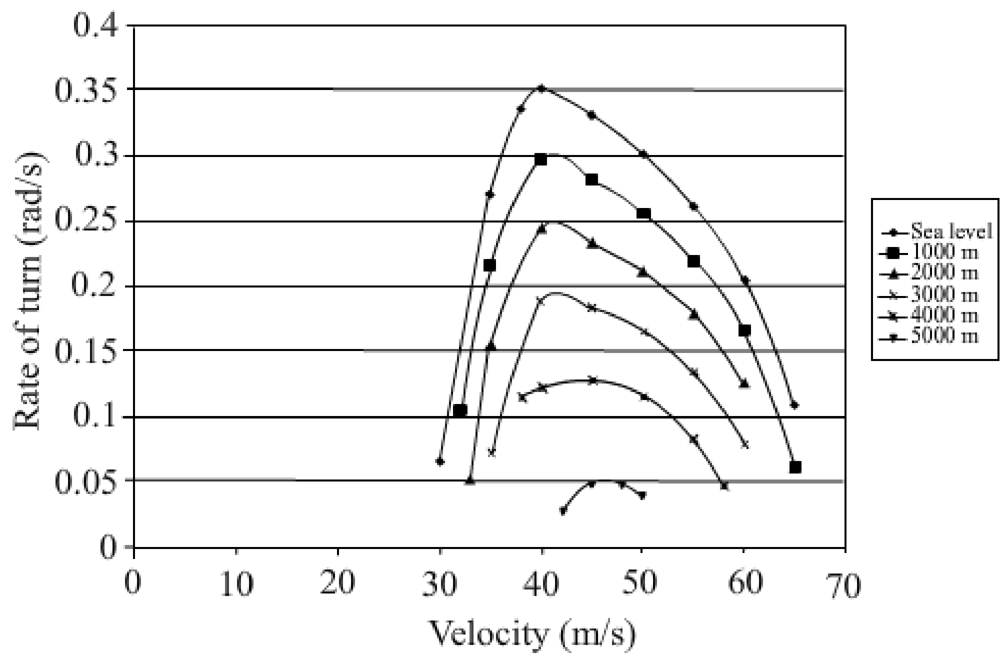

Fig.23 Variations of rate of turn with velocity at various altitudes

From Figs.22 and 23 the values of r_{min} , $\dot{\psi}_{max}$ **,** $\mathbf{V}_{rmin}$, $V_{\dot{\psi}max}$ can be obtained at various altitudes. The variations are presented in Table 10 and Figs.24, 25 and 26.

h (m)	r_{min}(m)	$\dot{\psi}_{max}$ (rad/s)	V_{rmin}(m/s)	$V_{\dot{\psi}max}$ (m/s)
0	110	0.351	38	40.0
1000	135	0.301	39	41.2
2000	163	0.248	39.5	41.5
3000	198	0.194	40.5	41.7
4000	324	0.128	41	44.0
5000	918	0.048	45.7	46.0

Table 10 Turning performance

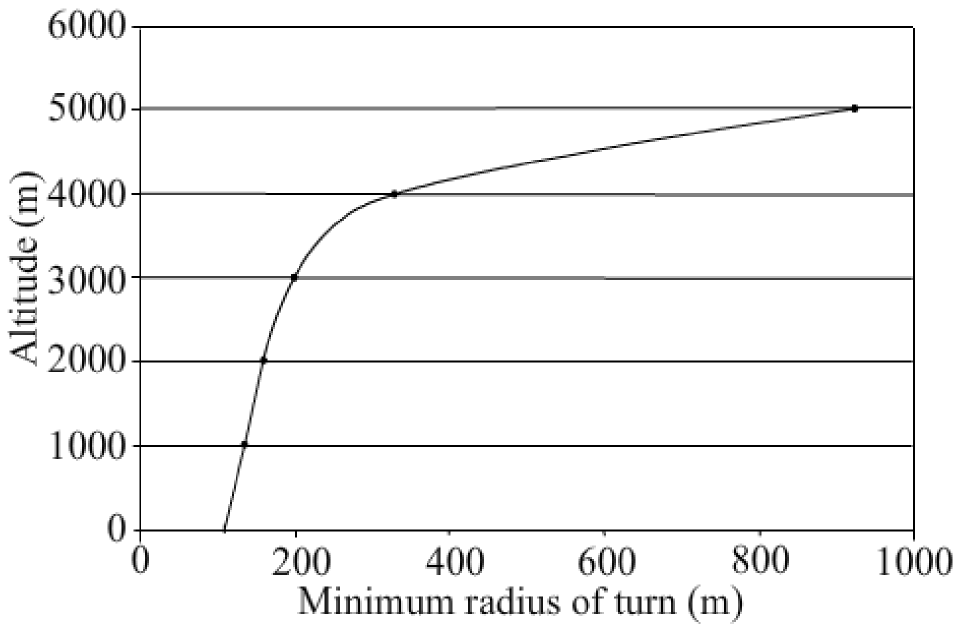

Fig.24 Variation of minimum radius of turn with altitude

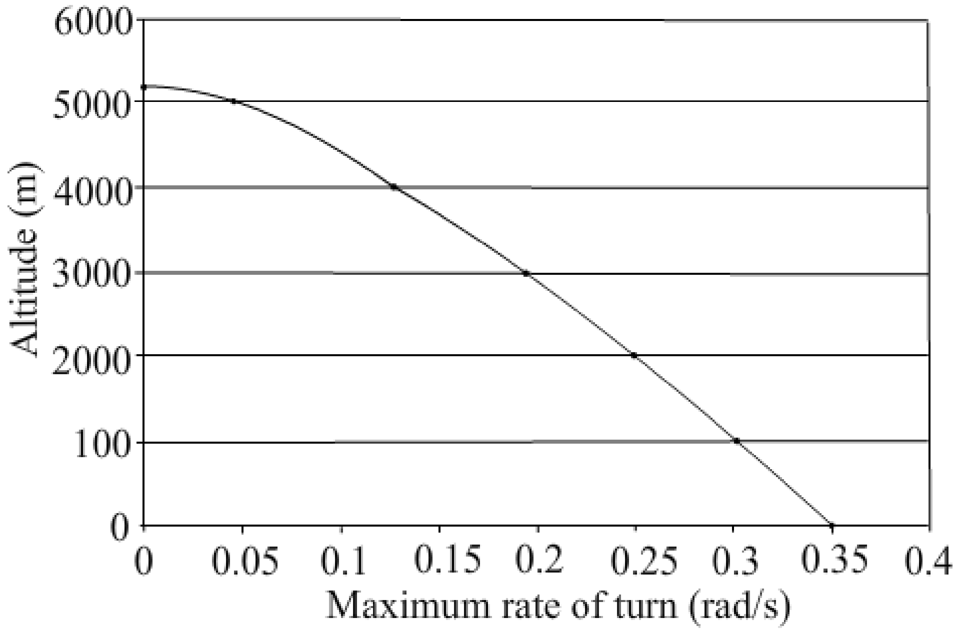

Fig.25 Variation of maximum rate of turn with altitude

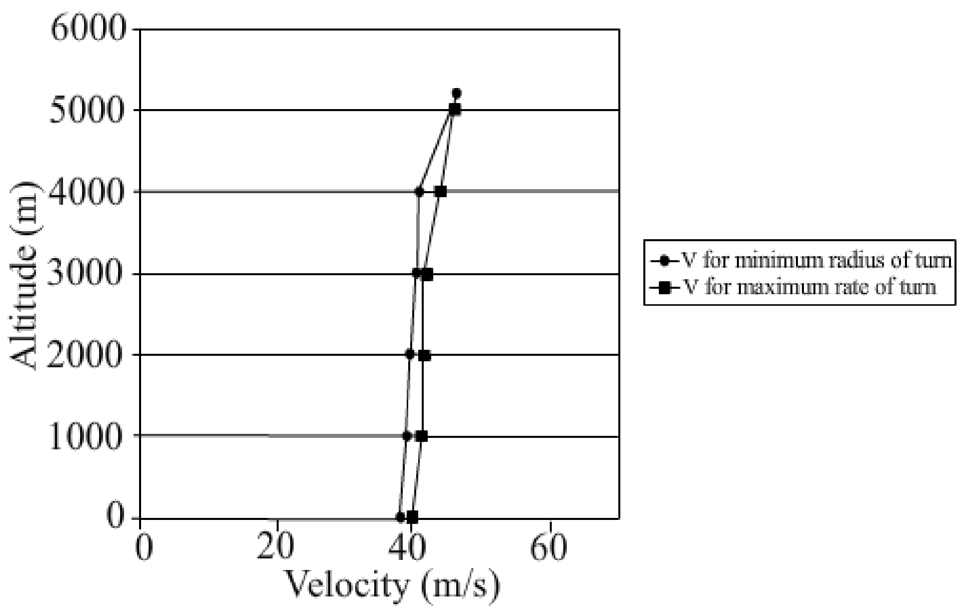

Fig.26 Variations of V_{rmin} and $V_{\psi max}$ with altitude

Remark:

The minimum radius of turn at sea level is about 110 m at flight speed of about 38 m/s. The maximum rate of turn at sea level is about 0.35 rad/sec at flight speed of about 40 m/s.

8 Take-off and landing distance estimates

Take-off flight can be divided into three phases: take-off run or ground run, transition and climb (Fig.27).

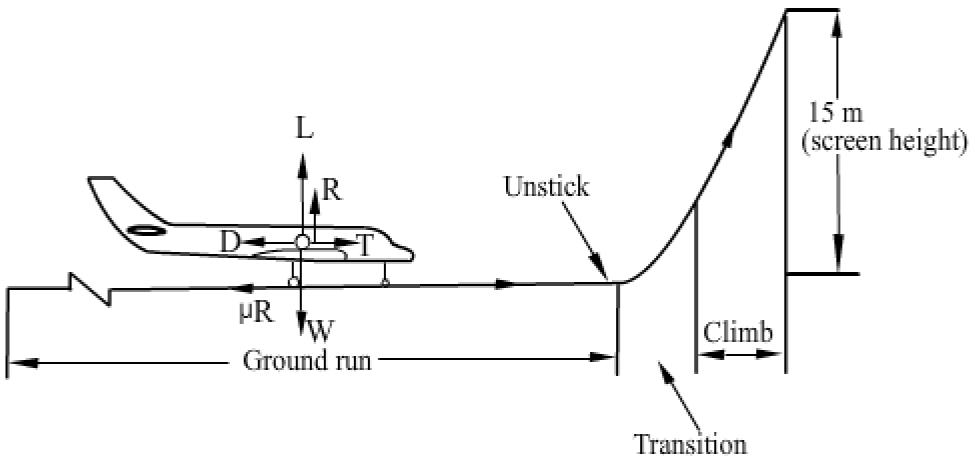

Fig.27 Phases of take-off flight

8.1 Distance covered during take-off run (s_1)

The equations of motion during the take off run are:

$$T - D - \mu R = \frac{W}{g}\frac{dV}{dt} \quad \text{and} \quad R = W - L \tag{34}$$

where R is the ground reaction. The acceleration can be written as:

$$\frac{dV}{dt} = \frac{g}{W} \times [T\text{-}D\text{-}\mu(W\text{-}L)]$$

Writing $\frac{dV}{dt}$ as $\frac{dV}{ds} \times \frac{ds}{dt}$, gives :

$$ds = \frac{W}{g}\frac{VdV}{T - D - \mu(W - L)}$$

Further, at sea level, BHP = constant = 135kW at 2700 rpm. Thrust is given by :

$T = BHP \times \eta_p / V$.

The distance covered during the take-off run (s_1) can be expressed as:

$$s_1 = \int_0^{V_{TO}} \frac{W\,V}{g\,F}\, dV \tag{35}$$

where F is the accelerating force given by:

$$F = \frac{BHP \times \eta_p}{V} - D - \mu(W-L)$$

Since η_p is a function of velocity, an accurate way to estimate s_1 is to evaluate the integrand in Eq.(35) at several values of V and carry-out a numerical integration. Simpson's rule is used for this purpose. Various quantities needed for the purpose are estimated below.

$V_{TO} = 1.2\ V_s$, where V_s is stalling speed, given by :

$$V_s = \sqrt{\frac{2W}{\rho S C_{Lmax}}}$$

During the take-off, flap deflection (δ_f) is 10°, hence $C_{Lmax} = 1.42$. It is assumed that the coefficient of friction is 0.02.

The take-off weight is W =10673.28 N

S = wing planform area =14.864 m^2

Density $\rho_{sl} = 1.225\ kg/m^3$

Thus, $V_s = 28.73 m/s$ and $V_{TO} = 34.48\ m/s$

To estimate C_L and C_D during take -off run it is noted that the airplane has a nose wheel type of landing gear and hence the airplane axis can be considered as horizontal and the wing produces lift corresponding to the wing setting angle (see Section 10.3.1 of the main text of the course material).

From Sec. 1.4, the average wing incidence is the average of incidence at root (4.62°) and that at tip (2.62°) i.e. 3.62°. The slope of the lift curve of the wing ($C_{L\alpha W}$) is approximately given by:

$$C_{L\alpha W} = 2\pi \frac{A}{A+2} = 2\pi \frac{5.625}{7.625} = 4.63 / rad = 0.0808 / deg$$

The angle of zero lift (α_{0L}) for the airfoil NACA $65_2 - 415$, from Ref. 5, is -2.6°.

Hence, lift coefficient during take-off run due to wing lift is :

$0.0808(3.62 + 2.6) = 0.502$

Since, the flaps are deflected during the take-off run the lift coefficient will be increased by (1.42 – 1.33 = 0.09). Hence, C_L during take-off run (C_{Ltr}) is :

$C_{Ltr} = 0.502 + 0.09 = 0.592$

The drag coefficient during take-off run (C_{Dtr}) , using the drag polar corresponding to take-off, is:

$$C_{Dtr} = 0.0389 + 0.0755 \times 0.592^2 = 0.0654$$

For applying Simpson's rule in this case, the various quantities are evaluated at seven points in the speed range of 0 to 34.48. The calculations are shown in Table 11.

V (m/s)	η_p	T (N)	D (N)	L (N)	F (N)	WV/gF (s)
0	0	*	0	0	-	0.000
5.75	0.153	3589.94	19.68	132.9	3364.45	1.859
11.49	0.283	3329.77	78.60	711.5	3051.9	4.096
17.24	0.392	3072.83	176.9	1601.3	2714.5	6.910
22.99	0.484	2843.74	314.7	2848.7	2372.5	10.54
28.73	0.562	2642.65	491.4	4448.2	2026.7	15.42
34.48	0.629	2464.68	707.8	6407.0	1671.6	22.42

* The value of thrust (T) at V = 0 is not zero. It can be evaluated using propeller charts. However, it is not needed in the present calculation, as the integrand is zero when V is zero.

Table 11 Evaluation of integrand in Eq.(35).

Using the values of integrand in Table 11 and employing Simpson's rule the ground run (s_1) is given by :

$$s_1 = \frac{5.747}{3}\left[0 + 4(1.859 + 6.91 + 15.42) + 2(4.096 + 10.54) + 22.42\right] = 284.4 \text{ m}$$

8.2 Distance covered during transition (s_2)

The entire power of the engine is assumed to be used to overcome the drag and to accelerate to a velocity V_2 given by $V_2 = 1.1\ V_{T0}$. The height attained during the transition phase is ignored.

Hence, $Ts_2 = Ds_2 + \frac{W}{2g}(V_2^2 - V_{TO}^2)$

$$s_2 = \frac{W}{2g} \frac{(V_2^2 - V_{TO}^2)}{T - D}$$

where T and D are evaluated at a speed which is mean (V_{avg}) of V_2 and V_{TO}

$$V_2 = 1.1 \times 34.48 = 37.93 \text{ m/s}$$

$$V_{avg} = \frac{V_{TO} + V_2}{2} = \frac{34.48 + 37.93}{2} = 36.71 \text{m/s}$$

$$T = \frac{\eta_p \times BHP \times 1000}{V_{avg}}$$

From Eq.(15), η_p at a speed of 36.71 m/s is 0.65285

Hence, $T = \frac{0.65285 \times 135 \times 1000}{36.71} = 2400.5\,N$

Further, $C_L = \frac{2 \times 10673.28}{1.225 \times 14.864 \times 36.71^2} = 0.87$

$$C_D = 0.0389 + 0.0755 \times 0.87^2 = 0.096$$

$$D = \frac{1}{2} \times 1.225 \times 36.71^2 \times 14.864 \times 0.096 = 1178.4\,N$$

Hence, $s_2 = \frac{W}{2g}\left(\frac{V_2^2 - V_{TO}^2}{T-D}\right) = \frac{10673.28(37.93^2 - 34.48^2)}{2 \times 9.81(2400.5 - 1178.4)} = 111.3\,m$

8.3 Distance covered during climb phase (s_3)

The airplane is assumed to climb to screen height (15m) at an angle of climb γ, where the climb angle γ is given by:

$$\gamma = \left(\frac{T-D}{W}\right)$$

For the climb phase, T and D are evaluated at V_2 which is equal to 37.93 m/s

From Eq.(15), η_p at a speed of 37.93 m/s is 0.665. Hence,

$$T = \frac{0.665 \times 135 \times 1000}{37.93} = 2366.86\,N$$

$$C_L = \frac{2 \times 10673.28}{1.225 \times 14.864 \times 37.93^2} = 0.82$$

$$C_D = 0.0389 + 0.0755 \times 0.82^2 = 0.0897$$

$$D = \frac{1}{2} \times 1.225 \times 36.71^2 \times 14.864 \times 0.0897 = 1174.5\,N$$

$$\sin\gamma = \frac{T - D}{W} = \frac{2366.86 - 1174.5}{10673.28} = 0.1117 \text{ or } \gamma = 6.41^o$$

Hence, $s_3 = \frac{15}{\tan\gamma} = \frac{15}{0.1124} = 133.4\,m$

Total takeoff distance is given by:

$$s = s_1 + s_2 + s_3 = 284.4 + 111.3 + 133.4 = 529.1 \text{ m}$$

Remarks:

i) The above estimation of take off distance is based on several assumptions. Reference 8 has compiled data on take-off distances of many propeller driven airplanes. This take-off distance is based on FAR 23 specifications and can be denoted by s_{to23} . Based on this data the following formula is obtained for s_{to23} in terms of a parameter called take-off parameter and denoted by TOP_{23} . In SI units the relationship is given as (See Guidelines for take-off distance in Section 10.4.7 of the main text of the course) .

$$S_{To23} = 8.681 \times 10^{-3} \times TOP_{23} + 5.566 \times 10^{-8} \times TOP_{23}^2$$

where, $TOP_{23} = \dfrac{(\frac{W}{S}) \times (\frac{W}{P})}{\sigma C_{LTO}}$; (W/S) is in N/m^2, W in N and P in kW.

σ is density ratio at the altitude of take-off.

In the present case:

W/S = 10673.28 / 14.864 = 718.1 N/m^2, W/P = 10673.28 / 135 = 79.06 N/ kW

σ = 1.0 and C_{LTO} = 1.42.

Consequently,

$$TOP_{23} = \frac{718.1 \times 79.06}{1.42} = 39981$$

$$s_{to23} = 8.681 \times 10^{-3} \times 39981 + 5.566 \times 10^{-8} \times 39981^2 = 347 + 89 = 436 \text{ m}$$

ii) The estimated take-off distance of 530 m is somewhat higher than the actual take-off distance of 488 m (section 1.10). This may be because the height attained during the transition phase has been ignored.

8.4 Estimation of landing distance

The landing distance can be calculated in a manner similar to that for take-off distance. However, due to uncertainty associated with piloting techniques during landing, the following formula is used.

$$s_{land} = -\frac{V_a^2}{2a}$$

where, $V_a = 1.3 \times V_s$ in landing configuration

The weight of the airplane during the landing is taken same as that during the take-off. However, C_{Lmax} with landing flap setting is 1.86. The stalling speed in this configuration is 25.1 m/s.

Hence, approach speed is 32.6 m/s.Taking a = -1.22m/ s^2 for a simple braking system yields:

s_{land} = 436 m,

which is close to the value of 426 m given in Section 1.10.

9 Concluding remarks

1. The performance of a piston-engined airplane has been estimated for stalling speed, maximum speed, minimum speed, steady climb, range, endurance, turning, take-off and landing.

2. A reasonable agreement has been observed between the calculated performance and the actual performance of the airplane (PA – 28 – 181).

3. Figure 28 presents the variations, with altitude, of the characteristic velocities corresponding to:

- Stalling speed V_s
- Maximum speed V_{max}
- Minimum speed as dictated by power $(V_{min})_e$
- Maximum rate of climb $V_{R/C\ max}$
- Maximum angle of climb $V_{\gamma max}$
- Maximum rate of turn $V_{\dot{\psi} max}$
- Minimum radius of turn $V_{r\ min}$

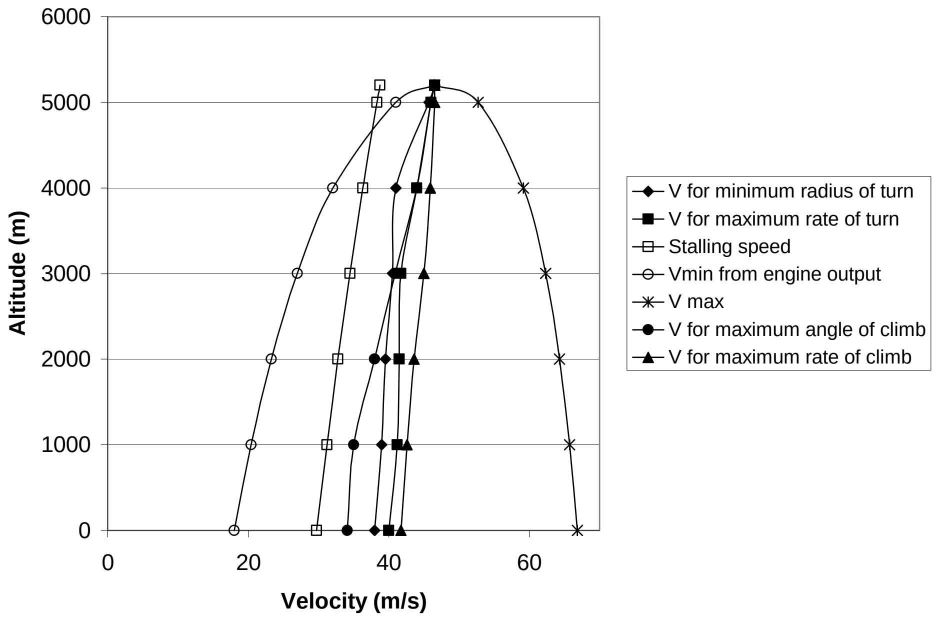

Fig.28 Variations of characteristic velocities with altitude